1). Reduce the following uncontrollable pair to a form which "partially resembles" the controllable companion form.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

(i) What is the controllability index (μ) of the controllable part?

(ii) Determine the uncontrollable modes & comment on the stabilizability of the system.

2. Refer class note for Q & L matrices (to form controller canonical form). Show that the absolute value of the determinant of QL is 1 i.e. $|QL| = 1$. Comment on the rank of the matrix Q.

3. Reduce the following pair to controllable companion form.

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(i) What are the controllability indices (d_i) & controllability index(M) of the above system?

4. Show that two controllable 4th order systems, the first with $d_1 = d_2 = 2$ & the second with $d_1 = 1$ & $d_2 = 3$ can not be equivalent (A_1 & A_2 are called equivalent iff $A_1 = Q A_2 Q^{-1}$). What can you conclude in general regarding the controllability indices of equivalent systems?

5. Find a state feedback gain matrix F, which places the closed loop poles of the system having

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 3 & 0 & -3 & 1 \\ -1 & 1 & 4 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

at $-1 \pm 2i$ & $-2 \pm 3i$.

(i) Show all steps for pole placement. Varify your result.

(ii) Obtain F for the above system by using "place" command in MATLAB. Say the new feedback gain matrix $\tilde{F}$. Compare F & $\tilde{F}$. Is it unique?