Granting Agencies: Department of Science and Technology,
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Control of Multi-Agent Systems:
Time Optimal Feedback

Joint Work with

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TIME OPTIMAL FEEDBACK

Blind driver: very fast car

- Blind driver wants to go from Main Building to Main Gate in MINIMUM time

Blind driver: very fast car

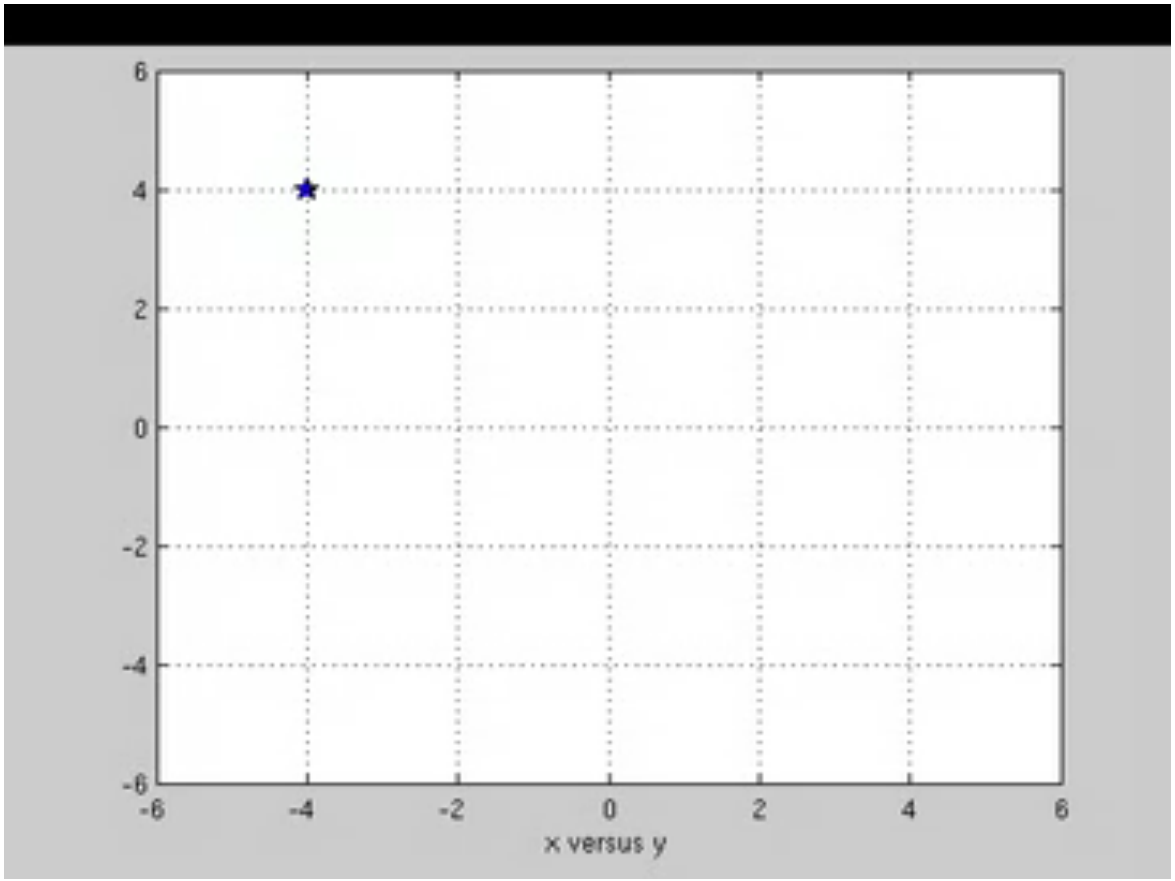
- Blind driver wants to go from Main Building to Main Gate in MINIMUM time
- Classical theorem (after lots of calculation) says
  - He must accelerate maximum until Gulmohar
  - Then decelerate maximum until Main Gate
- Being blind, he hits a stray bull in front of SOM and has to stop for a while
- All calculations become invalid: He does not know the minimum time trajectory anymore!

Feedback Control

- We have developed:

A closed form formula which can be used by the driver to EASILY (REAL TIME) RE-CALCULATE where to switch from max acceleration to max deceleration.

Time Optimal Feedback in R^2



Multi Agent: Minimum Time Consensus

Consensus: Many ‘agents’ try to reach a previously unspecified point autonomously

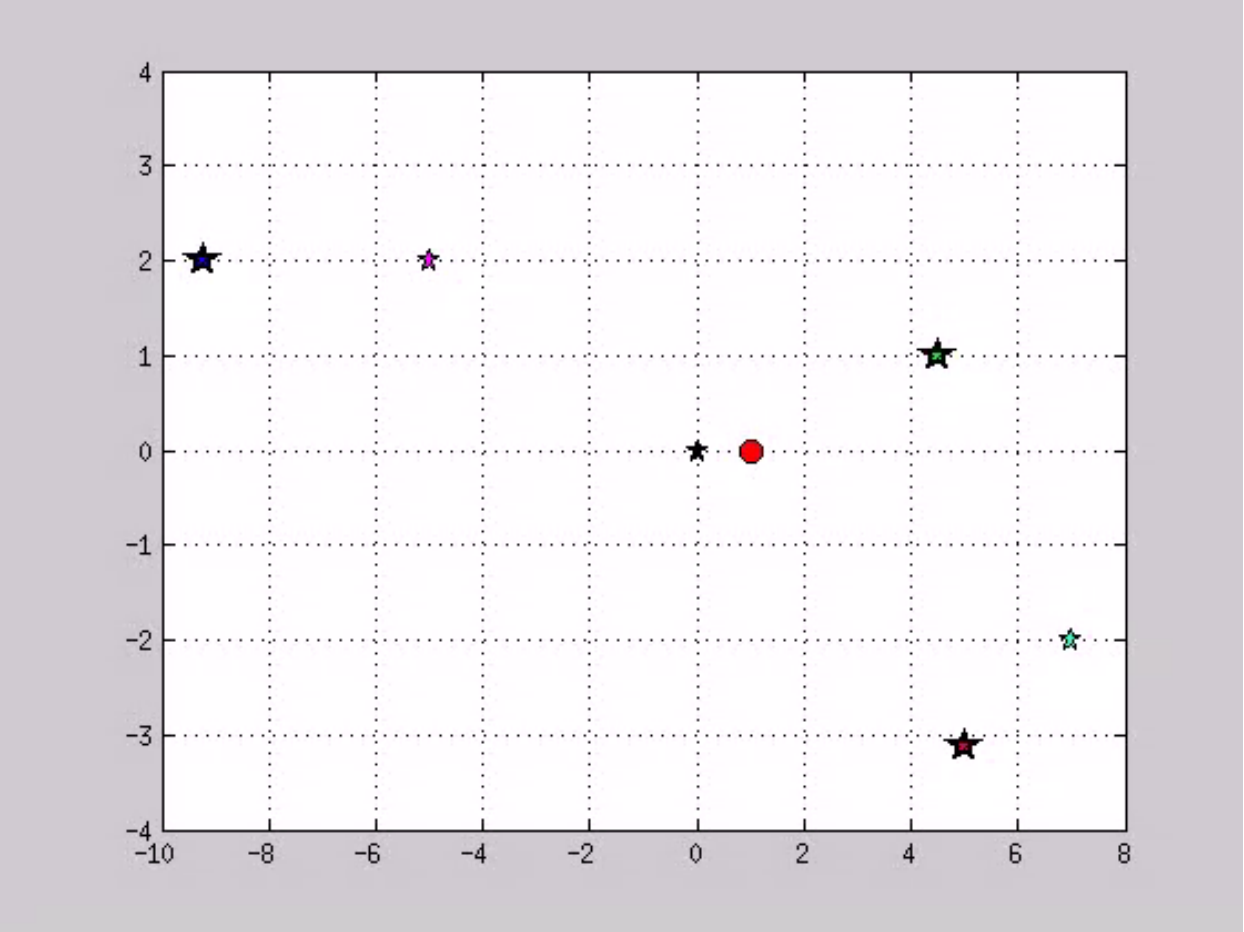
(Multi) Bovine Consensus

Minimum Time to Consensus

- All the IITB cows/bulls wants to gather in one UNSPECIFIED point on campus
- In minimum time
- We have (RE)-DISCOVERED (the cows knew it all along..) the solution to this problem:

Easily (REAL TIME) Re-calculable policy for each cow

Min time consensus on R^1



Pursuit Evasion Games

Leopard and Dog Games (Min Time)

- Leopard tries to catch dog in minimum time.
- Dog tries to evade leopard for longest possible time

Leopard and Dog Games (Min Time)

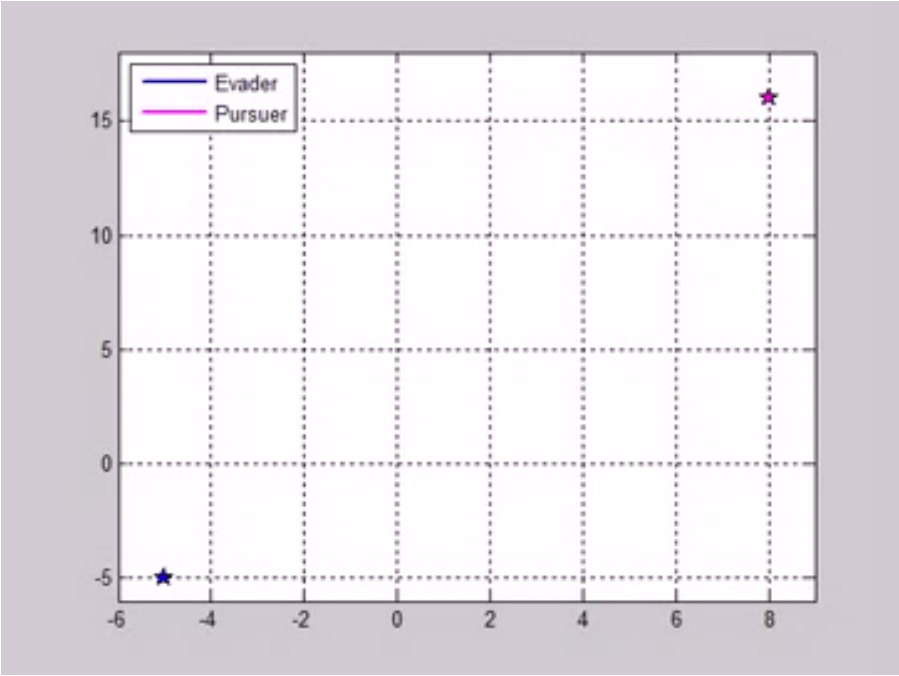
- Equal Acceleration
- Leopard is faster

- We have developed

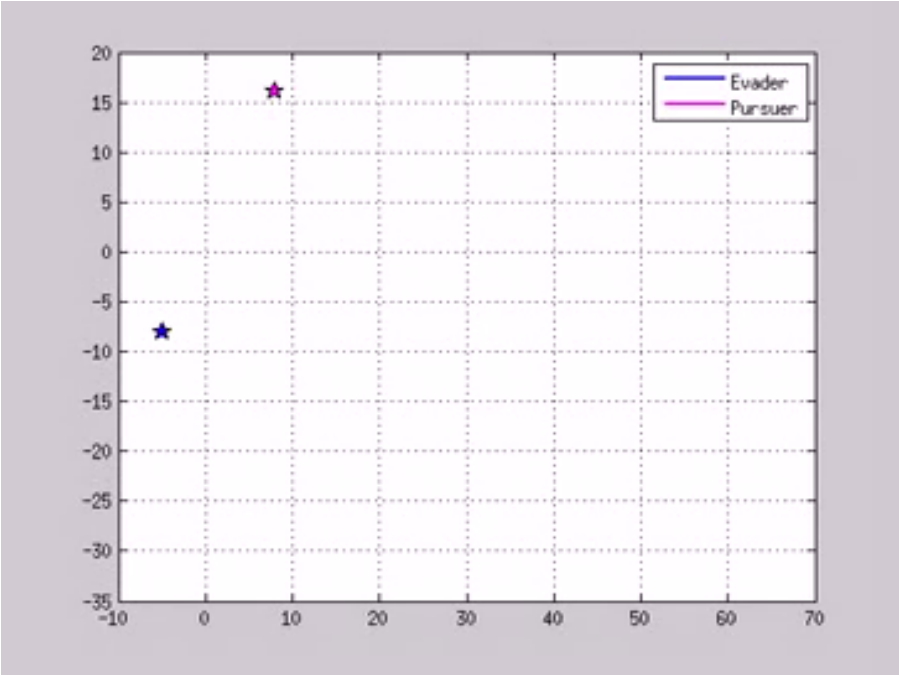
Closed form formula for the best (REAL-TIME RE-CALCULABLE) policies for both the leopard and the dog

Leopard Dog Games on R^2

- Independent Acceleration in both directions

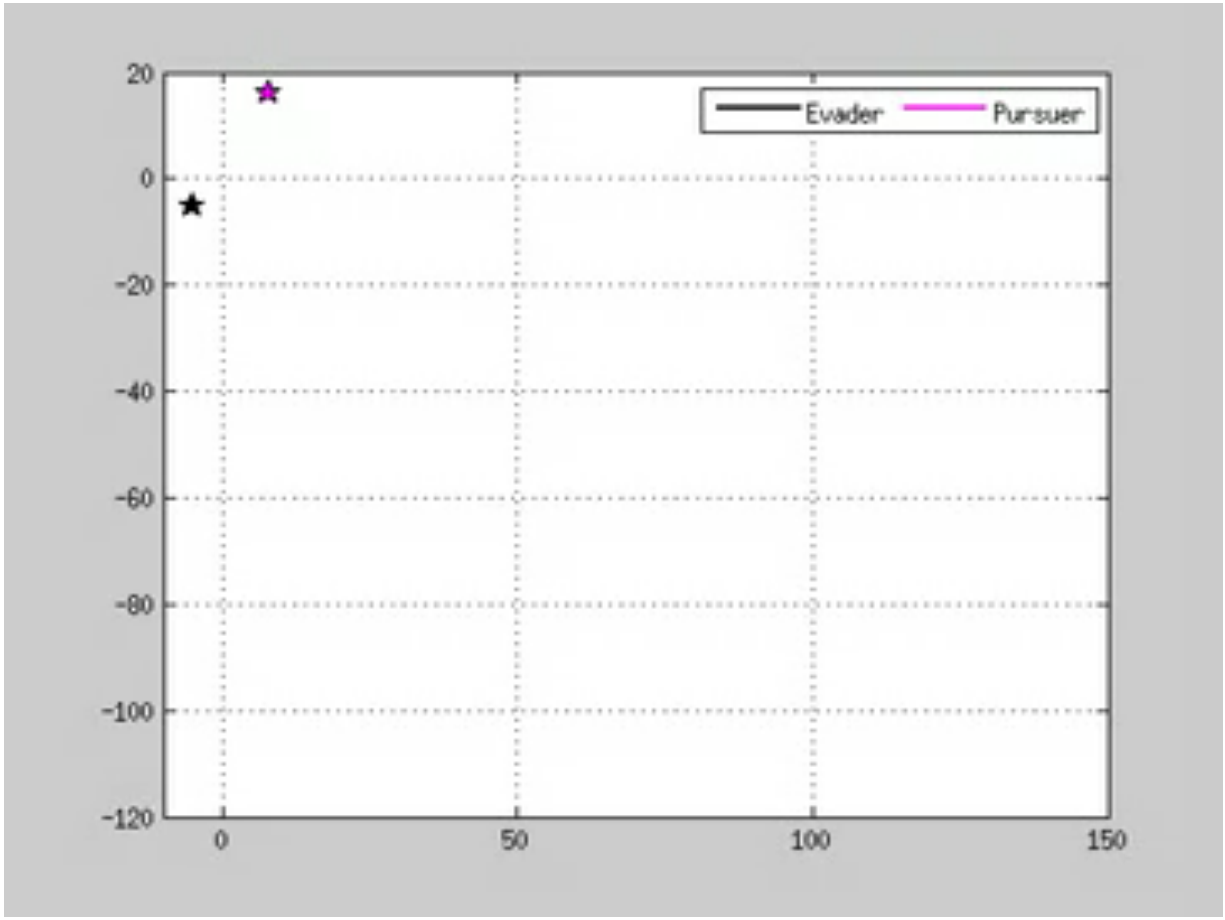


Slower and Not-so-bright dog

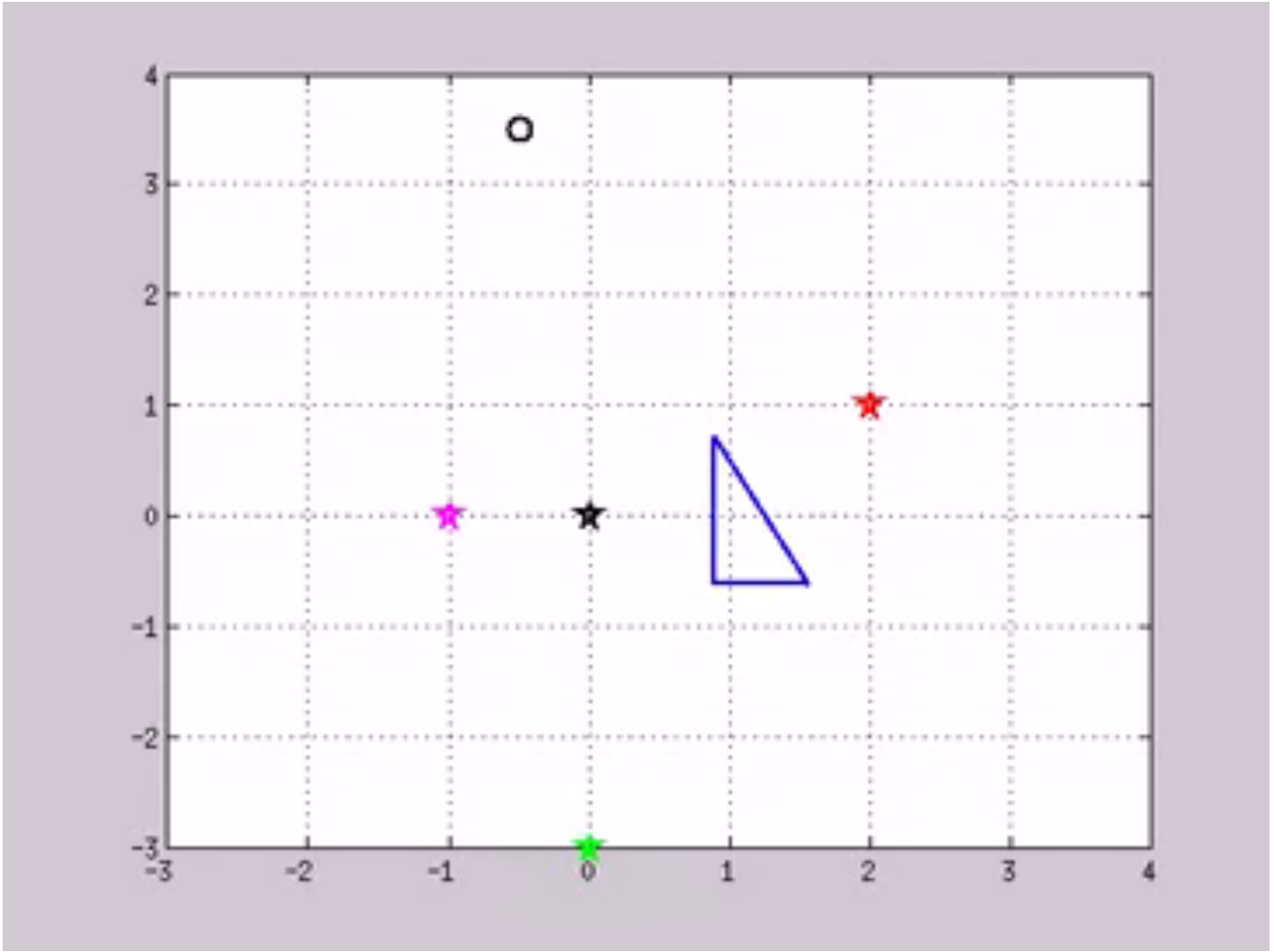


Slower dog – knows our formula

Dog as fast as leopard and knows our formula



Three Leopard one Dog - Unsolved



Min Time Consensus Tracking

Leader tracking in minimum time

- Leader sets the trajectory
- All followers need to converge onto that trajectory in minimum time using only local information (i.e. only looking at the duck in front)

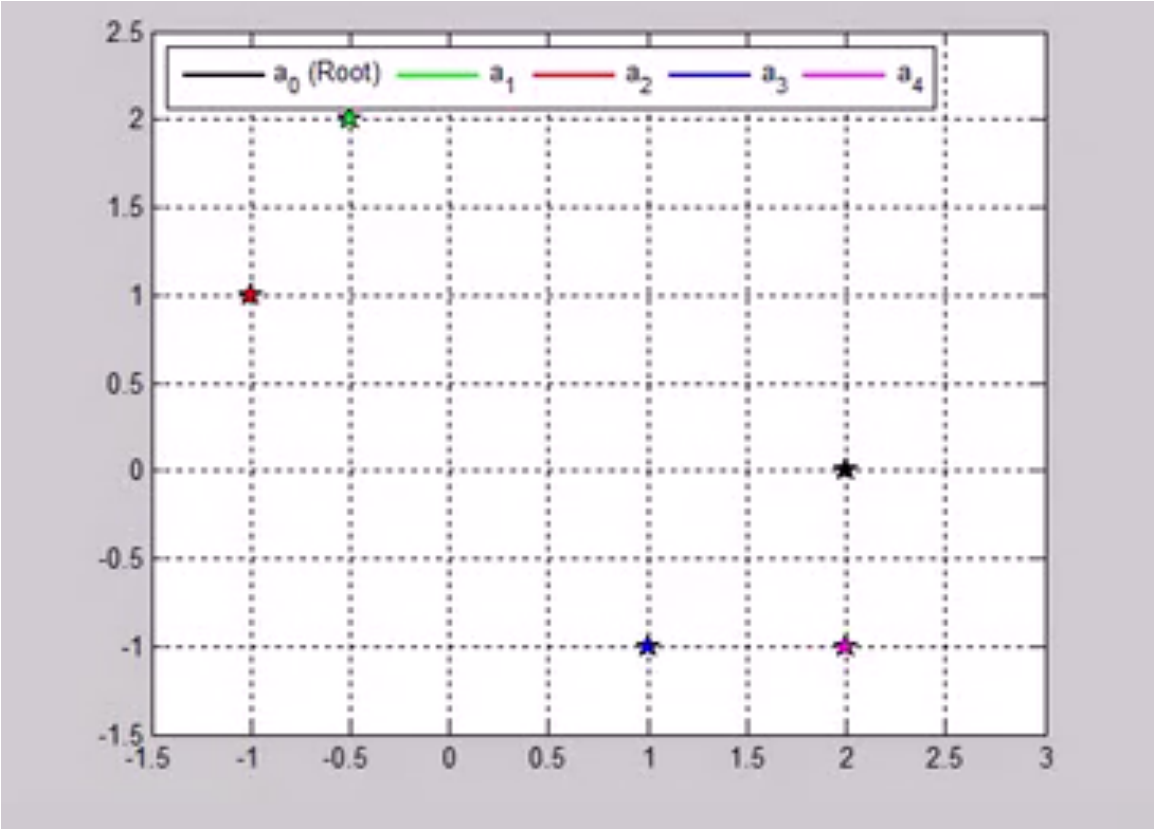
Min Time Leader Tracking

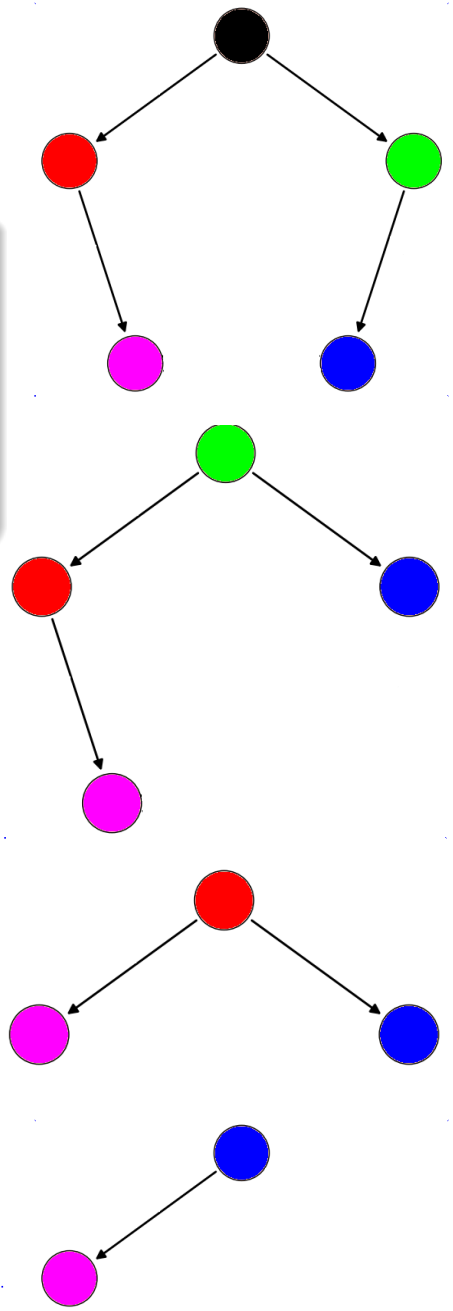
Example

5-agent systems communicating over a tree. Agent dynamics is given by

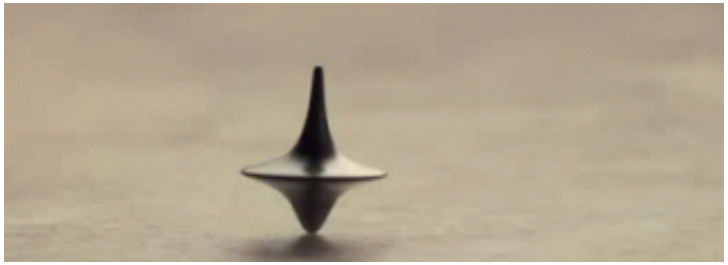
$$\dot{x}_i(t) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x_i(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_i(t) \qquad \text{for } i = 0, 1, ..., 4$$

$|u_0(t)| \leq 1$ and $|u_i(t)| \leq 3$ for $i = 1, ..., 4$





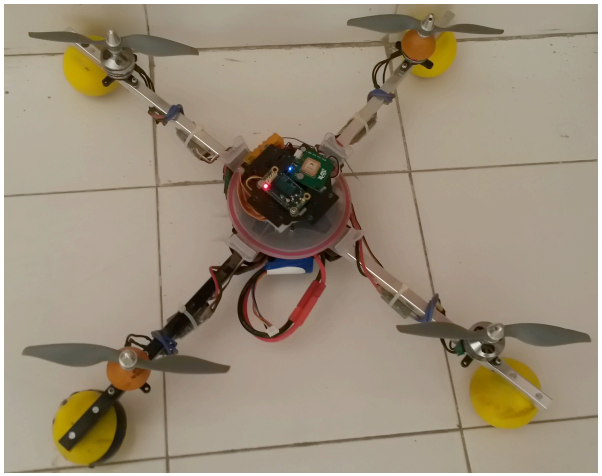
Are these real?



We are still not very sure

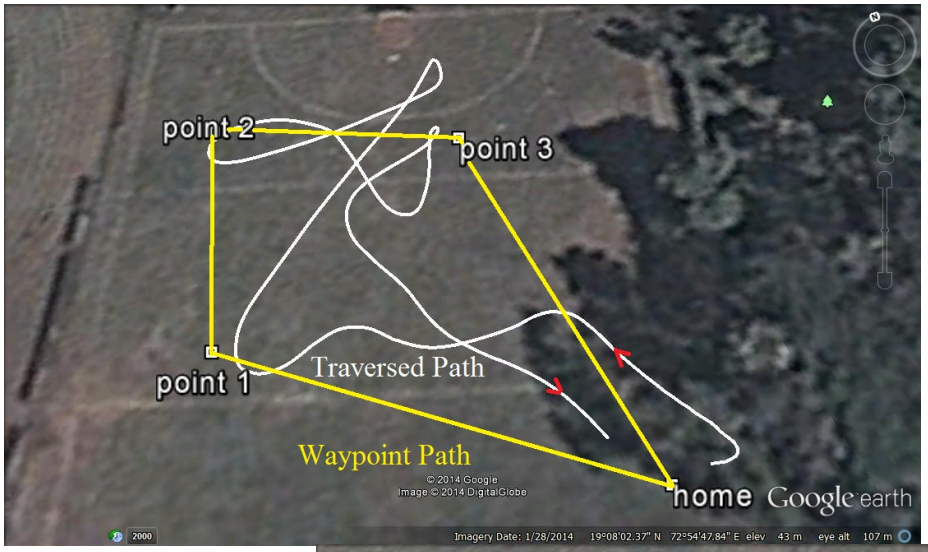
Quadcopter testbed

2012

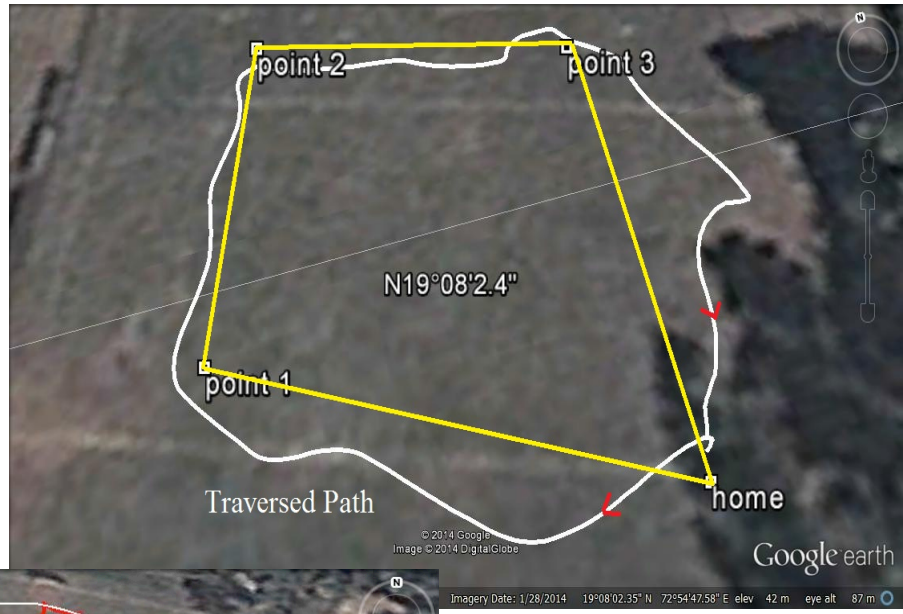


2012

GPS waypoint-Leader Follower



2013



2013

2014

Still long way to go before we can catch up with the leopard, duck or even cows

Thank You