

EE101: Resonance in RLC circuits



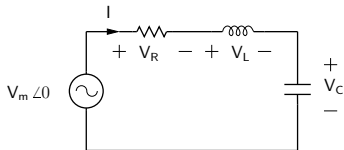
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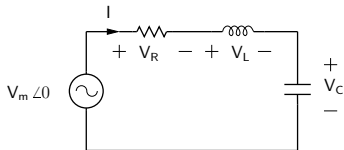
Resonance in series RLC circuits



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta, \text{ where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1} \left[\frac{\omega L - 1/\omega C}{R} \right].$$

Resonance in series RLC circuits

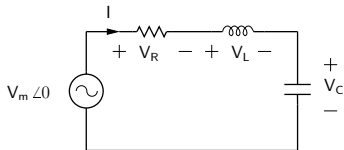


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* As ω is varied, both I_m and θ change.

Resonance in series RLC circuits

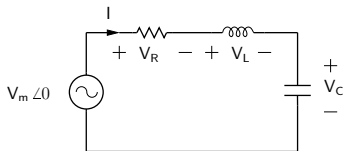


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- * When $\omega L = 1/\omega C$, I_m reaches its maximum value, $I_m^{max} = V_m/R$, and θ becomes 0, i.e., the current \mathbf{I} is *in phase* with the applied voltage.

Resonance in series RLC circuits



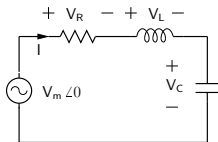
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- * When $\omega L = 1/\omega C$, I_m reaches its maximum value, $I_m^{max} = V_m/R$, and θ becomes 0, i.e., the current \mathbf{I} is *in phase* with the applied voltage.
- * The above condition is called “resonance,” and the corresponding frequency is called the “resonance frequency” (ω_0).

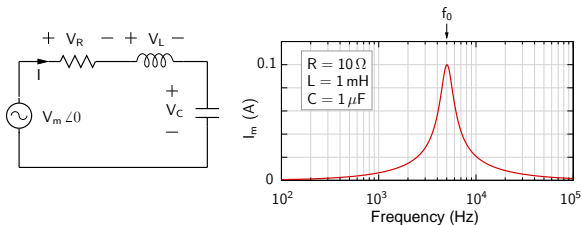
$$\omega_0 = 1/\sqrt{LC}$$

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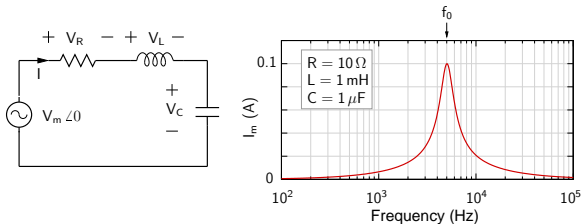
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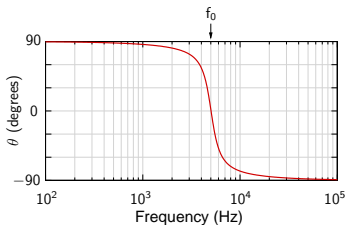
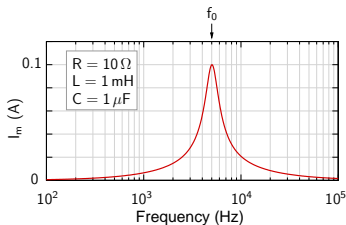
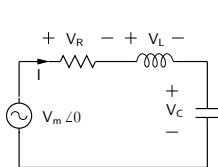
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- * As $\omega \rightarrow 0$, the term $1/\omega C$ dominates, and $\theta \rightarrow \pi/2$.

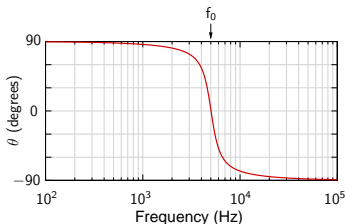
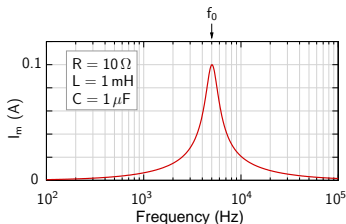
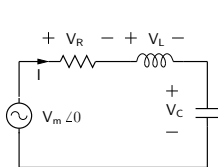
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Resonance in series RLC circuits

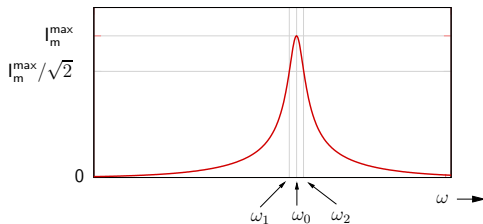
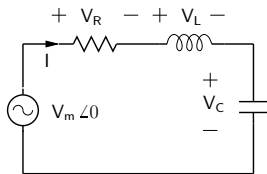


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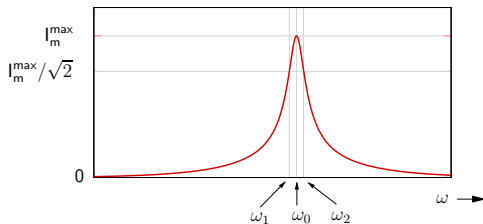
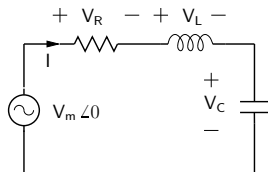
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Resonance in series RLC circuits



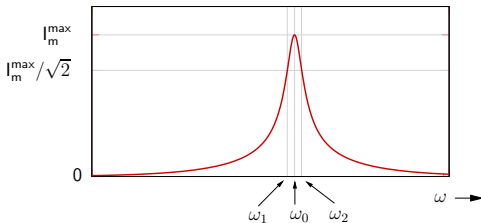
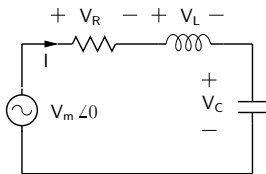
Resonance in series RLC circuits



* The maximum power that can be absorbed by the resistor is

$$P_{\max} = \frac{1}{2} (I_m^{\max})^2 R = \frac{1}{2} V_m^2 / R.$$

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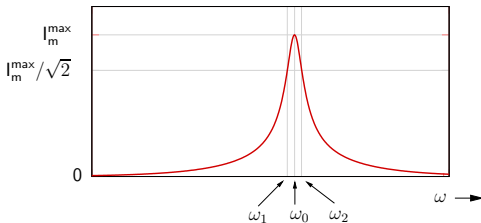
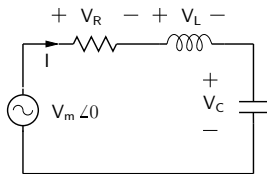


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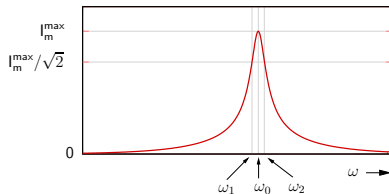
- * Define ω_1 and ω_2 (see figure) as frequencies at which $I_m = I_m^{\max} / \sqrt{2}$, i.e., the power absorbed by R is $P_{\max} / 2$.
- * The *bandwidth* of a resonant circuit is defined as $B = \omega_2 - \omega_1$, and the *quality factor* as $Q = \omega_0 / B$. Quality is a measure of the sharpness of the I_m versus frequency relationship.

Resonance in series RLC circuits

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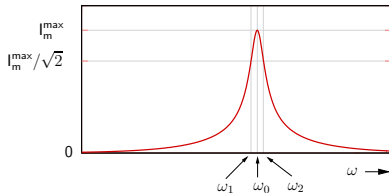
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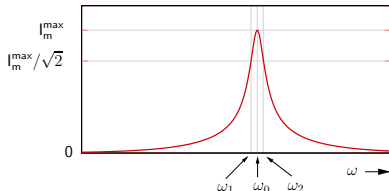
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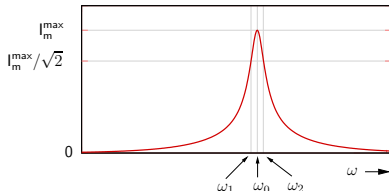


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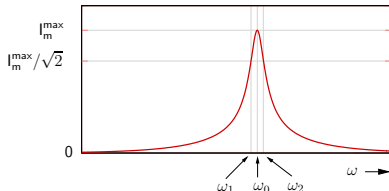
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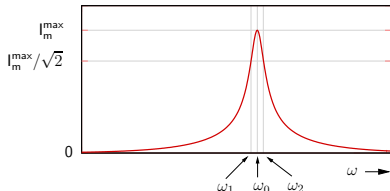
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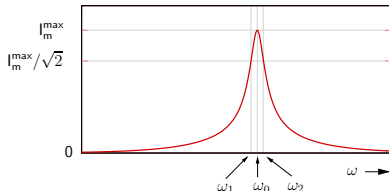
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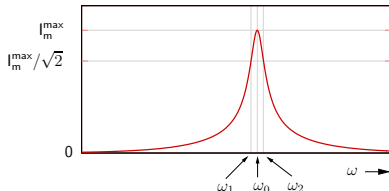
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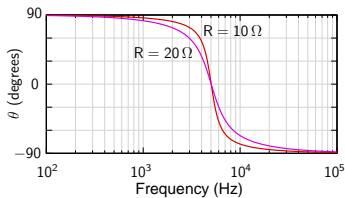
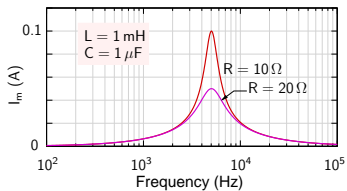
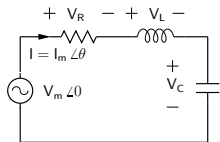
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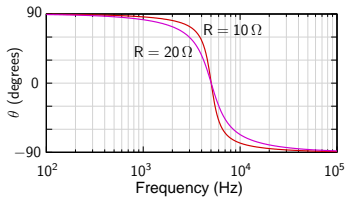
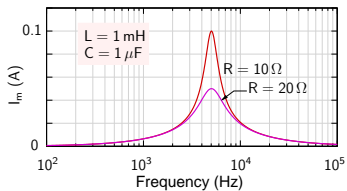
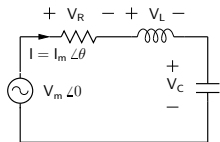
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- * Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$.

Resonance in series RLC circuits



As R is increased,

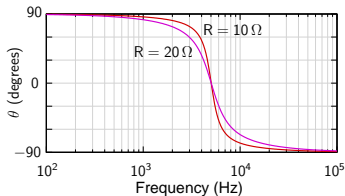
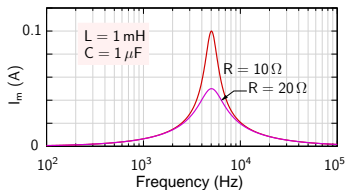
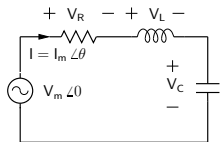
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As R is increased,

- * The quality factor $Q = \omega_0 L / R$ decreases, i.e., I_m versus ω curve becomes broader.

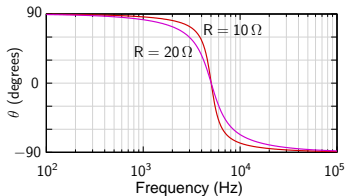
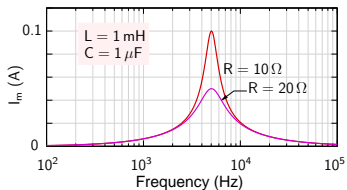
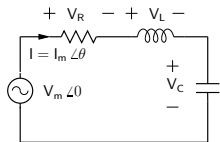
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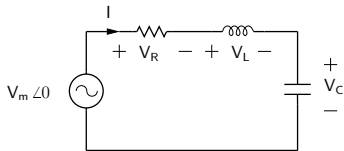
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- * The maximum current (at $\omega = \omega_0$) decreases (since $I_m^{max} = V_m / R$).
- * The resonance frequency ($\omega_0 = 1 / \sqrt{LC}$) is not affected.

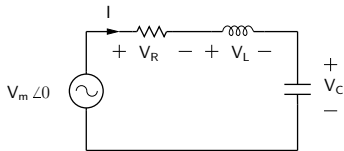
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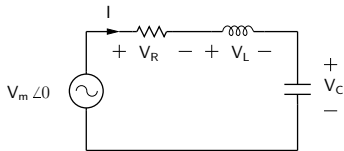


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- * For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.

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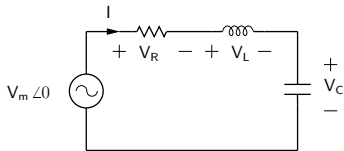


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$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1} \left[\frac{\omega L - 1/\omega C}{R} \right].$$

- * For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.
- * For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.

Resonance in series RLC circuits

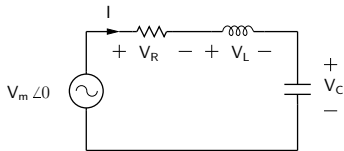


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- * For $\omega > \omega_0$, $\omega L > 1/\omega C$, the net impedance is inductive, and the current lags the applied voltage.

Resonance in series RLC circuits

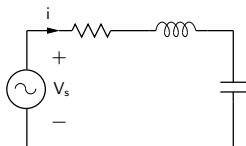


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- * Let us look at an example (next slide).

Resonance in series RLC circuits



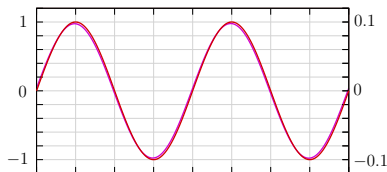
$$R = 10\ \Omega$$

$$L = 1\ \text{mH}$$

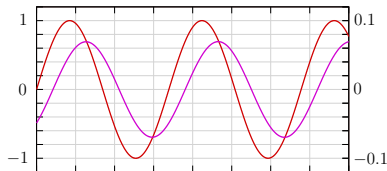
$$C = 1\ \mu\text{F}$$



$$f = 4.3\ \text{kHz}$$



$$f = 5\ \text{kHz} \approx f_0$$

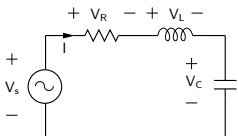


$$f = 5.9\ \text{kHz}$$

— V_s (V) (left axis)

— i (A) (right axis)

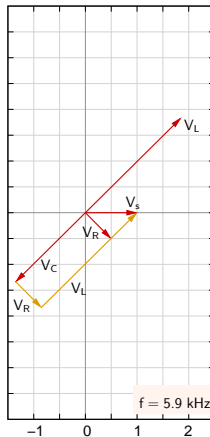
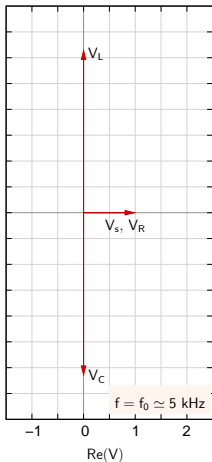
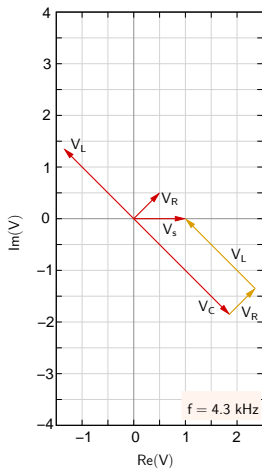
Resonance in series RLC circuits: phasor diagrams



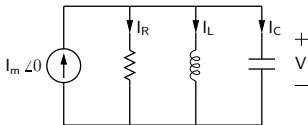
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Resonance in parallel RLC circuits

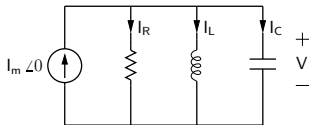


$$I_m \angle 0 = \mathbf{Y} \mathbf{V}, \text{ where } \mathbf{Y} = G + j\omega C + 1/j\omega L \quad (G = 1/R).$$

$$\mathbf{V} = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta, \text{ where}$$

$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1} \left[\frac{\omega C - 1/\omega L}{G} \right].$$

Resonance in parallel RLC circuits



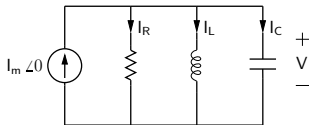
$I_m \angle 0 = \mathbf{Y} \mathbf{V}$, where $\mathbf{Y} = G + j\omega C + 1/j\omega L$ ($G = 1/R$).

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* As ω is varied, both V_m and θ change.

Resonance in parallel RLC circuits



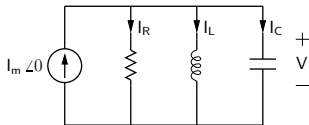
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- * When $\omega C = 1/\omega L$, V_m reaches its maximum value, $V_m^{max} = I_m/G = I_m R$, and θ becomes 0, i.e., the voltage \mathbf{V} is *in phase* with the source current.

Resonance in parallel RLC circuits



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- * The above condition is called “resonance,” and the corresponding frequency is called the “resonance frequency” (ω_0).

$$\omega_0 = 1/\sqrt{LC}$$

Resonance in parallel RLC circuits

Series RLC circuit: $I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$, $\theta = -\tan^{-1} \left[\frac{\omega L - 1/\omega C}{R} \right]$.

Parallel RLC circuit: $V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}$, $\theta = -\tan^{-1} \left[\frac{\omega C - 1/\omega L}{G} \right]$.

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- * The two situations are identical if we make the following substitutions:

$$\mathbf{I} \leftrightarrow \mathbf{V},$$

$$R \leftrightarrow 1/R,$$

$$L \leftrightarrow C.$$

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- * Show that $\omega_{1,2} = \mp \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$
 \Rightarrow Bandwidth $B = 1/RC$.

Resonance in parallel RLC circuits

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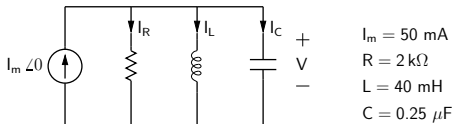
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- * Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$.

Resonance in parallel RLC circuits: home work



- * Calculate ω_0 , f_0 , B , Q .
- * Calculate I_R , I_L , I_C at $\omega = \omega_0$, ω_1 , ω_2 .
- * Verify graphically that $I_R + I_L + I_C = I_s$ in each case.
- * Plot the power absorbed by R as a function of frequency for $f_0/10 < f < 10 f_0$.