M.Tech 1st Semester Seminar

FIR Filter Design Techniques

By

Arojit Roychowdhury

Supervisor : Prof P. C. Pandey

EE Dept, IIT Bombay

November ' 02

Presentation overview

- Introduction to FIR filters
- Different methods of FIR filter design
- Advantage and Disadvantage of each method
- Summary

Introduction to FIR filters

- Filters with finite length coefficients

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + \ldots + h(N-1)z^{-(N-1)}$$

- Stable filters
- Provides a linear phase response
- Contains all poles at the origin

FIR filter design methods

- The Window Method
- The Frequency Sampling Technique
- Optimal Filter Design Methods

The Window method

Steps

- **Desired frequency response, $H_d(w)$ provided**
- **$h_d(n)$ calculated using formula**

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(w) e^{jwn} dw$$

- **$h_d(n)$ obtained infinite in length and anti-causal**
- **Window function, w(n) of length N selected**
- **Now finite length h(n) calculated**

$$h(n) = h_d(n)\, w(n)$$

- **Rectangular window used**

Other windows

- Bartlett window(Triangular window)
  - reduces overshoot but spreads transition region
- Generalized cosine windows
  - Hanning
  - Hamming
  - Blackmann
  - provide smooth truncation of impulse response
- Kaiser window
  - compromise between overshoot reduction and transition region width spreading.

Effects of windowing on filter response

- Ripple in filter output due to truncation of $h_d(n)$
- Increasing N increases the ripple in the filter response
- Discontinuities in $H(w)$ lead to transition bands
- Smoother window function results in lower sidelobe
- Smoother window function increases the transition bandwidth
- Smoother windows eliminate ringing effects at the band edge

Advantages of window method

- Relative simplicity of method
- Ease of use
- Well-defined equations available for window functions

Limitations of window method

- $h_d(n)$ can be calculated only if $H_d(w)$ can be integrated
- Cannot be used for irregular shapes of $H_d(w)$
- Offers limited flexibility in design
- Only useful for design of prototype filters

The frequency sampling technique

Steps

- **Desired frequency response,** $H_d(w)$ **provided**
- **Given frequency response sampled at** N **points**
- N **samples represent** N**-point DFT of** $H_d(w)$
- **Frequency response,** $h(n)$ **can be calculated as follows**

$$h(n) = \frac{1}{N} \sum_{n=0}^{N-1} H(k) e^{j(2\pi n/N)k}$$

Techniques to reduce the error in frequency sampling technique

- Make some of the frequency samples as unconstrained variables
- Frequency samples in the transition band chosen as unconstrained variables
- Samples can be taken at $f_k = k/N$ or at $f_k = (k + ½)/N$
- Second set provides additional flexibility

Advantages

- Can be used for any $H_d(w)$
- $H_d(w)$ need not be a function whose integration can be evaluated

Disadvantages

- Frequency response equal to desired response only at the sampled points
- At other points, there will be finite error

Rabiner's method

- Desired frequency response, $H_d(w)$ provided
- Given frequency response sampled at N points
- N samples represent N-point DFT of $H_d(w)$
- $h(n)$ rotated by $N/2$ or $(N-1)/2$ samples
- $15N$ zero-valued samples symmetrically placed
- Other method is to split $h(n)$ around $N/2^{nd}$ sample
- $15N$ zero-valued samples placed between 2 pieces of response
- Frequency response obtained

Optimal filter design methods

Basic Idea

- Iterative method to design the filter coefficients so as to reduce a given criteria of error

Various optimization methods

- Least squared error design
- Weighted Chebyshev approximation
- Nonlinear equation solution for maximal ripple FIR filters
- Polynomial interpolation solution for maximal ripple FIR filters

Least squared error frequency domain design

- Controls the filter response by considering samples greater than order of filter
- *L*-samples of given frequency response taken(*L*>*N*)
- Error (*E*) is defined as follows

$$Error(E) = \sum |H(w_k) - H_d(w_k)|^2$$

where $w_k = (2\pi k)/L$

- *h*(*n*) calculated using inverse DFT formula

- Obtained response symmetrically truncated to length ***N***
- Frequency response calculated using

$$H(w) = \sum_{n=0}^{N-1} h(n) e^{-jwn}$$

- This method carried on until the error(***E***) is minimized
- Error can be reduced by defining a transition region and taking samples in that band
- Overall least square error is reduced

Weighted Chebyshev approximation

- An error function is defined as follows

$$E(w) = W(w)[H_d(w) - H^*(w)]$$

where *W*(*w*) is frequency response of a weighting function
$H_d(w)$ is frequency response of desired filter
*H**(*w*) is frequency response of designed filter

- *H*(*w*) can be written as a product as a sine or cosine term in w and a product of the type

$$\Sigma\, a(n)\cos(wn)$$

- The coefficients *a*(*n*) calculated to minimize *E*(*w*)

Nonlinear equation solution for maximal ripple FIR filters

- Extrema of *H*(*w*) represents the maximum ripple points at the filter output
- At N_e unknown frequencies *H*(*w*) attains maximum value of δ
- At these N_e frequencies derivative of *H*(*w*) will be zero.
- Set of $2N_e$ equations are formed and solved using nonlinear optimization procedure
- Peak error δ not minimized

Polynomial interpolation solution for maximal ripple FIR filters

- Iterative technique for producing a polynomial $H(w)$
- Initial guess affects the number of iterations
- Lagrange interpolation formula used for finding the polynomial
- New values of w taken where extrema of 1st iteration present
- Again new polynomial calculated
- Whole procedure repeated

Summary

- Window method useful for prototype filters only
- Frequency sampling technique used for design of non-prototype filter
- But this method introduces error at the non-sampled frequency points
- Optimization algorithms are used to reduce the errors caused due to frequency sampling technique