HW 8

1) Prove that $\{A, B\}$ is controllable if & only if $\{A - BK, B\}$ is controllable for all $m \times n$ matrices K.

2) Let $N(s) = N_0 s^r + N_1 s^{r-1} + \cdots + N_r$. Show that the block controller realiz^n of $H(s) = N(s)/s^r$ will be observable if N_r has full rank.

3) If $\{P(s), U(s)\}$ are polynomial matrices, with $U(s)$ unimodular, show that pair $\{P(s), U(s)\}$ is always coprime (left or right as the case may be.)

4) Check in several different ways whether following pair of matrices are right coprime.

$$\begin{bmatrix} s & 0 \\ -s & s^2 \end{bmatrix} \quad \& \quad \begin{bmatrix} 0 & -(s+1)^2(s+2) \\ (s+2)^2 & (s+2) \end{bmatrix}$$

5) Suppose $N(s)D^{-1}(s)$ is proper. Show that $[D'(s)\ N'(s)]'$ will be column reduced if & only if $D(s)$ is column reduced.

6) If the elements of a matrix P are just real or complex numbers, show that the Smith form has $\lambda_1 = \lambda_2 = \cdots = \lambda_r = 1$, $\lambda_{r+i} = 0$ where r is the rank of P.

7) Find controller-, observer- & controllability-type realiz^n of

$$H(s) = \begin{bmatrix} \dfrac{1}{(s-1)^2} & \dfrac{1}{(s-1)(s+3)} \\ \dfrac{-6}{(s-1)(s+3)^2} & \dfrac{s-2}{(s+3)^2} \end{bmatrix}$$

8) a) Prove by direct calcul^n that $(sI - A_c^o)^{-1} B_c^o = \Psi(s) S^{-1}(s)$

= transfer function from $u(s)$ to $\xi(s)$

b) If we apply feedback & make an input

transformation as in $u(s) = D_{hc}^{-1} [v(s) - D_{lc} \xi(s)]$

Find the new transfer function from $u(s)$ to $\xi(s)$