

# EE101: RC and RL Circuits (with DC sources)

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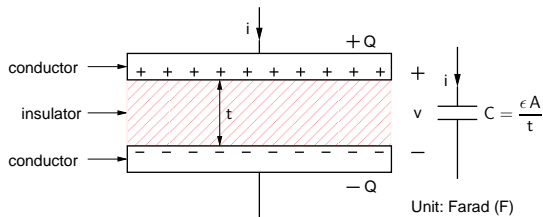
**M. B. Patil**

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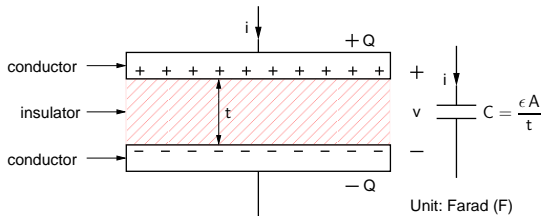
[www.ee.iitb.ac.in/~sequel](http://www.ee.iitb.ac.in/~sequel)

Department of Electrical Engineering  
Indian Institute of Technology Bombay

# Capacitors

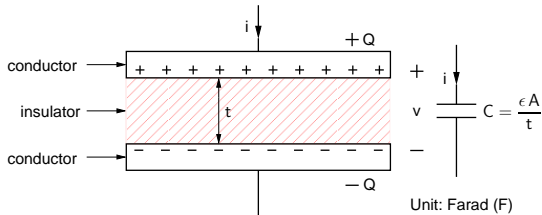


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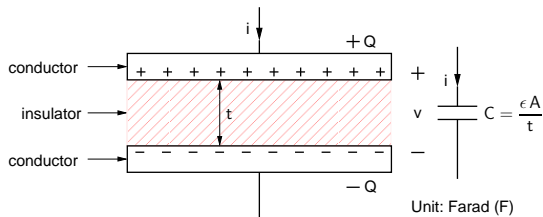
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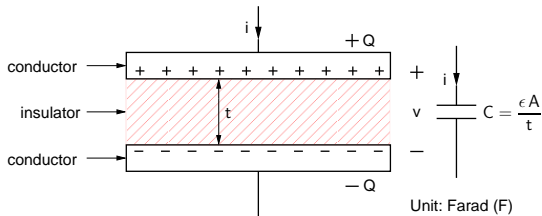
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- \* If  $v = \text{constant}$ ,  $i = 0$ , i.e., a capacitor behaves like an open circuit in DC conditions as one would expect from two conducting plates separated by an insulator.

## Example

Plot  $v$ ,  $p$ , and  $W$  versus time for the given source current. Assume  $v(0) = 0$  V,  $C = 5$  mF.



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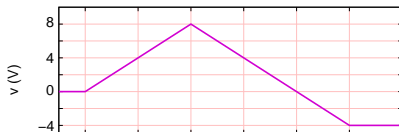
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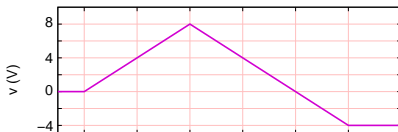
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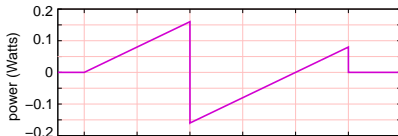
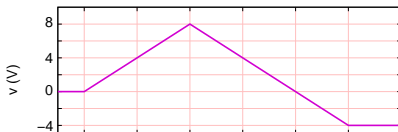
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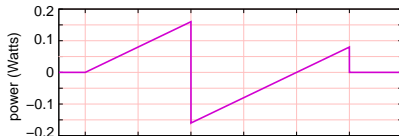
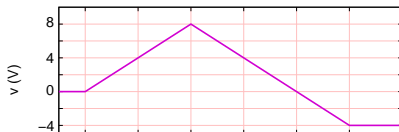


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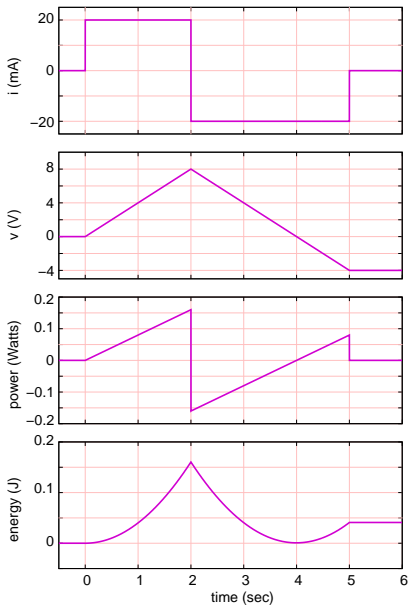


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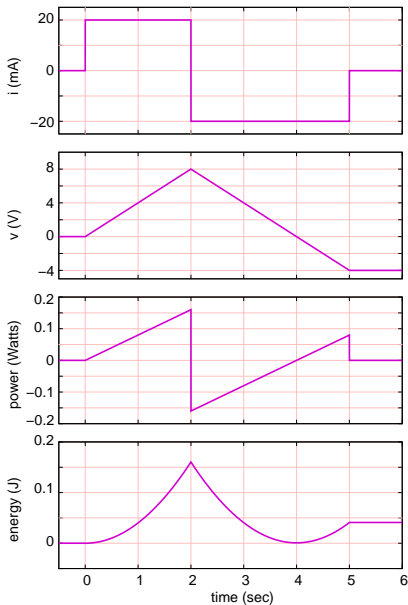
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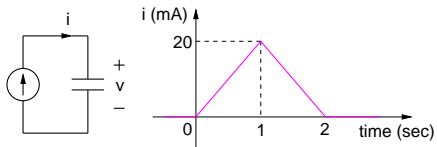
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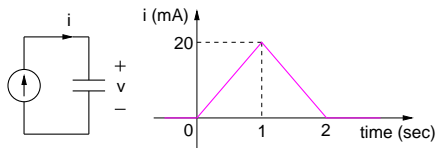
$$\begin{aligned} W(t) &= \int p(t) dt \\ &= C \int v \frac{dv}{dt} dt \\ &= C \int v dv \\ &= \frac{1}{2} C v^2 \end{aligned}$$



# Home work



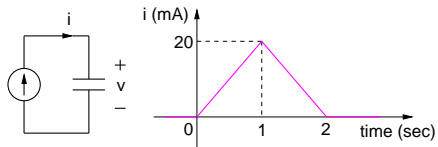
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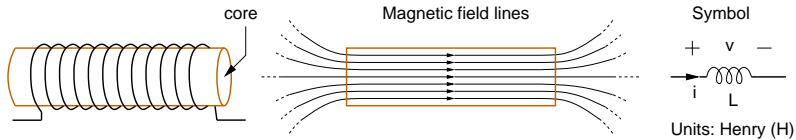


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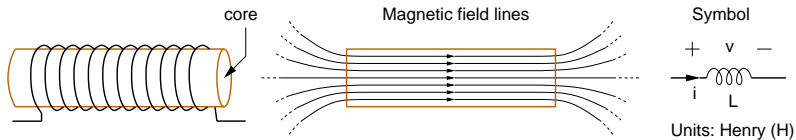


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- \* Verify your results with circuit simulation.

# Inductors

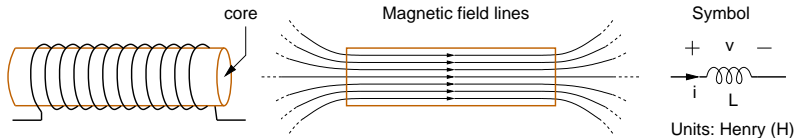


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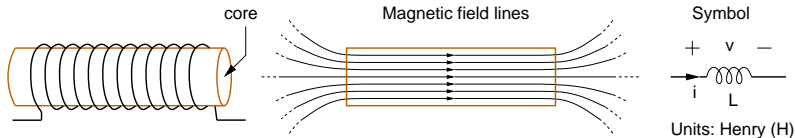
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Compare with  $v = L \frac{di}{dt}$ .

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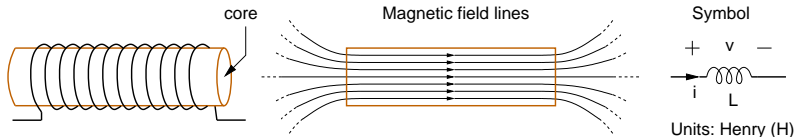
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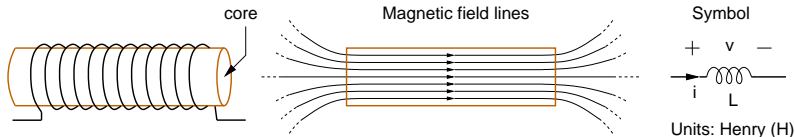
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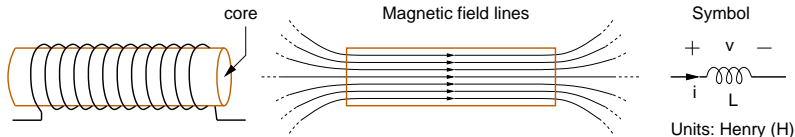
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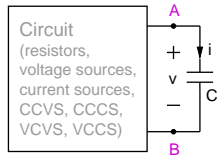
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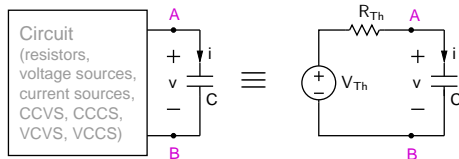
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- \* Note:  $B = \mu H$  is an approximation. In practice,  $B$  may be a nonlinear function of  $H$ , depending on the core material.



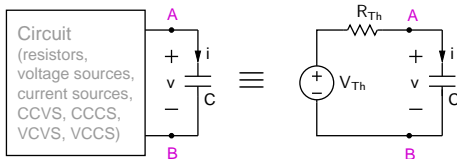
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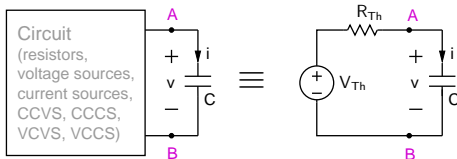


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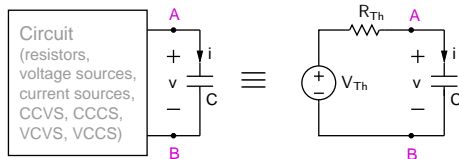
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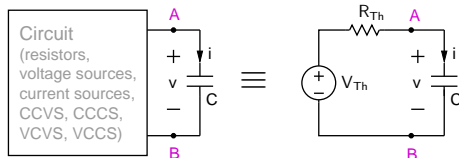
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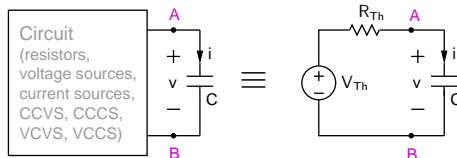
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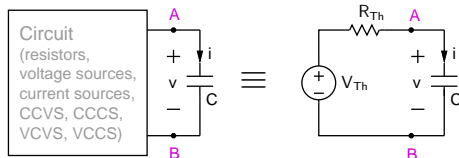
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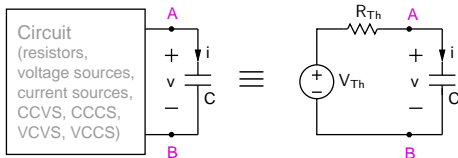
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$$* v = v^{(h)} + v^{(p)} = K \exp(-t/\tau) + V_{Th}.$$

\* In general,  $v(t) = A \exp(-t/\tau) + B$ , where  $A$  and  $B$  can be obtained from known conditions on  $v$ .

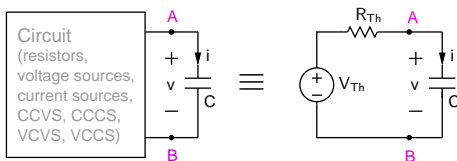


## RC circuits with DC sources (continued)



- \* If all sources are DC (constant), we have  $v(t) = A \exp(-t/\tau) + B$ ,  $\tau = RC$ .

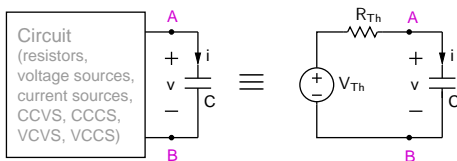
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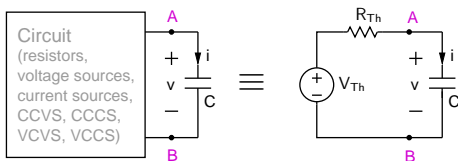
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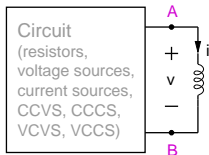
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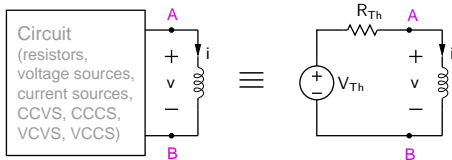


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- \* As  $t \rightarrow \infty$ ,  $i \rightarrow 0$ , i.e., the capacitor behaves like an open circuit since all derivatives vanish.
- \* Since the circuit in the black box is linear, any variable (current or voltage) in the circuit can be expressed as  $x(t) = K_1 \exp(-t/\tau) + K_2$ , where  $K_1$  and  $K_2$  can be obtained from suitable conditions on  $x(t)$ .

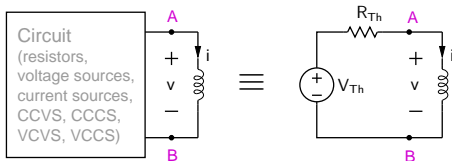
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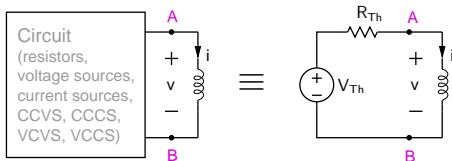


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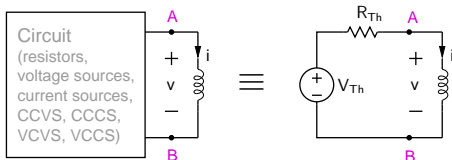


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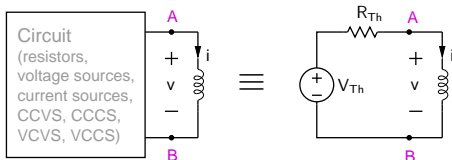
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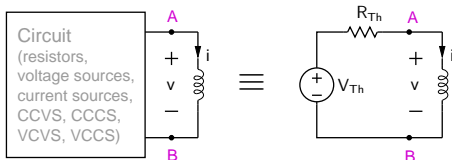
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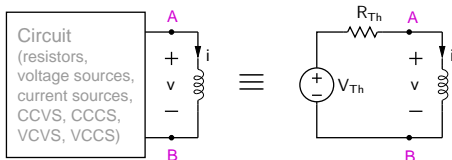
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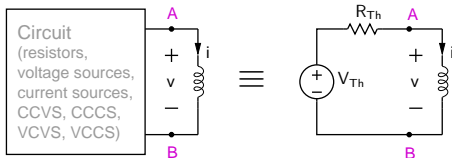
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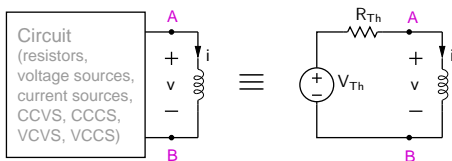
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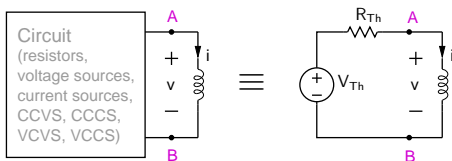
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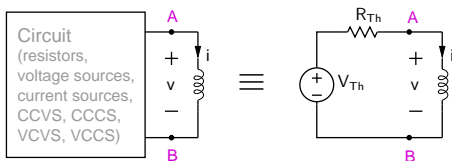
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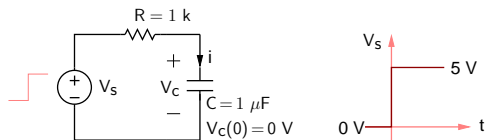
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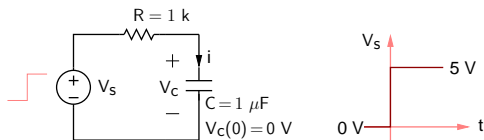
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- \* Since the circuit in the black box is linear, any variable (current or voltage) in the circuit can be expressed as  
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where  $K_1$  and  $K_2$  can be obtained from suitable conditions on  $x(t)$ .



## RC circuits: Can $V_C$ change “suddenly?”

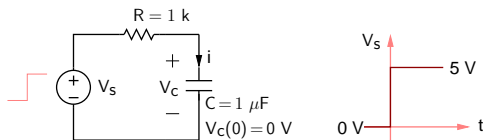


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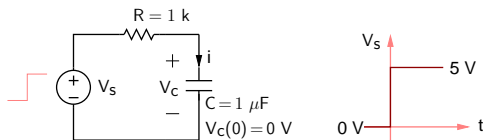
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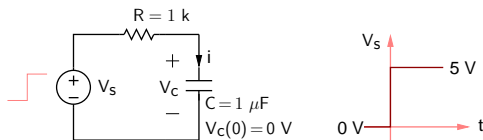
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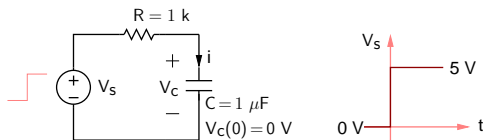
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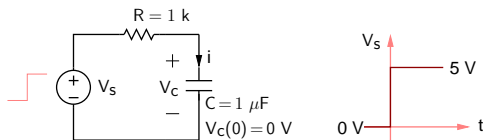
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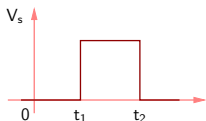
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- \* Similarly, an inductor does not allow abrupt changes in  $i_L$ .

## Analysis of $RC/RL$ circuits with a piece-wise constant source

- \* Identify intervals in which the source voltages/currents are constant.  
For example,



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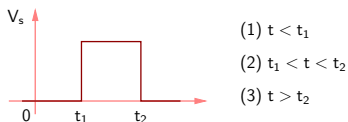
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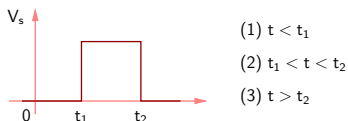
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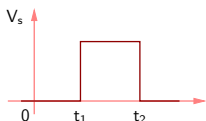
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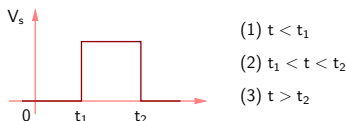
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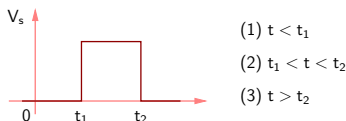
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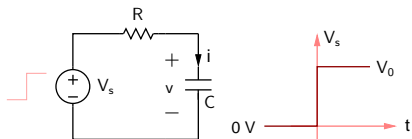
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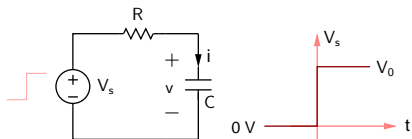


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- \* Compute  $A_1, B_1, \dots$  using the conditions on  $x(t)$ .

## RC circuits: charging and discharging transients

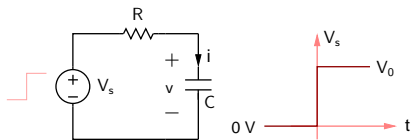


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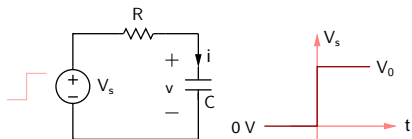
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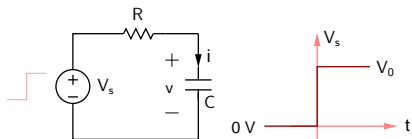
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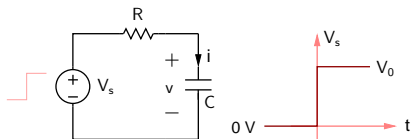
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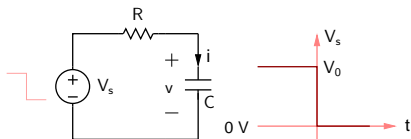
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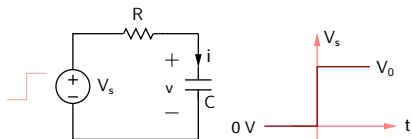
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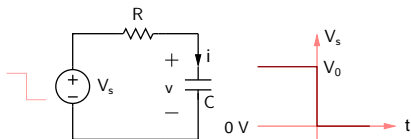
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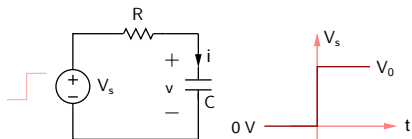
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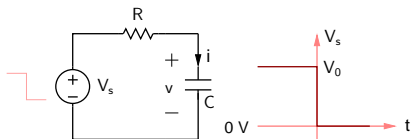
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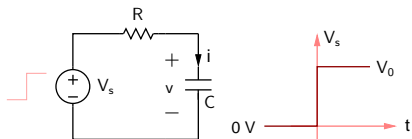
$$(1) \quad v(0^-) = V_S(0^-) = V_0$$

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$$(2) \quad \text{As } t \rightarrow \infty, i \rightarrow 0 \rightarrow v(\infty) = V_S(\infty) = 0 \text{ V}$$

## RC circuits: charging and discharging transients



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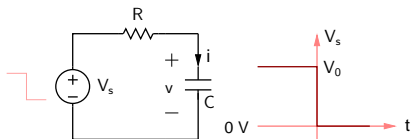
Imposing (1) and (2) on Eq. (A), we get

$$t = 0^+: 0 = A + B,$$

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$$\text{i.e., } A = V_0, B = -V_0$$

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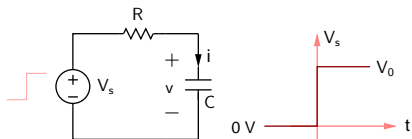
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## RC circuits: charging and discharging transients



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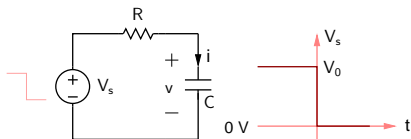
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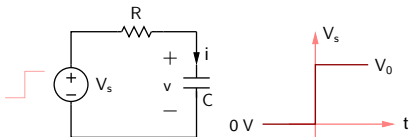
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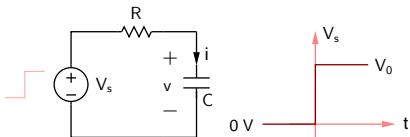
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Compute  $i(t)$ ,  $t > 0$ .



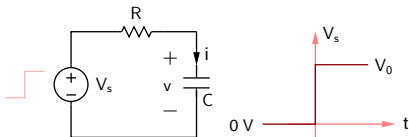
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Compute  $i(t)$ ,  $t > 0$ .

$$\begin{aligned} \text{(A) } i(t) &= C \frac{d}{dt} V_0 [1 - \exp(-t/\tau)] \\ &= \frac{CV_0}{\tau} \exp(-t/\tau) = \frac{V_0}{R} \exp(-t/\tau) \end{aligned}$$

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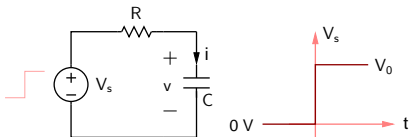
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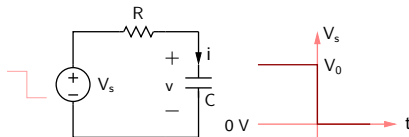
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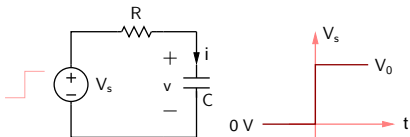
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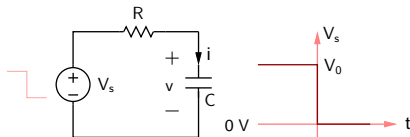
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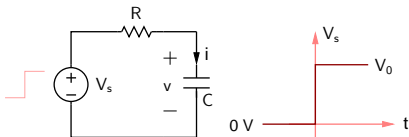
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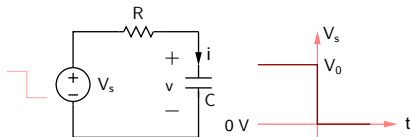
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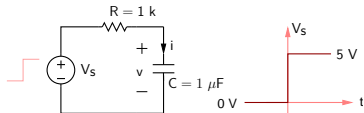
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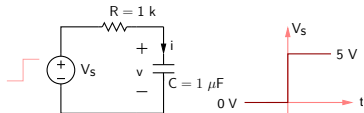
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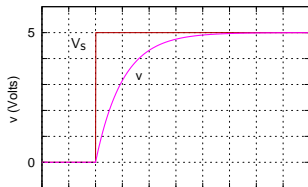
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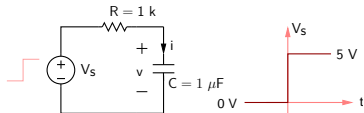


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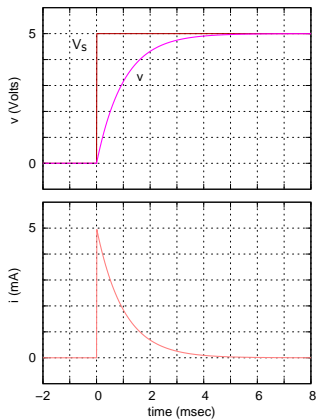


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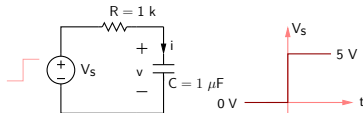
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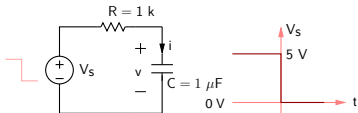
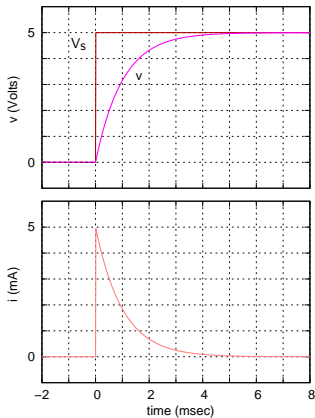


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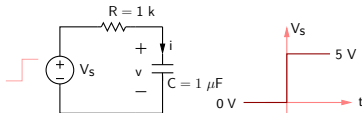
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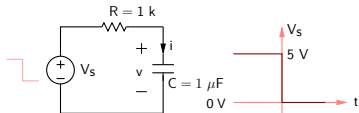
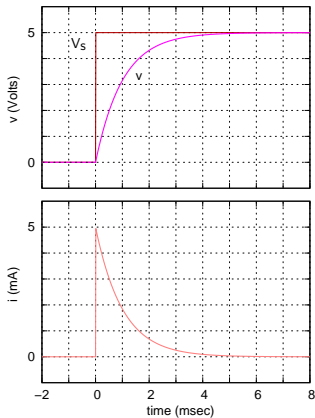
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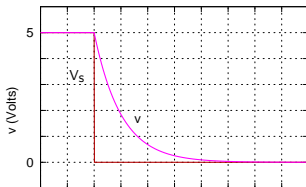
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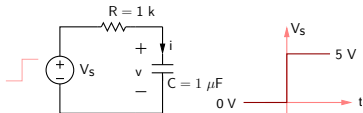


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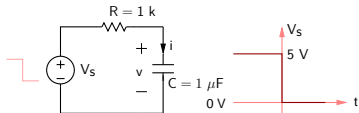
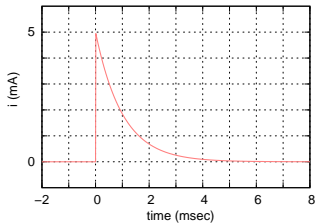
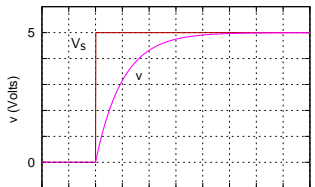


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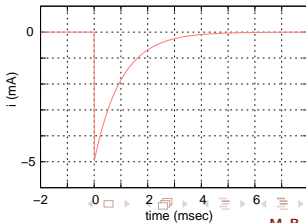
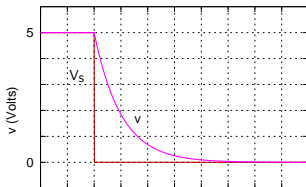
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## Significance of the time constant ( $\tau$ )

| $x$ | $e^{-x}$                | $1 - e^{-x}$ |
|-----|-------------------------|--------------|
| 0.0 | 1.0                     | 0.0          |
| 1.0 | 0.3679                  | 0.6321       |
| 2.0 | 0.1353                  | 0.8647       |
| 3.0 | $4.9787 \times 10^{-2}$ | 0.9502       |
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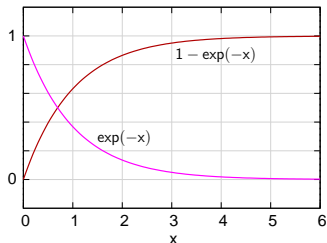
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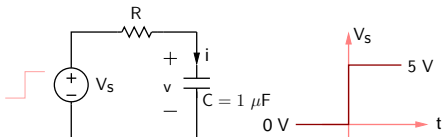
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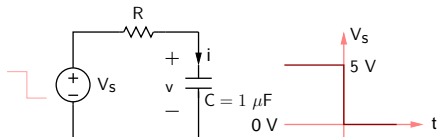
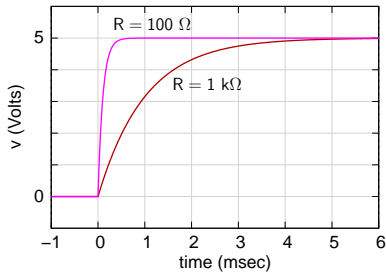


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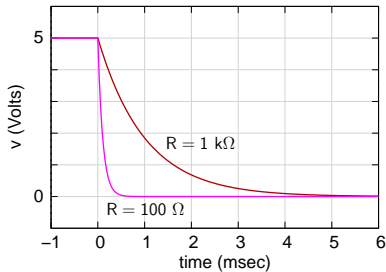
## RC circuits: charging and discharging transients



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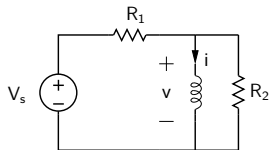


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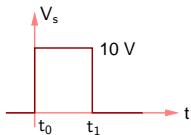




## RL circuit: example



$i(0) = 0$  A, Find  $i(t)$ .



$$R_1 = 10 \Omega$$

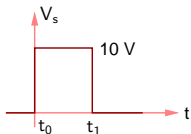
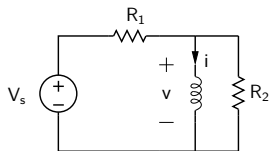
$$R_2 = 40 \Omega$$

$$L = 0.8 \text{ H}$$

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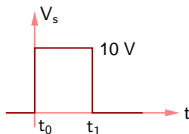
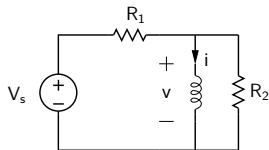
There are three intervals of constant  $V_s$ :

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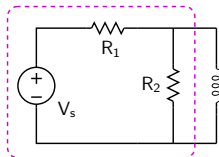
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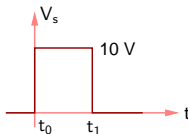
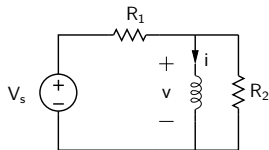
$R_{Th}$  seen by  $L$  is the same in all intervals:



$$R_{Th} = R_1 \parallel R_2 = 8 \Omega$$

$$\begin{aligned}\tau &= L/R_{Th} \\ &= 0.8 \text{ H}/8 \Omega \\ &= 0.1 \text{ s}\end{aligned}$$

## RL circuit: example



$$R_1 = 10 \Omega$$

$$R_2 = 40 \Omega$$

$$L = 0.8 \text{ H}$$

$$t_0 = 0$$

$$t_1 = 0.1 \text{ s}$$

$i(0) = 0 \text{ A}$ , Find  $i(t)$ .

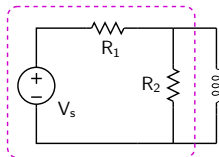
There are three intervals of constant  $V_s$ :

(1)  $t < t_0$

(2)  $t_0 < t < t_1$

(3)  $t > t_1$

$R_{Th}$  seen by  $L$  is the same in all intervals:



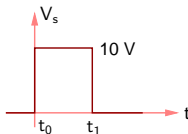
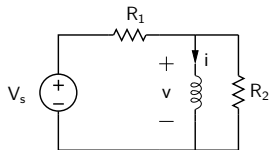
$$R_{Th} = R_1 \parallel R_2 = 8 \Omega$$

$$\begin{aligned} \tau &= L/R_{Th} \\ &= 0.8 \text{ H}/8 \Omega \\ &= 0.1 \text{ s} \end{aligned}$$

At  $t = t_0^-$ ,  $v = 0 \text{ V}$ ,  $V_s = 0 \text{ V}$ .

$\Rightarrow i(t_0^-) = 0 \text{ A} \Rightarrow i(t_0^+) = 0 \text{ A}$ .

## RL circuit: example



$$R_1 = 10 \Omega$$

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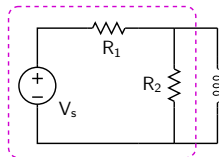
There are three intervals of constant  $V_s$ :

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(2)  $t_0 < t < t_1$

(3)  $t > t_1$

$R_{Th}$  seen by  $L$  is the same in all intervals:



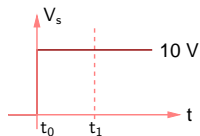
$$R_{Th} = R_1 \parallel R_2 = 8 \Omega$$

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At  $t = t_0^-$ ,  $v = 0 \text{ V}$ ,  $V_s = 0 \text{ V}$ .

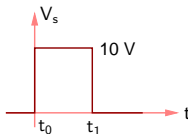
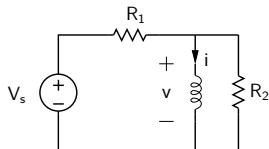
$\Rightarrow i(t_0^-) = 0 \text{ A} \Rightarrow i(t_0^+) = 0 \text{ A}$ .

If  $V_s$  did not change at  $t = t_1$ ,  
we would have



$v(\infty) = 0 \text{ V}$ ,  $i(\infty) = 10 \text{ V}/10 \Omega = 1 \text{ A}$ .

## RL circuit: example



$$R_1 = 10 \Omega$$

$$R_2 = 40 \Omega$$

$$L = 0.8 \text{ H}$$

$$t_0 = 0$$

$$t_1 = 0.1 \text{ s}$$

$i(0) = 0 \text{ A}$ , Find  $i(t)$ .

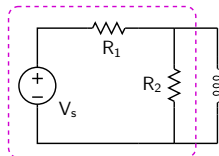
There are three intervals of constant  $V_s$ :

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(3)  $t > t_1$

$R_{Th}$  seen by  $L$  is the same in all intervals:

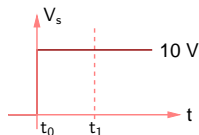


$$R_{Th} = R_1 \parallel R_2 = 8 \Omega$$

$$\begin{aligned} \tau &= L/R_{Th} \\ &= 0.8 \text{ H}/8 \Omega \\ &= 0.1 \text{ s} \end{aligned}$$

At  $t = t_0^-$ ,  $v = 0 \text{ V}$ ,  $V_s = 0 \text{ V}$ .  
 $\Rightarrow i(t_0^-) = 0 \text{ A} \Rightarrow i(t_0^+) = 0 \text{ A}$ .

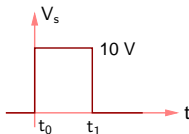
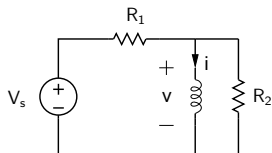
If  $V_s$  did not change at  $t = t_1$ ,  
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$v(\infty) = 0 \text{ V}$ ,  $i(\infty) = 10 \text{ V}/10 \Omega = 1 \text{ A}$ .

Using  $i(t_0^+)$  and  $i(\infty)$ , we can obtain  
 $i(t)$ ,  $t > 0$  (See next slide).

## RL circuit: example



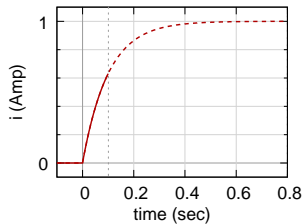
$$R_1 = 10\ \Omega$$

$$R_2 = 40\ \Omega$$

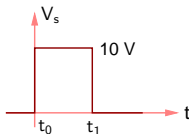
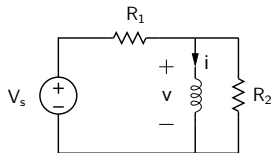
$$L = 0.8\ \text{H}$$

$$t_0 = 0$$

$$t_1 = 0.1\ \text{s}$$



## RL circuit: example



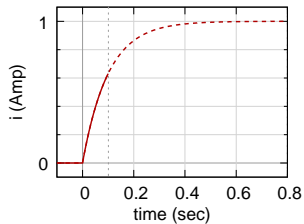
$$R_1 = 10 \Omega$$

$$R_2 = 40 \Omega$$

$$L = 0.8 \text{ H}$$

$$t_0 = 0$$

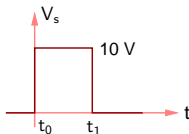
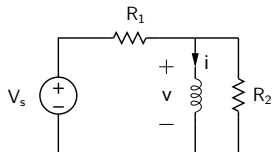
$$t_1 = 0.1 \text{ s}$$



In reality,  $V_s$  changes at  $t = t_1$ ,  
and we need to work out the  
solution for  $t > t_1$  separately.



## RL circuit: example



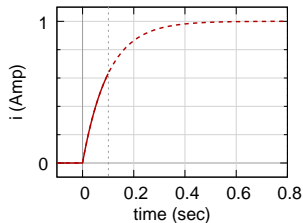
$$R_1 = 10 \Omega$$

$$R_2 = 40 \Omega$$

$$L = 0.8 \text{ H}$$

$$t_0 = 0$$

$$t_1 = 0.1 \text{ s}$$

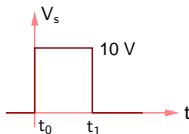
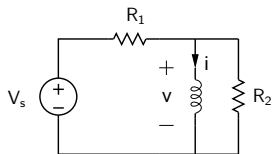


For  $t_0 < t < t_1$ ,  $i(t) = 1 - \exp(-t/\tau)$  Amp.

Consider  $t > t_1$ .

In reality,  $V_s$  changes at  $t = t_1$ ,  
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## RL circuit: example



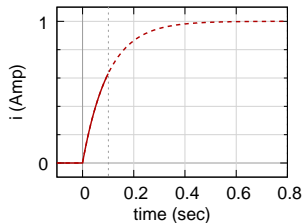
$$R_1 = 10 \Omega$$

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$$t_0 = 0$$

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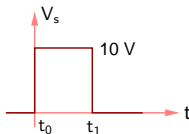
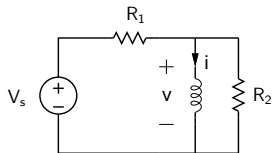
Consider  $t > t_1$ .

$$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632 \text{ A (Note: } t_1/\tau = 1).$$

$$i(\infty) = 0 \text{ A.}$$

In reality,  $V_s$  changes at  $t = t_1$ ,  
and we need to work out the  
solution for  $t > t_1$  separately.

## RL circuit: example



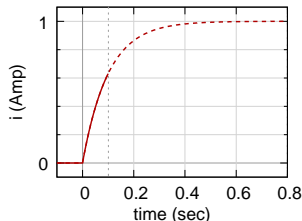
$$R_1 = 10 \Omega$$

$$R_2 = 40 \Omega$$

$$L = 0.8 \text{ H}$$

$$t_0 = 0$$

$$t_1 = 0.1 \text{ s}$$



In reality,  $V_s$  changes at  $t = t_1$ ,  
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For  $t_0 < t < t_1$ ,  $i(t) = 1 - \exp(-t/\tau)$  Amp.

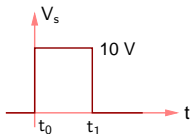
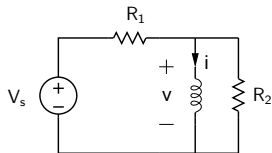
Consider  $t > t_1$ .

$$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632 \text{ A (Note: } t_1/\tau = 1).$$

$$i(\infty) = 0 \text{ A.}$$

$$\text{Let } i(t) = A \exp(-t/\tau) + B.$$

## RL circuit: example



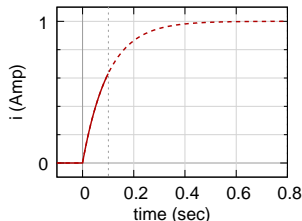
$$R_1 = 10 \Omega$$

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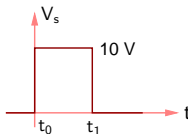
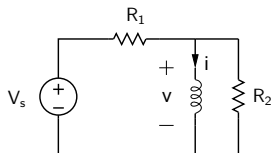
$$i(\infty) = 0 \text{ A.}$$

$$\text{Let } i(t) = A \exp(-t/\tau) + B.$$

It is convenient to rewrite  $i(t)$  as

$$i(t) = A' \exp[-(t - t_1)/\tau] + B.$$

## RL circuit: example



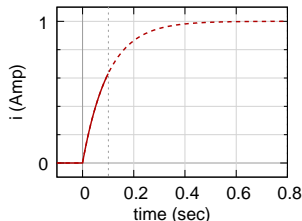
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In reality,  $V_s$  changes at  $t = t_1$ , and we need to work out the solution for  $t > t_1$  separately.

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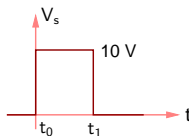
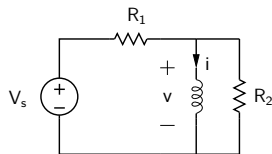
It is convenient to rewrite  $i(t)$  as

$$i(t) = A' \exp[-(t - t_1)/\tau] + B.$$

Using  $i(t_1^+)$  and  $i(\infty)$ , we get

$$i(t) = 0.693 \exp[-(t - t_1)/\tau] \text{ A.}$$

## RL circuit: example



$$R_1 = 10 \Omega$$

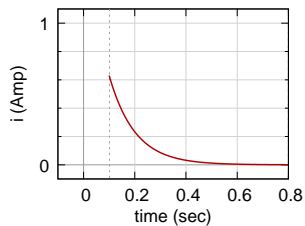
$$R_2 = 40 \Omega$$

$$L = 0.8 \text{ H}$$

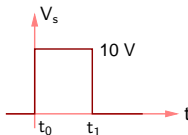
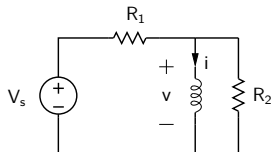
$$t_0 = 0$$

$$t_1 = 0.1 \text{ s}$$

$$i(t) = 0.693 \exp[-(t - t_1)/\tau] \text{ A.}$$



## RL circuit: example



$$R_1 = 10 \Omega$$

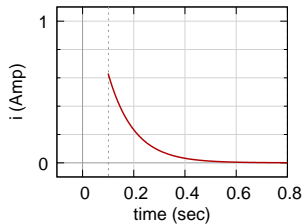
$$R_2 = 40 \Omega$$

$$L = 0.8 \text{ H}$$

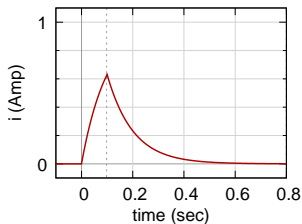
$$t_0 = 0$$

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$$i(t) = 0.693 \exp[-(t - t_1)/\tau] \text{ A.}$$

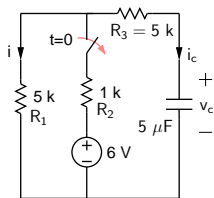


Combining the solutions for  $t_0 < t < t_1$  and  $t > t_1$ , we get



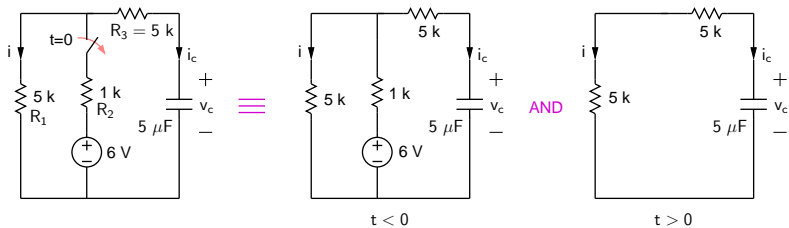
(SEQUEL file: ee101\_rl1.sqproj)

## RC circuit: example

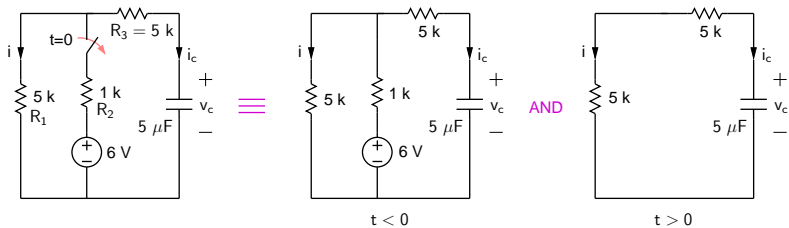




## RC circuit: example

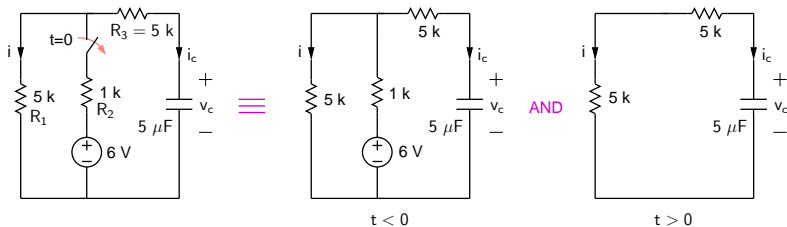


## RC circuit: example



$t = 0^-$ : capacitor is an open circuit,  $\Rightarrow i(0^-) = 6\text{ V} / (5\text{ k} + 1\text{ k}) = 1\text{ mA}$ .

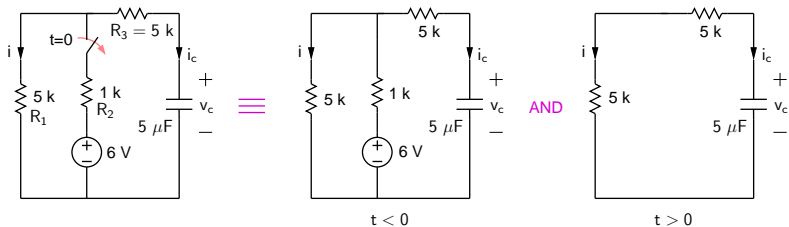
## RC circuit: example



$t = 0^-$ : capacitor is an open circuit,  $\Rightarrow i(0^-) = 6\text{ V} / (5\text{ k} + 1\text{ k}) = 1\text{ mA}$ .

$v_c(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_c(0^+) = 5\text{ V}$ .

## RC circuit: example

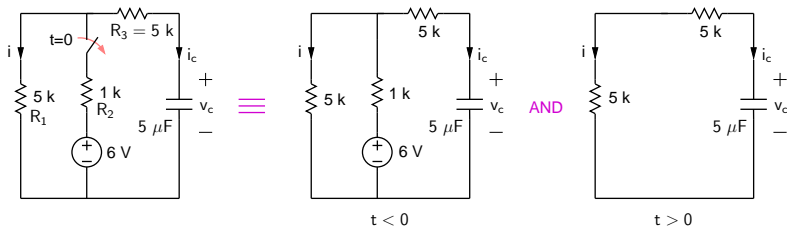


$t = 0^-$ : capacitor is an open circuit,  $\Rightarrow i(0^-) = 6 \text{ V} / (5 \text{ k} + 1 \text{ k}) = 1 \text{ mA}$ .

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$\Rightarrow i(0^+) = 5 \text{ V} / (5 \text{ k} + 5 \text{ k}) = 0.5 \text{ mA}$ .

## RC circuit: example



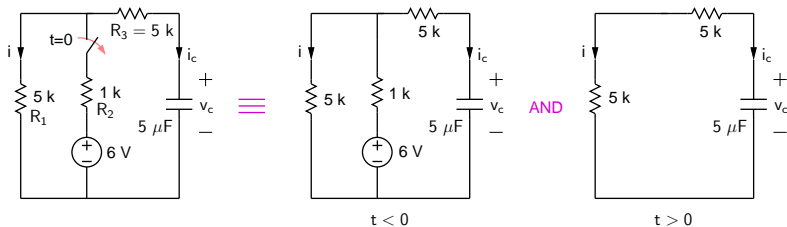
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$v_c(0^-) = 6 \text{ V} - 1 \text{ mA} \times R_2 = 5 \text{ V} \Rightarrow v_c(0^+) = 5 \text{ V}$ .

$\Rightarrow i(0^+) = 5 \text{ V} / (5 \text{ k} + 5 \text{ k}) = 0.5 \text{ mA}$ .

Let  $i(t) = A \exp(-t/\tau) + B$  for  $t > 0$ , with  $\tau = 10 \text{ k} \times 5 \mu\text{F} = 50 \text{ ms}$ .

## RC circuit: example



$t = 0^-$ : capacitor is an open circuit,  $\Rightarrow i(0^-) = 6\text{ V} / (5\text{ k} + 1\text{ k}) = 1\text{ mA}$ .

$v_c(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_c(0^+) = 5\text{ V}$ .

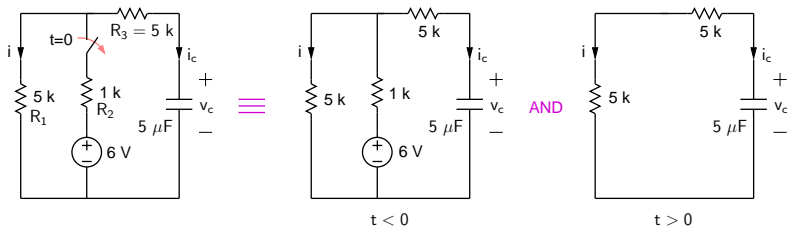
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Using  $i(0^+)$  and  $i(\infty) = 0\text{ A}$ , we get

$i(t) = 0.5 \exp(-t/\tau)\text{ mA}$ .

## RC circuit: example



$t = 0^-$ : capacitor is an open circuit,  $\Rightarrow i(0^-) = 6 \text{ V} / (5 \text{ k} + 1 \text{ k}) = 1 \text{ mA}$ .

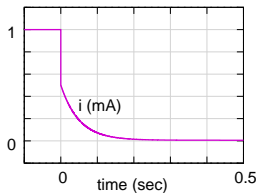
$v_c(0^-) = 6 \text{ V} - 1 \text{ mA} \times R_2 = 5 \text{ V} \Rightarrow v_c(0^+) = 5 \text{ V}$ .

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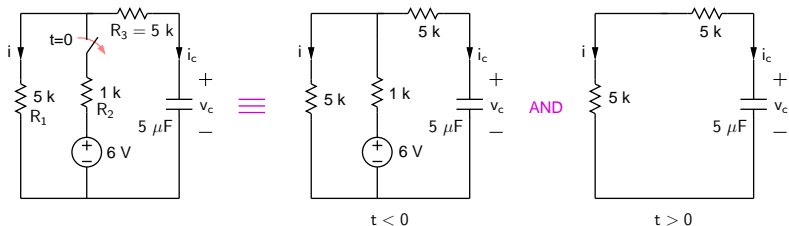
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## RC circuit: example



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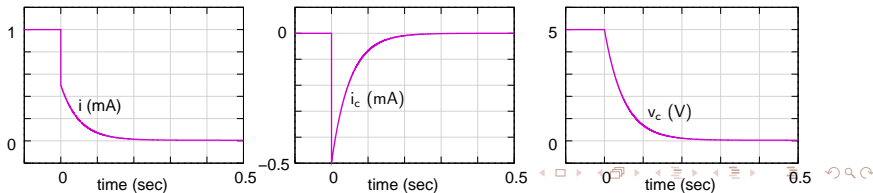
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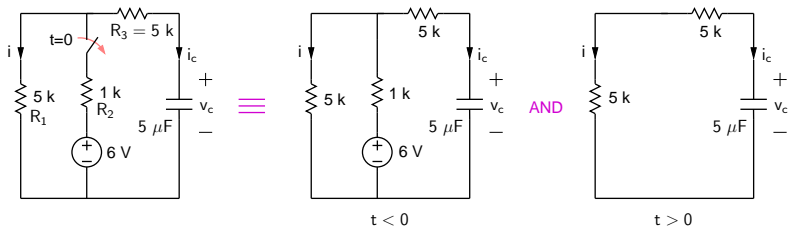
Using  $i(0^+)$  and  $i(\infty) = 0 \text{ A}$ , we get

$i(t) = 0.5 \exp(-t/\tau) \text{ mA}$ .





## RC circuit: example



$t = 0^-$ : capacitor is an open circuit,  $\Rightarrow i(0^-) = 6 \text{ V} / (5 \text{ k} + 1 \text{ k}) = 1 \text{ mA}$ .

$v_c(0^-) = 6 \text{ V} - 1 \text{ mA} \times R_2 = 5 \text{ V} \Rightarrow v_c(0^+) = 5 \text{ V}$ .

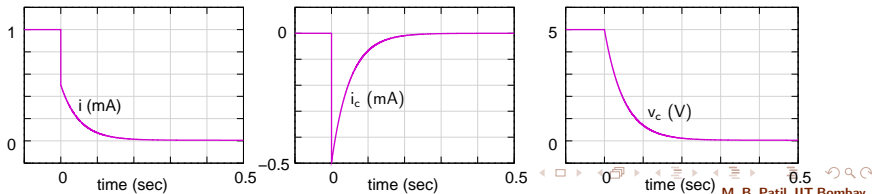
$\Rightarrow i(0^+) = 5 \text{ V} / (5 \text{ k} + 5 \text{ k}) = 0.5 \text{ mA}$ .

Let  $i(t) = A \exp(-t/\tau) + B$  for  $t > 0$ , with  $\tau = 10 \text{ k} \times 5 \mu\text{F} = 50 \text{ ms}$ .

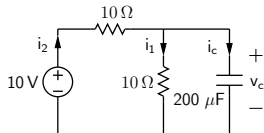
Using  $i(0^+)$  and  $i(\infty) = 0 \text{ A}$ , we get

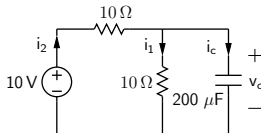
$i(t) = 0.5 \exp(-t/\tau) \text{ mA}$ .

(SEQUEL file: ee101\_rc2.sqproj)

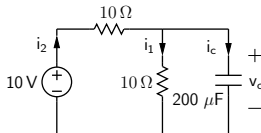


## RC circuits: home work

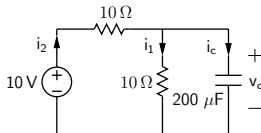




- \* Given  $v_c(0) = 0$  V, find  $v_c(t)$  for  $t > 0$ . Using this  $v_c(t)$ , find  $i_1$ ,  $i_2$ ,  $i_c$  for  $t > 0$ . Plot  $v_c$ ,  $i_1$ ,  $i_2$ ,  $i_c$  versus  $t$ .

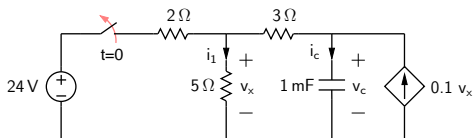


- \* Given  $v_c(0) = 0$  V, find  $v_c(t)$  for  $t > 0$ . Using this  $v_c(t)$ , find  $i_1$ ,  $i_2$ ,  $i_c$  for  $t > 0$ . Plot  $v_c$ ,  $i_1$ ,  $i_2$ ,  $i_c$  versus  $t$ .
- \* Find  $i_1$ ,  $i_2$ ,  $i_c$  directly (i.e., without getting  $v_c$ ) by finding the initial and final conditions for each of them ( $i_1(0^+)$  and  $i_1(\infty)$ , etc.) and then using them to compute the coefficients in the general expression,  $x(t) = A \exp(-t/\tau) + B$ .

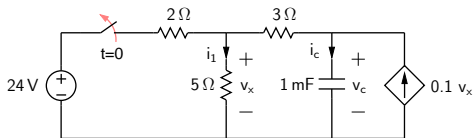


- \* Given  $v_c(0) = 0$  V, find  $v_c(t)$  for  $t > 0$ . Using this  $v_c(t)$ , find  $i_1$ ,  $i_2$ ,  $i_c$  for  $t > 0$ . Plot  $v_c$ ,  $i_1$ ,  $i_2$ ,  $i_c$  versus  $t$ .
- \* Find  $i_1$ ,  $i_2$ ,  $i_c$  directly (i.e., without getting  $v_c$ ) by finding the initial and final conditions for each of them ( $i_1(0^+)$  and  $i_1(\infty)$ , etc.) and then using them to compute the coefficients in the general expression,  $x(t) = A \exp(-t/\tau) + B$ .
- \* Verify your results with SEQUEL (file: ee101\_rc3.sqproj).

## RC circuits: home work

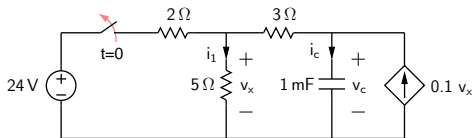


## RC circuits: home work



\* Find  $v_c(0^-)$ ,  $v_c(\infty)$ .

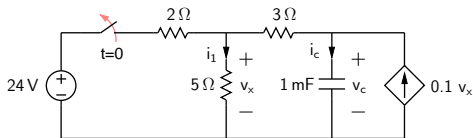
## RC circuits: home work



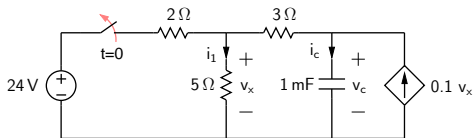
- \* Find  $v_c(0^-)$ ,  $v_c(\infty)$ .
- \* Find  $R_{Th}$  as seen by the capacitor for  $t > 0$ .



## RC circuits: home work

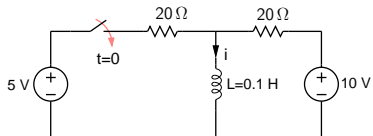


- \* Find  $v_c(0^-)$ ,  $v_c(\infty)$ .
- \* Find  $R_{Th}$  as seen by the capacitor for  $t > 0$ .
- \* Solve for  $v_c(t)$  and  $i_1(t)$ ,  $t > 0$ .

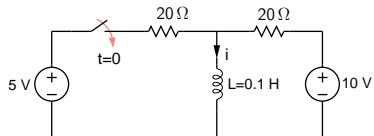


- \* Find  $v_c(0^-)$ ,  $v_c(\infty)$ .
- \* Find  $R_{Th}$  as seen by the capacitor for  $t > 0$ .
- \* Solve for  $v_c(t)$  and  $i_1(t)$ ,  $t > 0$ .
- \* Verify your results with SEQUEL (file: ee101\_rc4.sqproj).

## RL circuits: home work

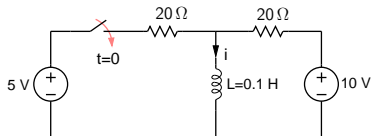


## RL circuits: home work



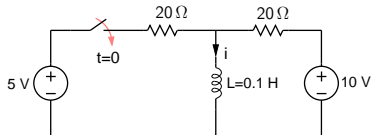
\* Find  $i(0^-)$ ,  $i(\infty)$ .

## RL circuits: home work

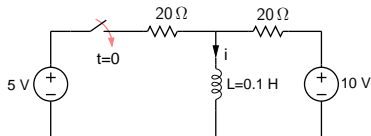


- \* Find  $i(0^-)$ ,  $i(\infty)$ .
- \* Find  $R_{Th}$  as seen by the inductor for  $t > 0$ .

## RL circuits: home work



- \* Find  $i(0^-)$ ,  $i(\infty)$ .
- \* Find  $R_{Th}$  as seen by the inductor for  $t > 0$ .
- \* Solve for  $i(t)$ ,  $t > 0$ .



- \* Find  $i(0^-)$ ,  $i(\infty)$ .
- \* Find  $R_{Th}$  as seen by the inductor for  $t > 0$ .
- \* Solve for  $i(t)$ ,  $t > 0$ .
- \* Verify your results with SEQUEL (file: ee101\_r12.sqproj).