EE-640 — Home Work 7

Note Title 24-07-2008

1) In state feedback design, we used feedback to the input according to $u(t) = v(t) - kx(t)$. We could do this for the observer $u(t) = v(t) + l(y(t) - \hat{y}(t))$, where of course, l is now a scalar. What can be achieved with such feedback? Why is it that we use feedback to the states in the observer problem but not in the controller problem?

2) Is the asymptotic observer of an observable system itself observable for all possible l? Give a proof.

3) Show that the observability of a realization $\{A, b, c\}$ is not invariant under general state feedback $(u \to v - kx)$ but is invariant under linear output feedback $\{u(t) \to v(t) - ky(t)\}$

4) <u>Another approach to Observers</u>
If $\dot{x}(t) = Ax(t) + bu(t)$, $y(t) = cx(t)$, $x(t_0) = x_0$, let $\hat{x}(\cdot)$ obey $\dot{\hat{x}}(t) = F\hat{x}(t) + gu(t) + hy(t)$, $\hat{x}(t_0) = \hat{x}_0$. The second equation can be said to define an observer for the first if $x_0 = \hat{x}_0 \Rightarrow x(t) = \hat{x}(t)$, $t \geq t_0$. Show that a necessary and sufficient condition for this is that $F = A - kc$, $h = k$, $g = b$, where k is an

arbitrary $n \times 1$ vector.

5) Design an observer for the oscillatory system $\dot{x}(t) = v(t)$, $\dot{v}(t) = -\omega_0^2 x(t)$ using measurements of the velocity $v(\cdot)$. Place both observer poles at $s = -\omega_0$.

6) The fact that $\tilde{x}(t) = x(t) - \hat{x}(t)$ is the error of the observer estimate, suggests that perhaps a more convenient set of state variables for the observer-controller system is $[x, \tilde{x}]$. Write state equations for these variables, and use them to calculate $G_{o-c}(s)$ & $a_{o-c}(s)$.

7) The observer-controller realization is clearly not minimal.
a) Show that this realization is not controllable. What are the un-controllable states?
b) Show that the realization will be un-observable iff at least one of the following holds:
(1) $\{k, A-lc\}$ is not observable
(2) $\{c, A-bk\}$ is not observable
(3) A pole of the observer cancels a zero of the original transfer function.

[Assume $\{A, b, c\}$ is minimal.]

8) In the combined controller-observer design we select k and ℓ so that $a_c(s)$ and $a_o(s)$ both have poles in the left half plane. Is it true that the resulting design must be stable even if the loop is broken open at y, for instance? Explain.

9) Consider the undamped harmonic oscillator $\dot{x}_1(t) = x_2(t)$, $\dot{x}_2(t) = -\omega_0^2 x_1(t) + u(t)$. Using an observation of velocity, $y(\cdot) = x_2(\cdot)$, design an observer/state-feedback compensator to control the position $x_1(\cdot)$. Place the state feedback controller poles at $s = -\omega_0 \pm j\omega_0$ and both observer poles at $s = -\omega_0$.

10) In the combined controller-observer design we select k and ℓ so that $a_c(s)$ and $a_o(s)$ both have poles in the left half plane. The open loop system may be stable or unstable.

a) Prove that if the loop is broken open at y, then the resulting design may be unstable? Write down the state equations for this scenario and explain.

b) Construct a 2-state example with a <u>stable</u> A which demonstrates your conclusion: namely, construct A, b, c, k and ℓ such that $\{A,b\}$ is controllable, $\{c,A\}$ is observable; A, $(A - bk)$ and $(A - \ell c)$ have stable eigenvalues; and with these quantities illustrate your answer to part (a).