

EE101: Sinusoidal steady state analysis



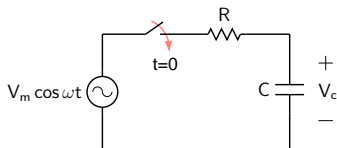
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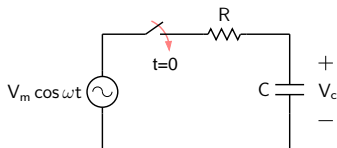
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Department of Electrical Engineering
Indian Institute of Technology Bombay

Sinusoidal steady state

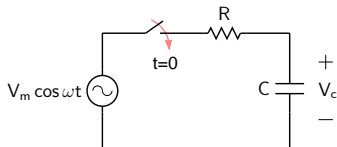


Sinusoidal steady state



$$R(C V'_c) + V_c = V_m \cos \omega t, \quad t > 0. \quad (1)$$

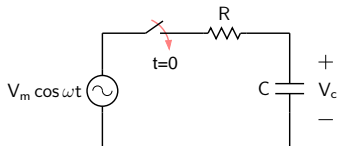
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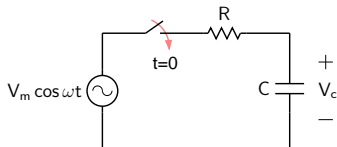
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The solution $V_c(t)$ is made up of two components, $V_c(t) = V_c^{(h)}(t) + V_c^{(p)}(t)$. $V_c^{(h)}(t)$ satisfies the homogeneous differential equation,

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from which, $V_c^{(h)}(t) = A \exp(-t/\tau)$, with $\tau = RC$.

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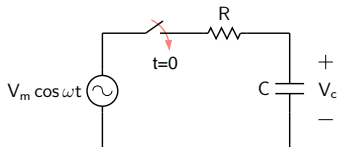
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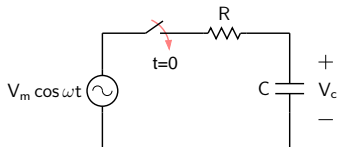
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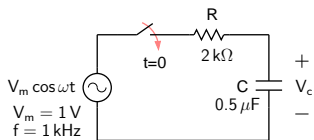
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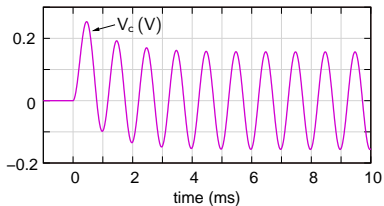
$$\omega R C (-C_1 \sin \omega t + C_2 \cos \omega t) + C_1 \cos \omega t + C_2 \sin \omega t = V_m \cos \omega t.$$

C_1 and C_2 can be found by equating the coefficients of $\sin \omega t$ and $\cos \omega t$ on the left and right sides.

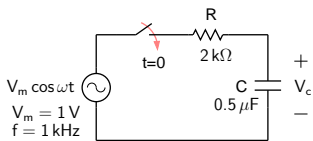
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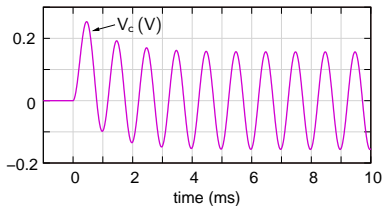
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Sinusoidal steady state

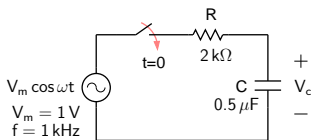


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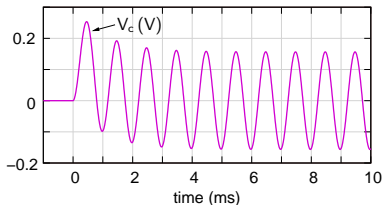


* The complete solution is $V_c(t) = A \exp(-t/\tau) + C_1 \cos \omega t + C_2 \sin \omega t$.

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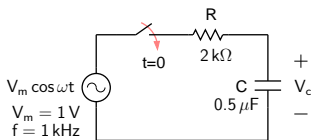


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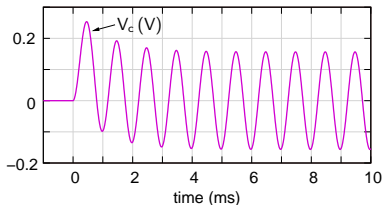


- * The complete solution is $V_c(t) = A \exp(-t/\tau) + C_1 \cos \omega t + C_2 \sin \omega t$.
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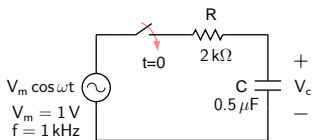


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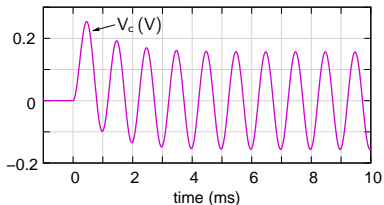


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- * This is known as the “sinusoidal steady state” response since all quantities (currents and voltages) in the circuit are sinusoidal in nature.
- * Any circuit containing resistors, capacitors, inductors, sinusoidal voltage and current sources (of the same frequency), dependent (linear) sources behaves in a similar manner, viz., each current and voltage in the circuit becomes purely sinusoidal as $t \rightarrow \infty$.

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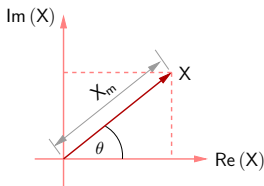
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- * Use of phasors substantially simplifies analysis of circuits in the sinusoidal steady state.
- * Note that a phasor can be written in the polar form or rectangular form,
 $\mathbf{X} = X_m \angle \theta = X_m \exp(j\theta) = X_m \cos \theta + j X_m \sin \theta$.

The term ωt is always *implicit*.



Phasors: examples

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Addition of phasors

Consider addition of two sinusoidal quantities:

$$\begin{aligned}v(t) &= v_1(t) + v_2(t) \\&= V_{m1} \cos(\omega t + \theta_1) + V_{m2} \cos(\omega t + \theta_2)\end{aligned}$$

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Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

$$\begin{aligned}\mathbf{V} &= \mathbf{V}_1 + \mathbf{V}_2 \\&= V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}\end{aligned}$$

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$$\begin{aligned}\tilde{v}(t) &= \text{Re} [\mathbf{V}e^{j\omega t}] \\&= \text{Re} [(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}) e^{j\omega t}]\end{aligned}$$

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$$\begin{aligned}v(t) &= v_1(t) + v_2(t) \\&= V_{m1} \cos(\omega t + \theta_1) + V_{m2} \cos(\omega t + \theta_2)\end{aligned}$$

Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

$$\begin{aligned}\mathbf{V} &= \mathbf{V}_1 + \mathbf{V}_2 \\&= V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}\end{aligned}$$

In the time domain, \mathbf{V} corresponds to $\tilde{v}(t)$, with

$$\begin{aligned}\tilde{v}(t) &= \text{Re} [\mathbf{V}e^{j\omega t}] \\&= \text{Re} [(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}) e^{j\omega t}] \\&= \text{Re} [V_{m1}e^{j(\omega t + \theta_1)} + V_{m2}e^{j(\omega t + \theta_2)}]\end{aligned}$$

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which is the same as $v(t)$.

- * Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.

Addition of phasors

- * Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.
- * The KCL and KVL equations,
 $\sum i_k(t) = 0$ at a node, and
 $\sum v_k(t) = 0$ in a loop,
amount to addition of sinusoidal quantities and can therefore be replaced by the corresponding phasor equations,
 $\sum \mathbf{I}_k = \mathbf{0}$ at a node, and
 $\sum \mathbf{V}_k = \mathbf{0}$ in a loop.

Impedance of a resistor

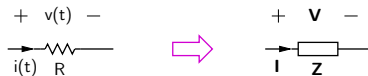


Impedance of a resistor



Let $i(t) = I_m \cos(\omega t + \theta)$.

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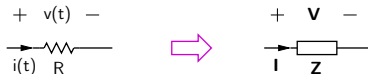
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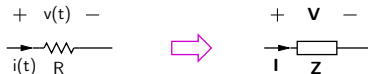
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The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,

$$\mathbf{I} = I_m \angle \theta, \quad \mathbf{V} = R \times I_m \angle \theta.$$

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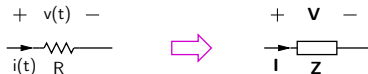
$$\equiv V_m \cos(\omega t + \theta).$$

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Thus, the *impedance* of a resistor, defined as, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is

$$\mathbf{Z} = R + j0$$

Impedance of a capacitor



Impedance of a capacitor



Let $v(t) = V_m \cos(\omega t + \theta)$.

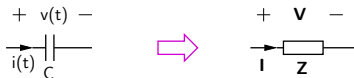
Impedance of a capacitor



Let $v(t) = V_m \cos(\omega t + \theta)$.

$$i(t) = C \frac{dv}{dt} = -C \omega V_m \sin(\omega t + \theta).$$

Impedance of a capacitor



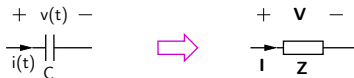
Let $v(t) = V_m \cos(\omega t + \theta)$.

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Using the identity, $\cos(\phi + \pi/2) = -\sin \phi$, we get

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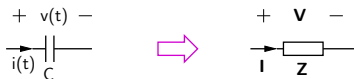
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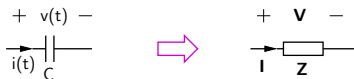
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In terms of phasors, $\mathbf{V} = V_m \angle \theta$, $\mathbf{I} = \omega C V_m \angle (\theta + \pi/2)$.

\mathbf{I} can be rewritten as,

$$\mathbf{I} = \omega C V_m e^{j(\theta + \pi/2)} = \omega C V_m e^{j\theta} e^{j\pi/2} = j\omega C (V_m e^{j\theta}) = j\omega C \mathbf{V}$$

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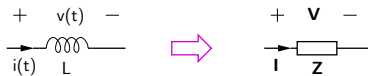
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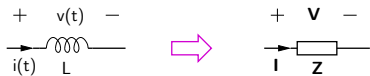
Thus, the *impedance* of a capacitor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = 1/(j\omega C)}$,

and the *admittance* of a capacitor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = j\omega C}$.

Impedance of an inductor



Impedance of an inductor



Let $i(t) = I_m \cos(\omega t + \theta)$.

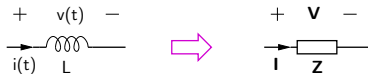
Impedance of an inductor



Let $i(t) = I_m \cos(\omega t + \theta)$.

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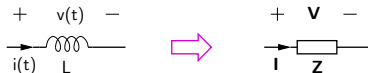
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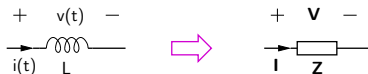
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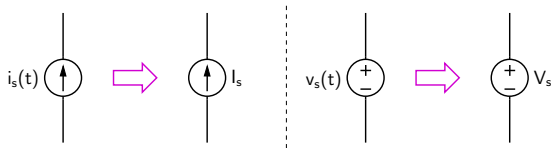
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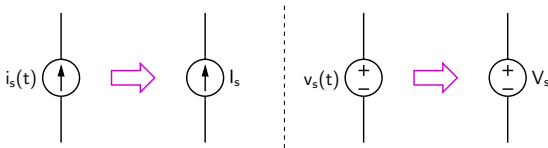
$$\mathbf{V} = \omega L I_m e^{j(\theta + \pi/2)} = \omega L I_m e^{j\theta} e^{j\pi/2} = j\omega L (I_m e^{j\theta}) = j\omega L \mathbf{I}$$

Thus, the *impedance* of an inductor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = j\omega L}$,

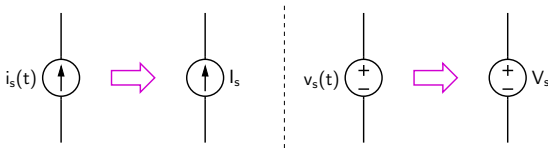
and the *admittance* of an inductor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = 1/(j\omega L)}$.

Sources

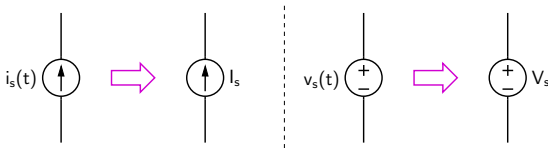




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- * Dependent (linear) sources can be treated in the sinusoidal steady state in the same manner as a resistor, i.e., by the corresponding phasor relationship. For example, for a CCVS, we have, $v(t) = r i_c(t)$ in the time domain.
 $\mathbf{V} = r \mathbf{I}_c$ in the frequency domain.

Use of phasors in circuit analysis

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- * The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.

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- * An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V}_s = \text{constant}$ (a complex number).
- * For dependent sources, a time-domain relationship such as $i(t) = \beta i_c(t)$ translates to $\mathbf{I} = \beta \mathbf{I}_c$ in the frequency domain.

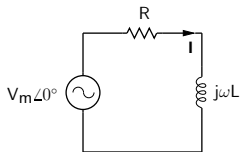
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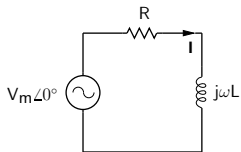
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- * Series/parallel formulas for resistors, nodal analysis, mesh analysis, Thevenin's and Norton's theorems can be directly applied to circuits in the sinusoidal steady state.

RL circuit



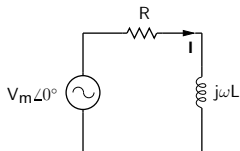
RL circuit



$$I = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle (-\theta),$$

$$\text{where } I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}, \text{ and } \theta = \tan^{-1}(\omega L/R).$$

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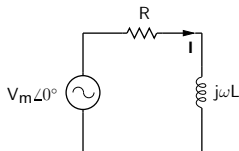


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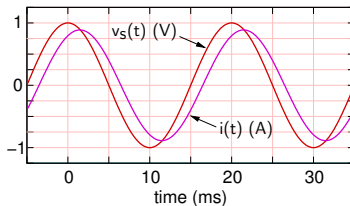
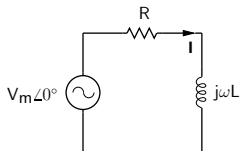
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For $R = 1 \Omega$, $L = 1.6 \text{ mH}$, $f = 50 \text{ Hz}$, $\theta = 26.6^\circ$, $t_{\text{lag}} = 1.48 \text{ ms}$.

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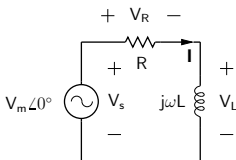
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For $R = 1 \Omega$, $L = 1.6 \text{ mH}$, $f = 50 \text{ Hz}$, $\theta = 26.6^\circ$, $t_{\text{lag}} = 1.48 \text{ ms}$.

(SEQUEL file: ee101_r1_ac_1.sqproj)

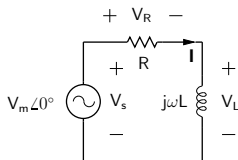
RL circuit



$$I = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle (-\theta),$$

$$\text{where } I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}, \text{ and } \theta = \tan^{-1}(\omega L/R).$$

RL circuit



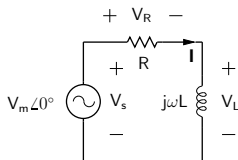
$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle (-\theta),$$

$$\text{where } I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}, \text{ and } \theta = \tan^{-1}(\omega L/R).$$

$$\mathbf{V}_R = \mathbf{I} \times R = R I_m \angle (-\theta),$$

$$\mathbf{V}_L = \mathbf{I} \times j\omega L = \omega I_m L \angle (-\theta + \pi/2),$$

RL circuit



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle (-\theta),$$

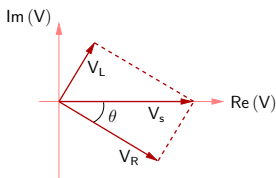
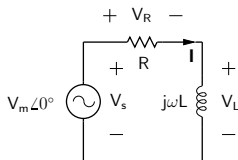
$$\text{where } I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}, \text{ and } \theta = \tan^{-1}(\omega L/R).$$

$$\mathbf{V}_R = \mathbf{I} \times R = R I_m \angle (-\theta),$$

$$\mathbf{V}_L = \mathbf{I} \times j\omega L = \omega I_m L \angle (-\theta + \pi/2),$$

The KVL equation, $\mathbf{V}_s = \mathbf{V}_R + \mathbf{V}_L$, can be represented in the complex plane by a “phasor diagram.”

RL circuit



$$I = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle (-\theta),$$

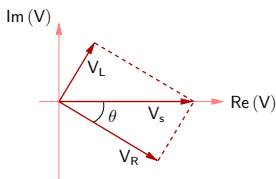
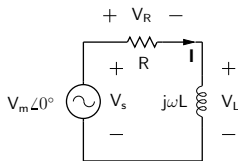
$$\text{where } I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}, \text{ and } \theta = \tan^{-1}(\omega L/R).$$

$$V_R = I \times R = R I_m \angle (-\theta),$$

$$V_L = I \times j\omega L = \omega I_m L \angle (-\theta + \pi/2),$$

The KVL equation, $V_s = V_R + V_L$, can be represented in the complex plane by a “phasor diagram.”

RL circuit



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle (-\theta),$$

$$\text{where } I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}, \text{ and } \theta = \tan^{-1}(\omega L/R).$$

$$\mathbf{V}_R = \mathbf{I} \times R = R I_m \angle (-\theta),$$

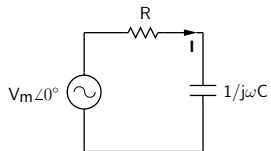
$$\mathbf{V}_L = \mathbf{I} \times j\omega L = \omega I_m L \angle (-\theta + \pi/2),$$

The KVL equation, $\mathbf{V}_s = \mathbf{V}_R + \mathbf{V}_L$, can be represented in the complex plane by a “phasor diagram.”

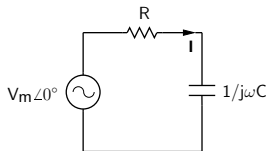
If $R \gg |j\omega L|$, $\theta \rightarrow 0$, $|\mathbf{V}_R| \simeq |\mathbf{V}_s| = V_m$.

If $R \ll |j\omega L|$, $\theta \rightarrow \pi/2$, $|\mathbf{V}_L| \simeq |\mathbf{V}_s| = V_m$.

RC circuit



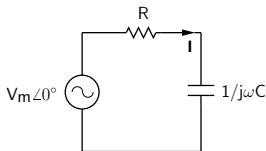
RC circuit



$$I = \frac{V_m \angle 0}{R + 1/j\omega C} \equiv I_m \angle \theta,$$

$$\text{where } I_m = \frac{\omega C V_m}{\sqrt{1 + (\omega RC)^2}}, \text{ and } \theta = \pi/2 - \tan^{-1}(\omega RC).$$

RC circuit

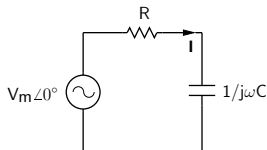


$$I = \frac{V_m \angle 0}{R + 1/j\omega C} \equiv I_m \angle \theta,$$

$$\text{where } I_m = \frac{\omega C V_m}{\sqrt{1 + (\omega RC)^2}}, \text{ and } \theta = \pi/2 - \tan^{-1}(\omega RC).$$

In the time domain, $i(t) = I_m \cos(\omega t + \theta)$, which *leads* the source voltage since the peak (or zero) of $i(t)$ occurs $t = \theta/\omega$ seconds before that of the source voltage.

RC circuit



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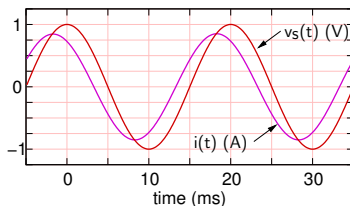
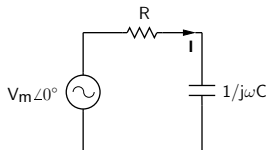
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For $R = 1 \Omega$, $C = 5.3 \text{ mF}$, $f = 50 \text{ Hz}$, $\theta = 31^\circ$, $t_{\text{lead}} = 1.72 \text{ ms}$.

(SEQUEL file: ee101_rc_ac_1.sqproj)

RC circuit



$$R = 1 \Omega$$
$$C = 5.3 \text{ mF}$$

$$I = \frac{V_m \angle 0}{R + 1/j\omega C} \equiv I_m \angle \theta,$$

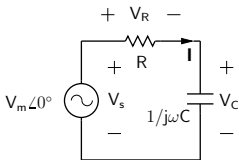
$$\text{where } I_m = \frac{\omega C V_m}{\sqrt{1 + (\omega RC)^2}}, \text{ and } \theta = \pi/2 - \tan^{-1}(\omega RC).$$

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(SEQUEL file: ee101_rc_ac_1.sqproj)

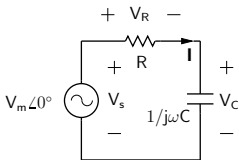
RC circuit



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$$\text{where } I_m = \frac{\omega C V_m}{\sqrt{1 + (\omega RC)^2}}, \text{ and } \theta = \pi/2 - \tan^{-1}(\omega RC).$$

RC circuit



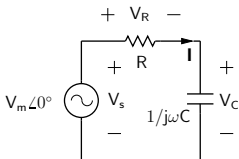
$$I = \frac{V_m \angle 0}{R + 1/j\omega C} \equiv I_m \angle \theta,$$

$$\text{where } I_m = \frac{\omega C V_m}{\sqrt{1 + (\omega RC)^2}}, \text{ and } \theta = \pi/2 - \tan^{-1}(\omega RC).$$

$$V_R = I \times R = R I_m \angle \theta,$$

$$V_C = I \times (1/j\omega C) = (I_m/\omega C) \angle (\theta - \pi/2),$$

RC circuit



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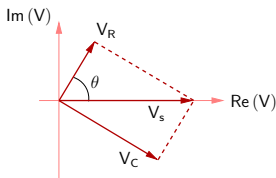
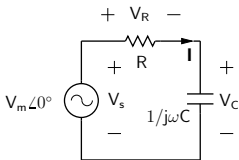
$$\text{where } I_m = \frac{\omega C V_m}{\sqrt{1 + (\omega RC)^2}}, \text{ and } \theta = \pi/2 - \tan^{-1}(\omega RC).$$

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RC circuit



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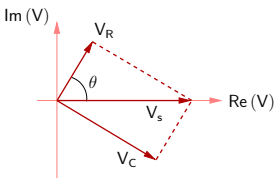
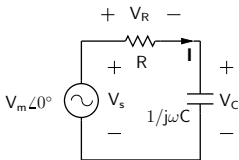
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$$\mathbf{V}_R = \mathbf{I} \times R = R I_m \angle \theta,$$

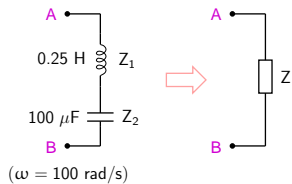
$$\mathbf{V}_C = \mathbf{I} \times (1/j\omega C) = (I_m/\omega C) \angle (\theta - \pi/2),$$

The KVL equation, $\mathbf{V}_s = \mathbf{V}_R + \mathbf{V}_C$, can be represented in the complex plane by a “phasor diagram.”

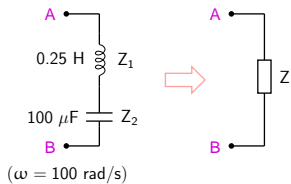
If $R \gg |1/j\omega C|$, $\theta \rightarrow 0$, $|\mathbf{V}_R| \simeq |\mathbf{V}_s| = V_m$.

If $R \ll |1/j\omega C|$, $\theta \rightarrow \pi/2$, $|\mathbf{V}_C| \simeq |\mathbf{V}_s| = V_m$.

Series/parallel connections



Series/parallel connections

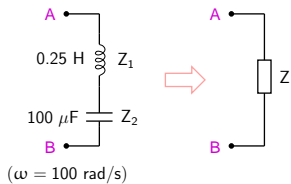


$$Z_1 = j \times 100 \times 0.25 = j 25 \Omega$$

$$Z_2 = -j / (100 \times 100 \times 10^{-6}) = -j 100 \Omega$$

$$Z = Z_1 + Z_2 = -j 75 \Omega$$

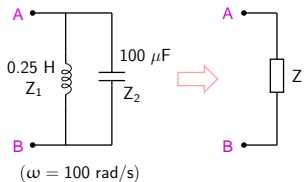
Series/parallel connections



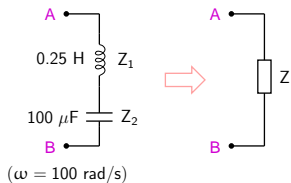
$$Z_1 = j \times 100 \times 0.25 = j 25 \Omega$$

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$$Z = Z_1 + Z_2 = -j 75 \Omega$$



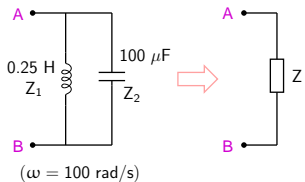
Series/parallel connections



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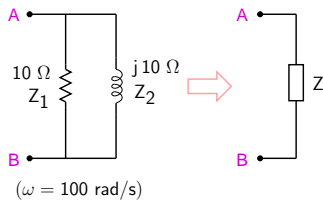
$$Z = Z_1 + Z_2 = -j 75 \Omega$$



$$\begin{aligned} Z &= \frac{Z_1 Z_2}{Z_1 + Z_2} \\ &= \frac{(j 25) \times (-j 100)}{j 25 - j 100} \\ &= \frac{25 \times 100}{-j 75} \\ &= j 33.3 \Omega \end{aligned}$$

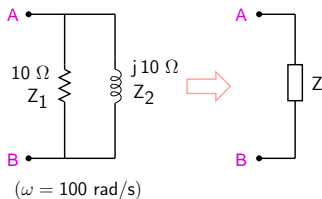
Impedance example

Obtain Z in polar form.



Impedance example

Obtain Z in polar form.



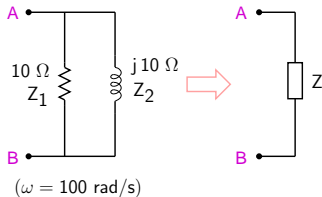
Method 1:

$$\begin{aligned} Z &= \frac{10 \times j10}{10 + j10} = \frac{j10}{1 + j} \\ &= \frac{j10}{1 + j} \times \frac{1 - j}{1 - j} \\ &= \frac{10 + j10}{2} = 5 + j5 \Omega \end{aligned}$$

Convert to polar form $\rightarrow Z = 7.07 \angle 45^\circ \Omega$

Impedance example

Obtain Z in polar form.



Method 1:

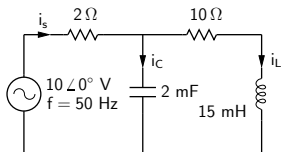
$$\begin{aligned} Z &= \frac{10 \times j10}{10 + j10} = \frac{j10}{1 + j} \\ &= \frac{j10}{1 + j} \times \frac{1 - j}{1 - j} \\ &= \frac{10 + j10}{2} = 5 + j5 \Omega \end{aligned}$$

Convert to polar form $\rightarrow Z = 7.07 \angle 45^\circ \Omega$

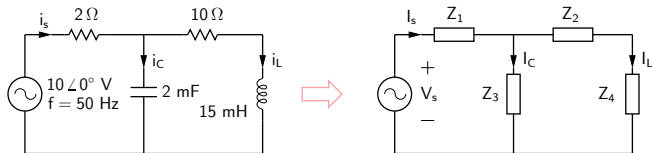
Method 2:

$$\begin{aligned} Z &= \frac{10 \times j10}{10 + j10} = \frac{100 \angle \pi/2}{10\sqrt{2} \angle \pi/4} \\ &= 5\sqrt{2} \angle (\pi/2 - \pi/4) = 7.07 \angle 45^\circ \Omega \end{aligned}$$

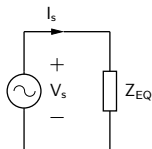
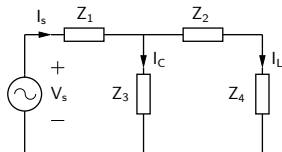
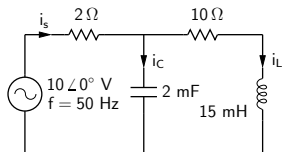
Circuit example



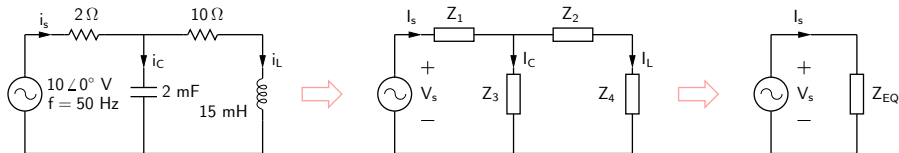
Circuit example



Circuit example

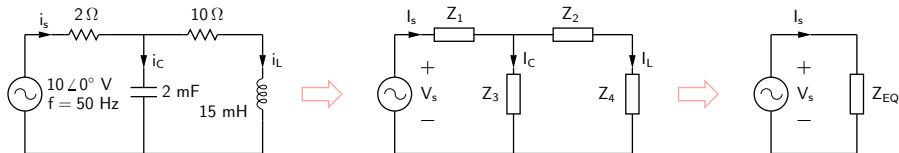


Circuit example



$$Z_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j1.6 \Omega$$

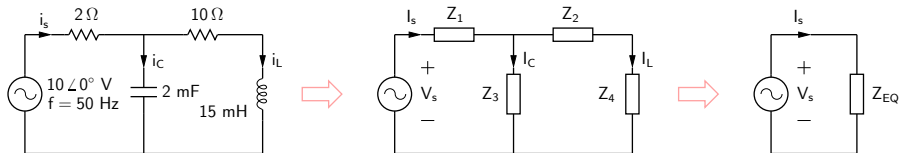
Circuit example



$$Z_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j 1.6 \Omega$$

$$Z_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j 4.7 \Omega$$

Circuit example

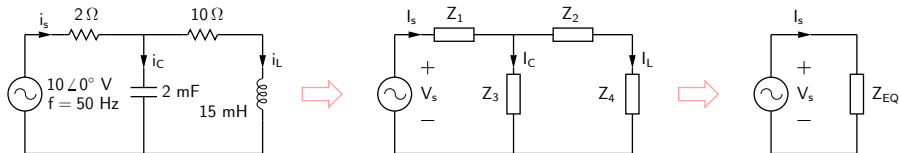


$$Z_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j1.6 \Omega$$

$$Z_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j4.7 \Omega$$

$$Z_{EQ} = Z_1 + Z_3 \parallel (Z_2 + Z_4)$$

Circuit example



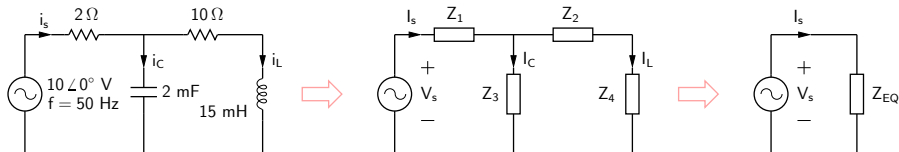
$$Z_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j1.6 \Omega$$

$$Z_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j4.7 \Omega$$

$$Z_{EQ} = Z_1 + Z_3 \parallel (Z_2 + Z_4)$$

$$= 2 + (-j1.6) \parallel (10 + j4.7) = 2 + \frac{(-j1.6) \times (10 + j4.7)}{-j1.6 + 10 + j4.7}$$

Circuit example



$$Z_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j1.6 \Omega$$

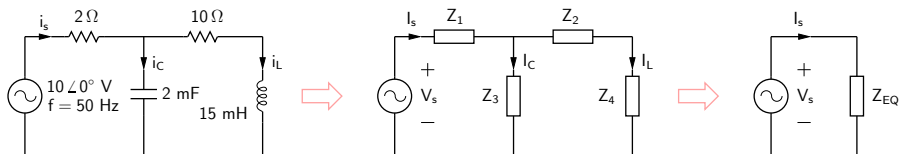
$$Z_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j4.7 \Omega$$

$$Z_{EQ} = Z_1 + Z_3 \parallel (Z_2 + Z_4)$$

$$= 2 + (-j1.6) \parallel (10 + j4.7) = 2 + \frac{(-j1.6) \times (10 + j4.7)}{-j1.6 + 10 + j4.7}$$

$$= 2 + \frac{1.6 \angle (-90^\circ) \times 11.05 \angle (25.2^\circ)}{10.47 \angle (17.2^\circ)} = 2 + \frac{17.7 \angle (-64.8^\circ)}{10.47 \angle (17.2^\circ)}$$

Circuit example



$$Z_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j1.6 \Omega$$

$$Z_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j4.7 \Omega$$

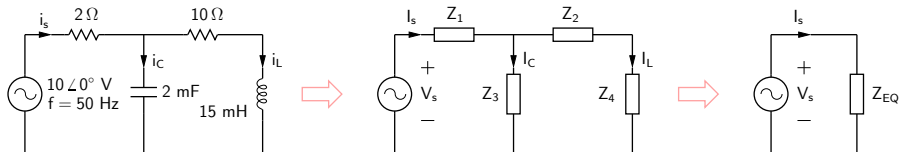
$$Z_{EQ} = Z_1 + Z_3 \parallel (Z_2 + Z_4)$$

$$= 2 + (-j1.6) \parallel (10 + j4.7) = 2 + \frac{(-j1.6) \times (10 + j4.7)}{-j1.6 + 10 + j4.7}$$

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$$= 2 + 1.69 \angle (-82^\circ) = 2 + (0.235 - j1.67)$$

Circuit example



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j1.6 \, \Omega$$

$$\mathbf{Z}_4 = 2\pi \times 50 \times 15 \times 10^{-3} = j4.7 \, \Omega$$

$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

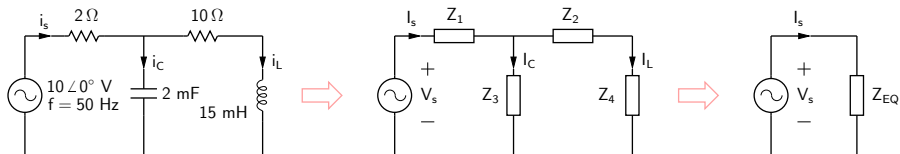
$$= 2 + (-j1.6) \parallel (10 + j4.7) = 2 + \frac{(-j1.6) \times (10 + j4.7)}{-j1.6 + 10 + j4.7}$$

$$= 2 + \frac{1.6 \angle (-90^\circ) \times 11.05 \angle (25.2^\circ)}{10.47 \angle (17.2^\circ)} = 2 + \frac{17.7 \angle (-64.8^\circ)}{10.47 \angle (17.2^\circ)}$$

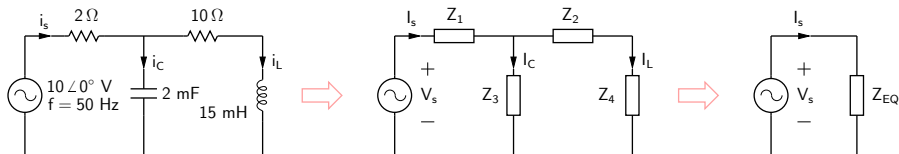
$$= 2 + 1.69 \angle (-82^\circ) = 2 + (0.235 - j1.67)$$

$$= 2.235 - j1.67 = 2.79 \angle (-36.8^\circ) \, \Omega$$

Circuit example (continued)

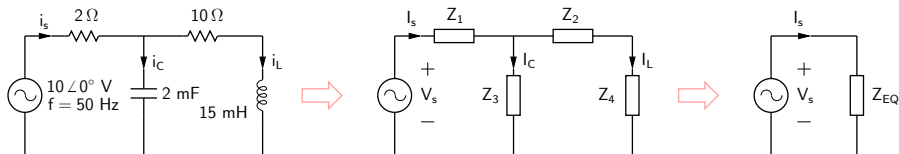


Circuit example (continued)



$$I_s = \frac{V_s}{Z_{EQ}} = \frac{10 \angle (0^\circ)}{2.79 \angle (-36.8^\circ)} = 3.58 \angle (36.8^\circ) \text{ A}$$

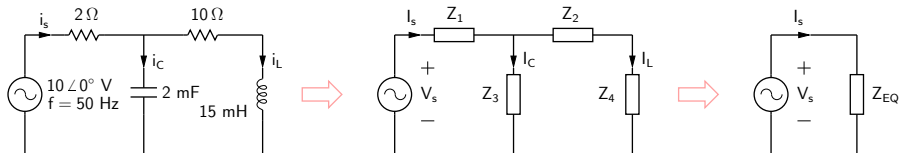
Circuit example (continued)



$$I_s = \frac{V_s}{Z_{EQ}} = \frac{10 \angle (0^\circ)}{2.79 \angle (-36.8^\circ)} = 3.58 \angle (36.8^\circ) \text{ A}$$

$$I_C = \frac{(Z_2 + Z_4)}{Z_3 + (Z_2 + Z_4)} \times I_s = 3.79 \angle (44.6^\circ) \text{ A}$$

Circuit example (continued)

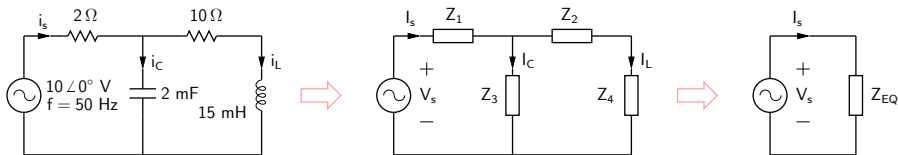


$$I_s = \frac{V_s}{Z_{EQ}} = \frac{10 \angle (0^\circ)}{2.79 \angle (-36.8^\circ)} = 3.58 \angle (36.8^\circ) \text{ A}$$

$$I_C = \frac{(Z_2 + Z_4)}{Z_3 + (Z_2 + Z_4)} \times I_s = 3.79 \angle (44.6^\circ) \text{ A}$$

$$I_L = \frac{Z_3}{Z_3 + (Z_2 + Z_4)} \times I_s = 0.546 \angle (-70.6^\circ) \text{ A}$$

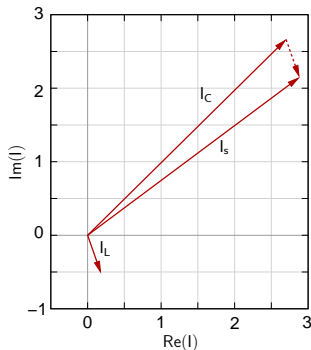
Circuit example (continued)



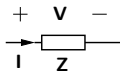
$$I_s = \frac{V_s}{Z_{EQ}} = \frac{10 \angle (0^\circ)}{2.79 \angle (-36.8^\circ)} = 3.58 \angle (36.8^\circ) \text{ A}$$

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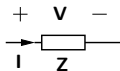
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Sinusoidal steady state: power computation



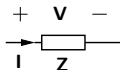
Sinusoidal steady state: power computation



Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle \theta$,

$i(t) = I_m \cos(\omega t + \phi)$, i.e., $\mathbf{I} = I_m \angle \phi$.

Sinusoidal steady state: power computation



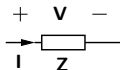
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The *instantaneous* power absorbed by \mathbf{Z} is,

$$\begin{aligned} P(t) &= v(t) i(t) \\ &= V_m I_m \cos(\omega t + \theta) \cos(\omega t + \phi) \\ &= \frac{1}{2} V_m I_m [\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)] \end{aligned} \tag{1}$$

Sinusoidal steady state: power computation



Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle \theta$,

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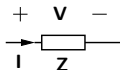
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The *average* power absorbed by Z is

$$P = \frac{1}{T} \int_0^T P(t) dt, \text{ where } T = 2\pi/\omega.$$

Sinusoidal steady state: power computation



Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle \theta$,

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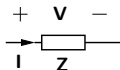
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The first term in Eq. (1) has an average value of zero and does not contribute to P .
Therefore,

Sinusoidal steady state: power computation



Let $v(t) = V_m \cos(\omega t + \theta)$, i.e., $\mathbf{V} = V_m \angle \theta$,

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The *instantaneous* power absorbed by \mathbf{Z} is,

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The *average* power absorbed by \mathbf{Z} is

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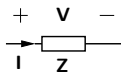
The first term in Eq. (1) has an average value of zero and does not contribute to P .

Therefore,

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

 gives the average power absorbed by \mathbf{Z} .

Average power for R , L , C

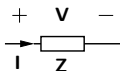


General formula:

$$V = V_m \angle \theta, I = I_m \angle \phi$$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

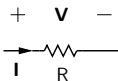
Average power for R , L , C



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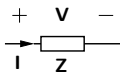


$$V = RI$$

$$\text{For } I = I_m \angle \alpha, V = RI_m \angle \alpha,$$

$$P = \frac{1}{2} (RI_m) I_m \cos(\alpha - \alpha) = \frac{1}{2} I_m^2 R = \frac{1}{2} V_m^2 / R$$

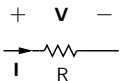
Average power for R , L , C



General formula:

$$V = V_m \angle \theta, I = I_m \angle \phi$$

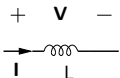
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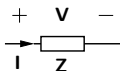


$$V = j\omega LI$$

$$\text{For } I = I_m \angle \alpha, V = \omega L I_m \angle (\alpha + \pi/2),$$

$$P = \frac{1}{2} V_m I_m \cos[(\alpha + \pi/2) - \alpha] = 0$$

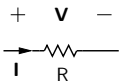
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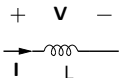
$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$



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$$\text{For } I = I_m \angle \alpha, V = RI_m \angle \alpha,$$

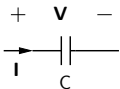
$$P = \frac{1}{2} (RI_m) I_m \cos(\alpha - \alpha) = \frac{1}{2} I_m^2 R = \frac{1}{2} V_m^2 / R$$



$$V = j\omega L I$$

$$\text{For } I = I_m \angle \alpha, V = \omega L I_m \angle (\alpha + \pi/2),$$

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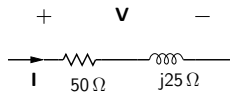


$$I = j\omega C V$$

$$\text{For } V = V_m \angle \alpha, I = \omega C V_m \angle (\alpha + \pi/2),$$

$$P = \frac{1}{2} V_m I_m \cos[\alpha - (\alpha + \pi/2)] = 0$$

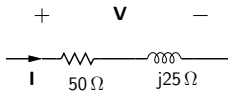
Average power: example



Given: $I = 2 \angle 45^\circ\text{ A}$

Find the average power absorbed.

Average power: example



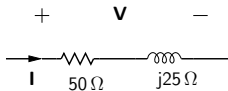
Given: $I = 2 \angle 45^\circ$ A

Find the average power absorbed.

Method 1:

$$\begin{aligned} V &= (50 + j25) \times 2 \angle 45^\circ \\ &= 55.9 \angle 26.6^\circ \times 2 \angle 45^\circ \\ &= 111.8 \angle (45^\circ + 26.6^\circ) \end{aligned}$$

Average power: example



Given: $I = 2 \angle 45^\circ$ A

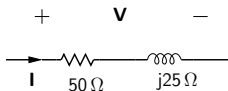
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$$P = \frac{1}{2} \times 111.8 \times 2 \times \cos(26.6^\circ) = 100\text{ W}.$$

Average power: example



Given: $I = 2 \angle 45^\circ\text{ A}$

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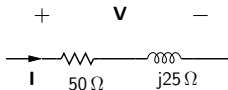
$$P = \frac{1}{2} \times 111.8 \times 2 \times \cos(26.6^\circ) = 100\text{ W}.$$

Method 2:

No average power is absorbed by the inductor.

$\Rightarrow P = P_R$ (average power absorbed by R)

Average power: example



Given: $I = 2 \angle 45^\circ\text{ A}$

Find the average power absorbed.

Method 1:

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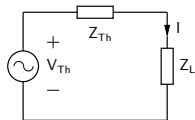
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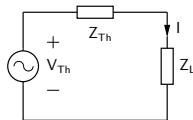
$$\begin{aligned} &= \frac{1}{2} I_m^2 R = \frac{1}{2} \times 2^2 \times 50 \\ &= 100\text{ W}. \end{aligned}$$

Maximum power transfer



Maximum power transfer

Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

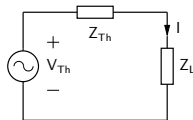


Maximum power transfer

Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by \mathbf{Z}_L is,

$$\begin{aligned} P &= \frac{1}{2} I_m^2 R_L \\ &= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L \\ &= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} R_L. \end{aligned}$$



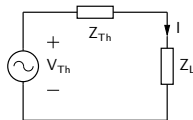
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For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.



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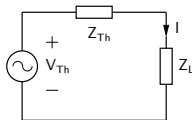
$$\begin{aligned} P &= \frac{1}{2} I_m^2 R_L \\ &= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L \\ &= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} R_L. \end{aligned}$$

For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.

With $X_L = -X_{Th}$, we have,

$$P = \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2} R_L,$$

which is maximum for $R_L = R_{Th}$.



Maximum power transfer

Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by \mathbf{Z}_L is,

$$\begin{aligned} P &= \frac{1}{2} I_m^2 R_L \\ &= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L \\ &= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} R_L. \end{aligned}$$

For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.

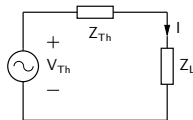
With $X_L = -X_{Th}$, we have,

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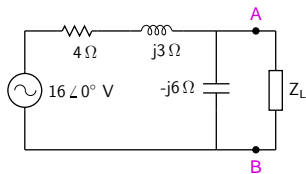
which is maximum for $R_L = R_{Th}$.

Therefore, for maximum power transfer to the load \mathbf{Z}_L , we need,

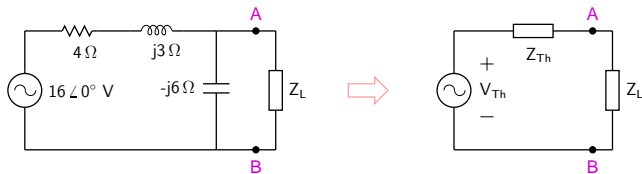
$$R_L = R_{Th}, X_L = -X_{Th}, \text{ i.e., } \boxed{\mathbf{Z}_L = \mathbf{Z}_{Th}^*}.$$



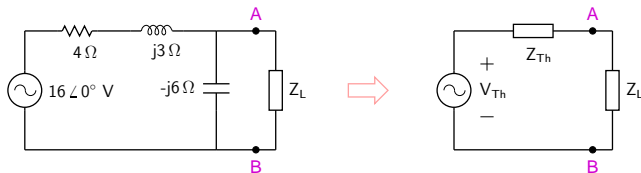
Maximum power transfer: example



Maximum power transfer: example

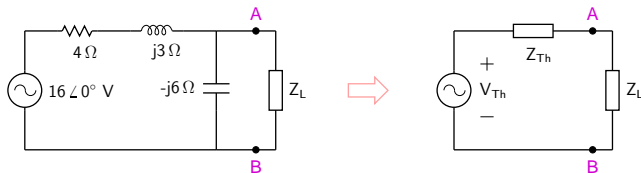


Maximum power transfer: example



$$\mathbf{Z}_{Th} = (-j6) \parallel (4 + j3) = 5.76 - j1.68 \Omega.$$

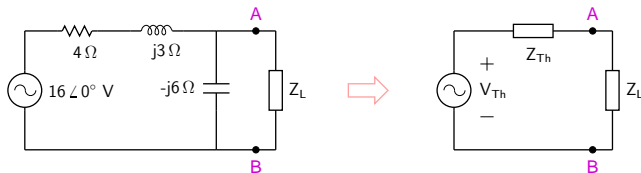
Maximum power transfer: example



$$\mathbf{Z}_{Th} = (-j6) \parallel (4 + j3) = 5.76 - j1.68 \Omega.$$

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 5.76 + j1.68 \Omega \equiv R_L + jX_L$.

Maximum power transfer: example

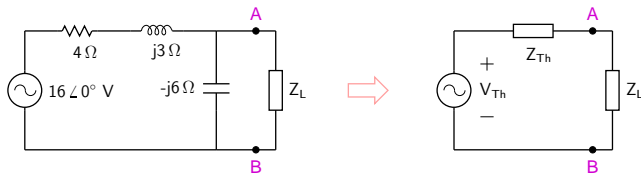


$$\mathbf{Z}_{Th} = (-j6) \parallel (4 + j3) = 5.76 - j1.68 \Omega.$$

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 5.76 + j1.68 \Omega \equiv R_L + jX_L$.

$$\mathbf{V}_{Th} = 16 \angle 0^\circ \times \frac{-j6}{(4 + j3) + (-j6)} = 19.2 \angle (-53.13^\circ).$$

Maximum power transfer: example



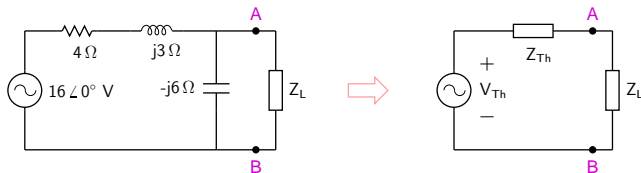
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For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 5.76 + j1.68 \Omega \equiv R_L + jX_L$.

$$\mathbf{V}_{Th} = 16 \angle 0^\circ \times \frac{-j6}{(4 + j3) + (-j6)} = 19.2 \angle (-53.13^\circ).$$

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} = \frac{\mathbf{V}_{Th}}{2R_L}.$$

Maximum power transfer: example



$$\mathbf{Z}_{Th} = (-j6) \parallel (4 + j3) = 5.76 - j1.68 \Omega.$$

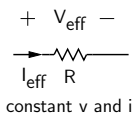
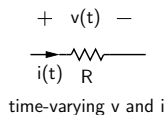
For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_{Th}^* = 5.76 + j1.68 \Omega \equiv R_L + jX_L$.

$$\mathbf{V}_{Th} = 16 \angle 0^\circ \times \frac{-j6}{(4 + j3) + (-j6)} = 19.2 \angle (-53.13^\circ).$$

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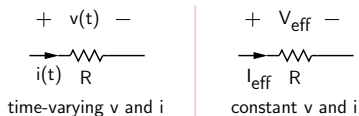
$$P = \frac{1}{2} I_m^2 R_L = \frac{1}{2} \left(\frac{19.2}{2R_L} \right)^2 \times R_L = \frac{1}{2} \frac{(19.2)^2}{4R_L} = 8 \text{ W}.$$

Effective (rms) values of voltage/current



Consider a periodic current $i(t)$ passing through R .

Effective (rms) values of voltage/current



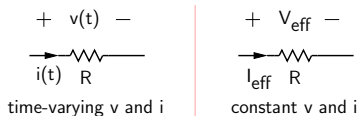
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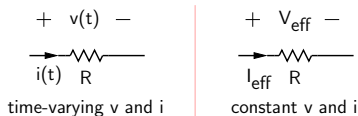
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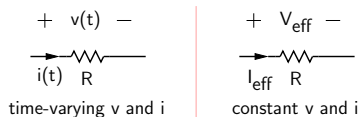
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I_{eff} , the *effective* value of $i(t)$, is defined such that $P_1 = P_2$, i.e.,

$$I_{\text{eff}}^2 R = \frac{1}{T} \int_0^T [i(t)]^2 R dt,$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T [i(t)]^2 dt}.$$

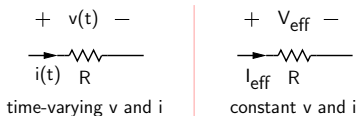
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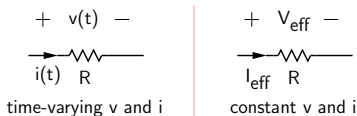
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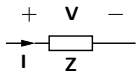
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Similarly, $V_{eff} = V_m/\sqrt{2}$.

Apparent power, real power, and power factor



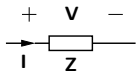
$$V = V_m \angle \theta$$

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The average ("real") power absorbed by Z is,

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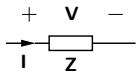
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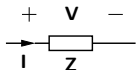
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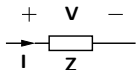
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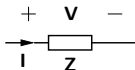
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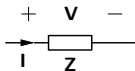
$(\theta - \phi) < 0$: $i(t)$ leads $v(t)$, the P.F. is called a *leading* P.F.
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Power factor: examples



1. Given: $V = 120 \angle 0^\circ$ V (rms), $I = 2 \angle (-36.9^\circ)$ A (rms).
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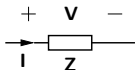
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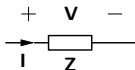


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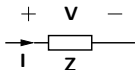
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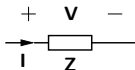
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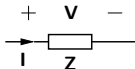
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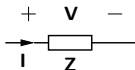
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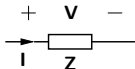
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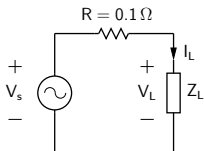
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Why is power factor important?

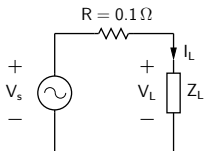


Consider a simplified model of a power system consisting of a generator (V_s), transmission line (R), and load (Z_L).

The load is specified as $P = 50 \text{ kW}$, P.F. = 0.6 (lagging), $V_L = 480 \angle 0^\circ \text{ V}$ (rms).

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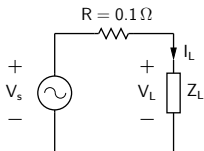
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$$P = 50 \times 10^3 \text{ W} = |V_L| \times |I_L| \times \text{P.F.} \Rightarrow |I_L| = \frac{50 \times 10^3}{480 \times 0.6} = 173.6 \text{ A (rms)}.$$

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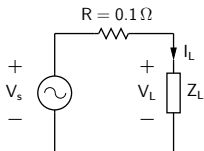
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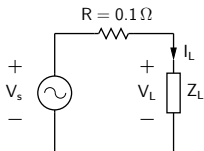
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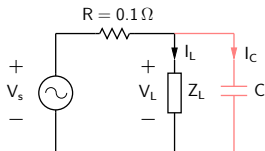
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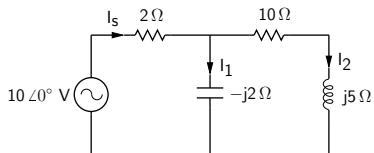
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The effective power factor of an inductive load can be improved by connecting a suitable capacitance in parallel.

Power computation: home work



- * Find I_1 , I_2 , I_s .
- * Compute the average power absorbed by each element.
- * Verify power balance.

(SEQUEL file: ee101_phasors_2.sqproj)