

EE101: Digital circuits (Part 5)



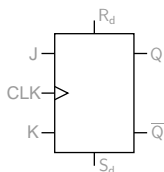
M. B. Patil

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Indian Institute of Technology Bombay

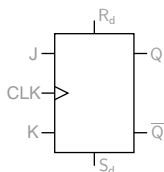
JK flip-flop: asynchronous inputs



S_d	R_d	CLK	J	K	Q_{n+1}
0	1	X	X	X	0
1	0	X	X	X	1
1	1	X	X	X	invalid
0	0	↑	0	0	Q_n
0	0	↑	0	1	0
0	0	↑	1	0	1
0	0	↑	1	1	$\overline{Q_n}$

} normal operation

JK flip-flop: asynchronous inputs

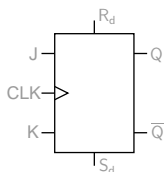


S_d	R_d	CLK	J	K	Q_{n+1}
0	1	X	X	X	0
1	0	X	X	X	1
1	1	X	X	X	invalid
0	0	↑	0	0	Q_n
0	0	↑	0	1	0
0	0	↑	1	0	1
0	0	↑	1	1	$\overline{Q_n}$

} normal operation

- * Clocked flip-flops are also provided with *asynchronous* or *direct* Set and Reset inputs, S_d and R_d , (also called Preset and Clear, respectively) which override all other inputs (J, K, CLK).

JK flip-flop: asynchronous inputs

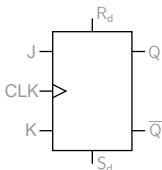


S_d	R_d	CLK	J	K	Q_{n+1}
0	1	X	X	X	0
1	0	X	X	X	1
1	1	X	X	X	invalid
0	0	↑	0	0	Q_n
0	0	↑	0	1	0
0	0	↑	1	0	1
0	0	↑	1	1	$\overline{Q_n}$

} normal operation

- * Clocked flip-flops are also provided with *asynchronous* or *direct* Set and Reset inputs, S_d and R_d , (also called Preset and Clear, respectively) which override all other inputs (J, K, CLK).
- * The S_d and R_d inputs may be active low; in that case, they are denoted by $\overline{S_d}$ and $\overline{R_d}$.

JK flip-flop: asynchronous inputs

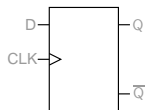


S_d	R_d	CLK	J	K	Q_{n+1}
0	1	X	X	X	0
1	0	X	X	X	1
1	1	X	X	X	invalid
0	0	↑	0	0	Q_n
0	0	↑	0	1	0
0	0	↑	1	0	1
0	0	↑	1	1	$\overline{Q_n}$

} normal operation

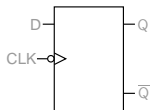
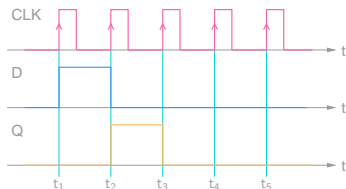
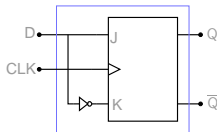
- * Clocked flip-flops are also provided with *asynchronous* or *direct* Set and Reset inputs, S_d and R_d , (also called Preset and Clear, respectively) which override all other inputs (J, K, CLK).
- * The S_d and R_d inputs may be active low; in that case, they are denoted by $\overline{S_d}$ and $\overline{R_d}$.
- * The asynchronous inputs are convenient for “starting up” a circuit in a known state.

D flip-flop



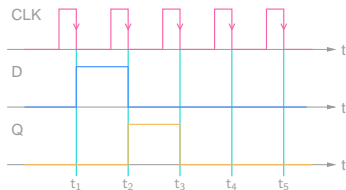
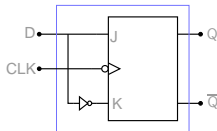
CLK	D	Q_{n+1}
↑	0	0
↑	1	1

positive edge-triggered D flip-flop

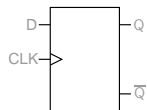


CLK	D	Q_{n+1}
↓	0	0
↓	1	1

negative edge-triggered D flip-flop

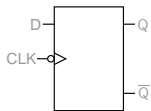
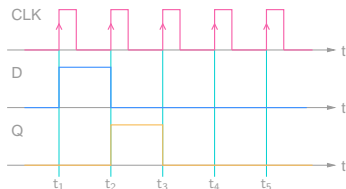
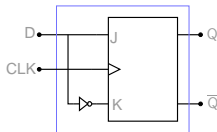


D flip-flop



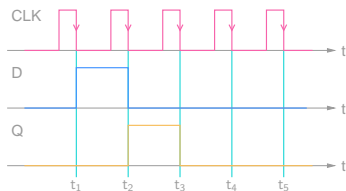
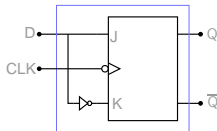
CLK	D	Q_{n+1}
↑	0	0
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positive edge-triggered D flip-flop



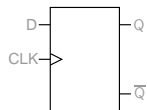
CLK	D	Q_{n+1}
↓	0	0
↓	1	1

negative edge-triggered D flip-flop



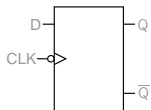
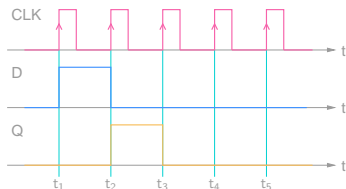
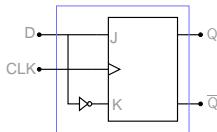
- * The D flip-flop can be used to *delay* the Data (D) signal by one clock period.

D flip-flop



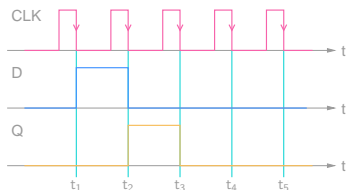
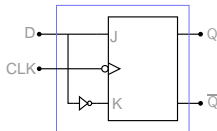
CLK	D	Q_{n+1}
↑	0	0
↑	1	1

positive edge-triggered D flip-flop



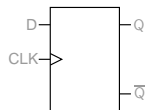
CLK	D	Q_{n+1}
↓	0	0
↓	1	1

negative edge-triggered D flip-flop



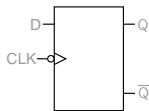
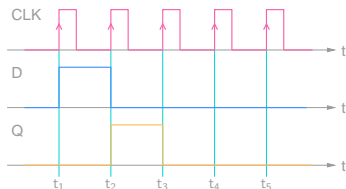
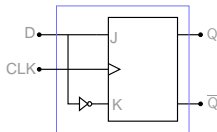
- * The D flip-flop can be used to *delay* the Data (D) signal by one clock period.
- * With $J = D$, $K = \overline{D}$, we have either $J = 0$, $K = 1$ or $J = 1$, $K = 0$; the next Q is 0 in the first case, 1 in the second case.

D flip-flop



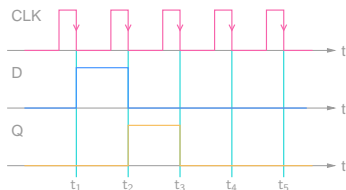
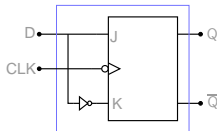
CLK	D	Q_{n+1}
↑	0	0
↑	1	1

positive edge-triggered D flip-flop



CLK	D	Q_{n+1}
↓	0	0
↓	1	1

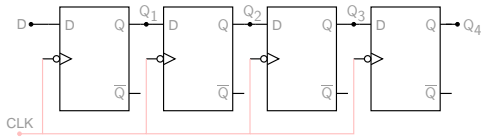
negative edge-triggered D flip-flop



- * The D flip-flop can be used to *delay* the Data (D) signal by one clock period.
- * With $J = D$, $K = \overline{D}$, we have either $J = 0$, $K = 1$ or $J = 1$, $K = 0$; the next Q is 0 in the first case, 1 in the second case.
- * Instead of a JK flip-flop, an RS flip-flop can also be used to make a D flip-flop, with $S = D$, $R = \overline{D}$.

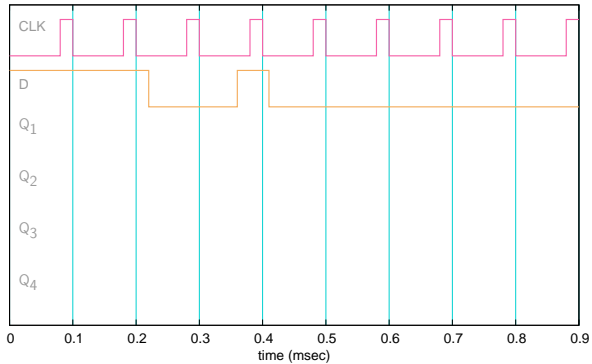
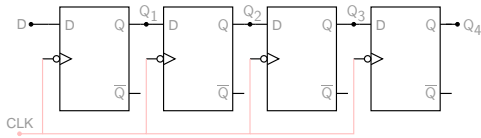
Shift register

Let $Q_1 = Q_2 = Q_3 = Q_4 = 0$ initially.



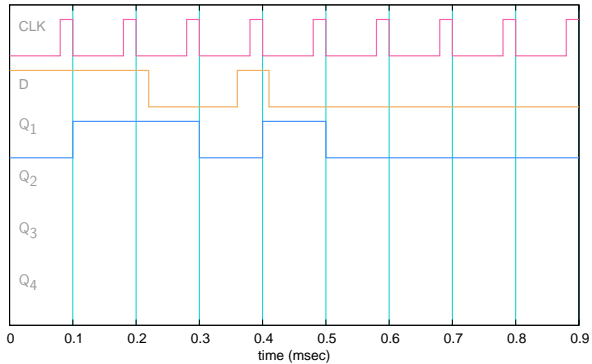
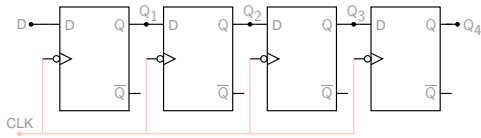
Shift register

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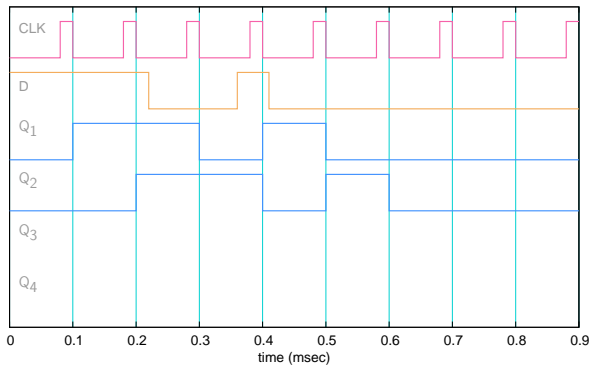
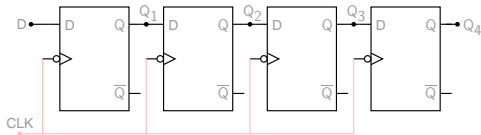
Shift register

Let $Q_1 = Q_2 = Q_3 = Q_4 = 0$ initially.



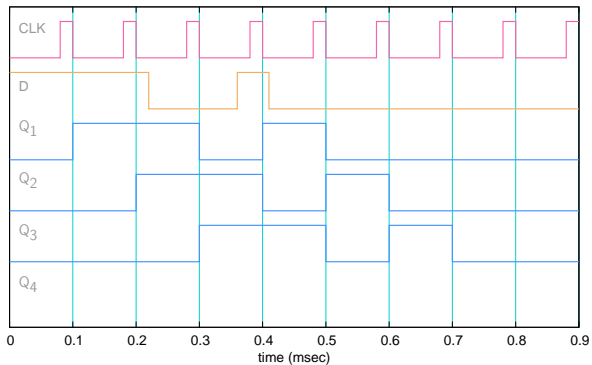
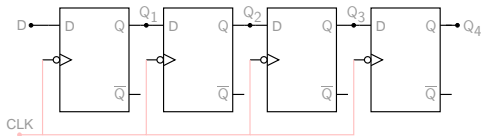
Shift register

Let $Q_1 = Q_2 = Q_3 = Q_4 = 0$ initially.



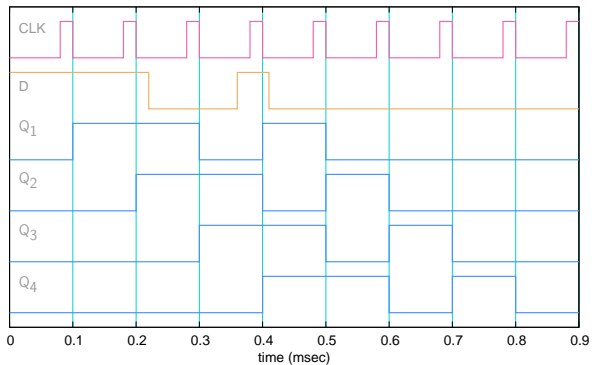
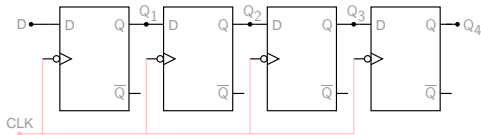
Shift register

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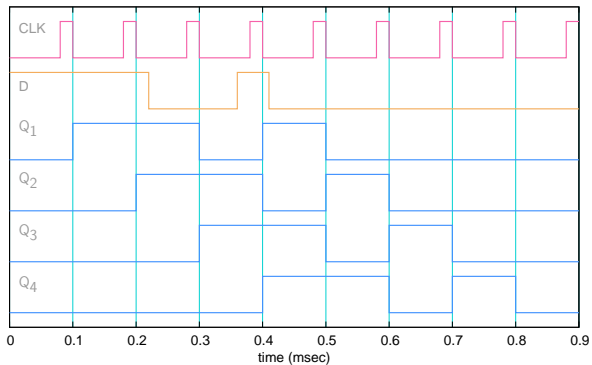
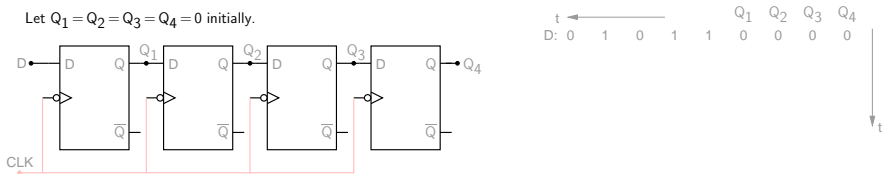
Shift register

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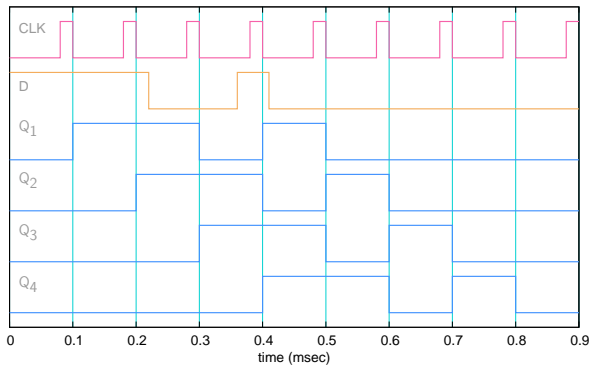
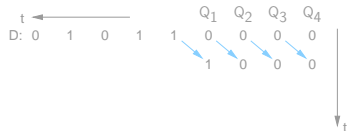
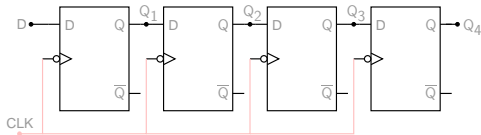
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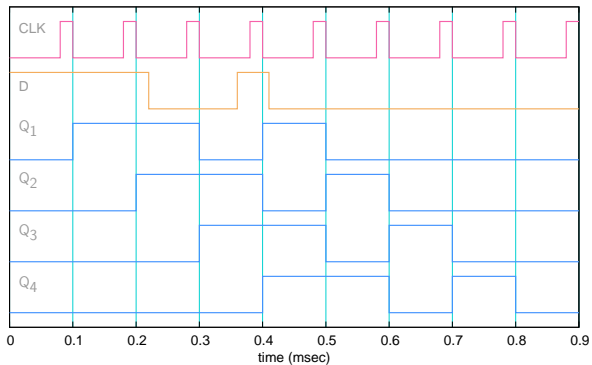
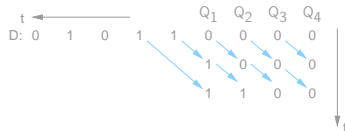
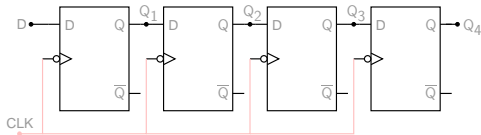
Shift register

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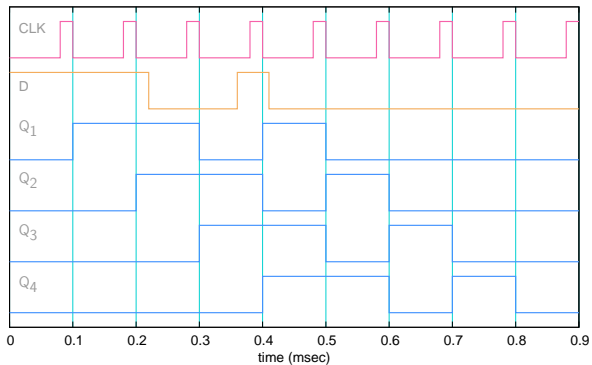
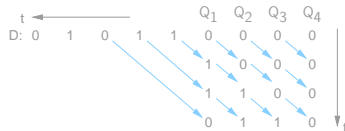
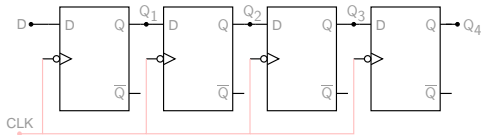
Shift register

Let $Q_1 = Q_2 = Q_3 = Q_4 = 0$ initially.



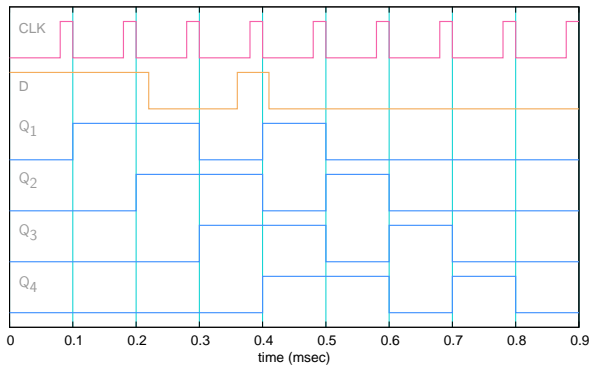
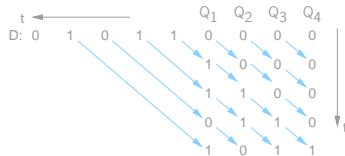
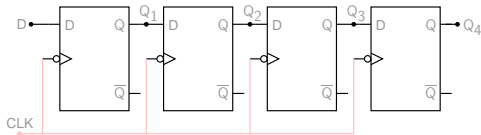
Shift register

Let $Q_1 = Q_2 = Q_3 = Q_4 = 0$ initially.



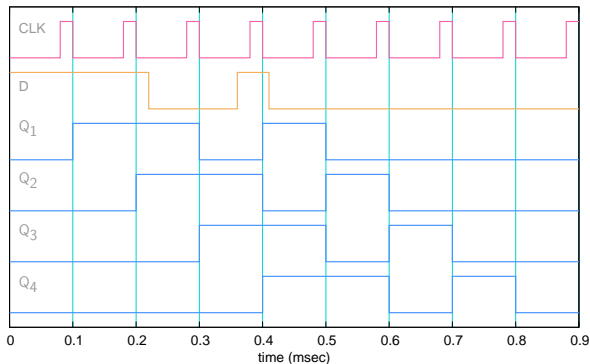
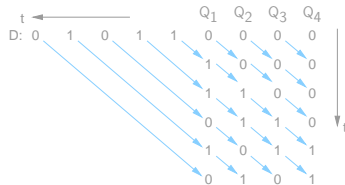
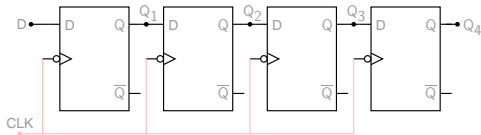
Shift register

Let $Q_1 = Q_2 = Q_3 = Q_4 = 0$ initially.



Shift register

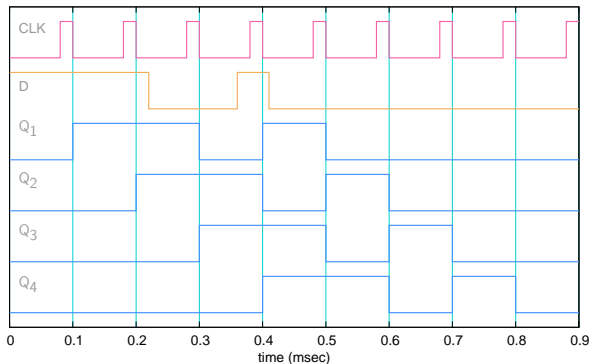
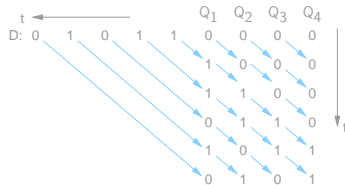
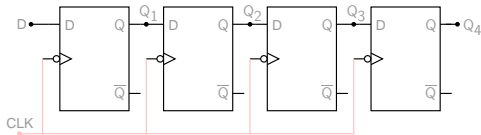
Let $Q_1 = Q_2 = Q_3 = Q_4 = 0$ initially.



(SEQUEL file: ee101_shift_reg_1.sqproj)

Shift register

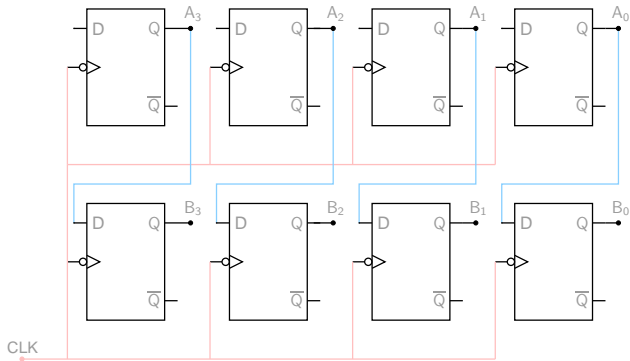
Let $Q_1 = Q_2 = Q_3 = Q_4 = 0$ initially.



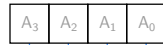
(SEQUEL file: ee101_shift_reg_1.sqproj)

- * The data (D) keeps shifting right after each active clock edge.

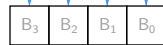
Parallel transfer between shift registers



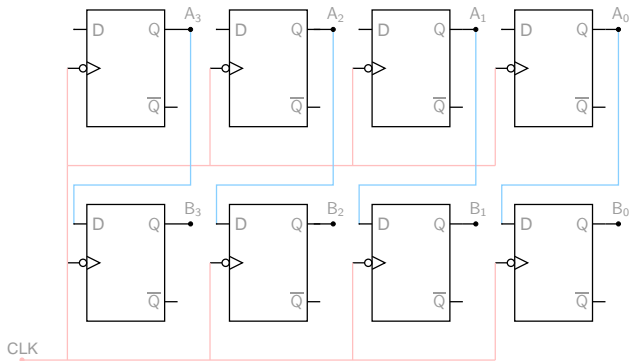
Register A



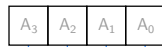
Register B



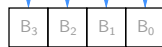
Parallel transfer between shift registers



Register A

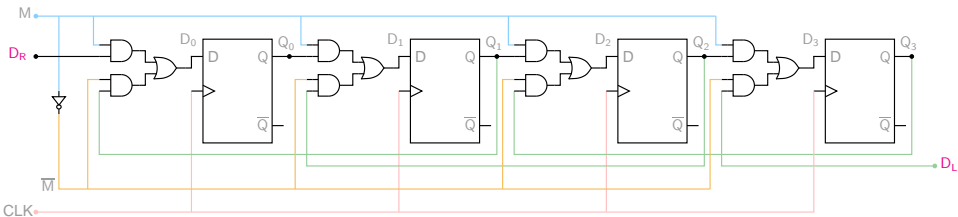


Register B

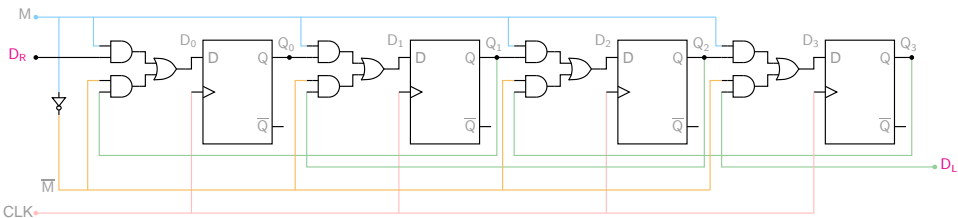


- * After the active clock edge, the contents of the A register ($A_3A_2A_1A_0$) are copied to the B register.

Bidirectional shift register

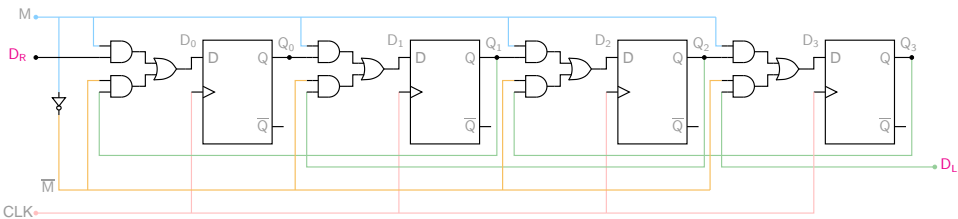


Bidirectional shift register



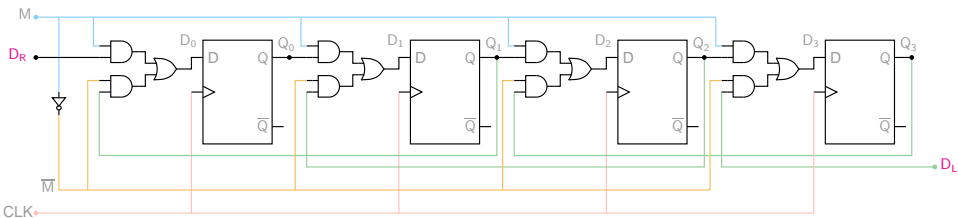
- * When the mode input (M) is 1, we have
 $D_0 = D_R, D_1 = Q_0, D_2 = Q_1, D_3 = Q_2$.

Bidirectional shift register



- * When the mode input (M) is 1, we have $D_0 = D_R$, $D_1 = Q_0$, $D_2 = Q_1$, $D_3 = Q_2$.
- * When the mode input (M) is 0, we have $D_0 = Q_1$, $D_1 = Q_2$, $D_2 = Q_3$, $D_3 = D_L$.

Bidirectional shift register



- * When the mode input (M) is 1, we have
 $D_0 = D_R$, $D_1 = Q_0$, $D_2 = Q_1$, $D_3 = Q_2$.
- * When the mode input (M) is 0, we have
 $D_0 = Q_1$, $D_1 = Q_2$, $D_2 = Q_3$, $D_3 = D_L$.
- * $M = 1 \rightarrow$ shift right operation.
 $M = 0 \rightarrow$ shift left operation.

Multiplication using shift and add

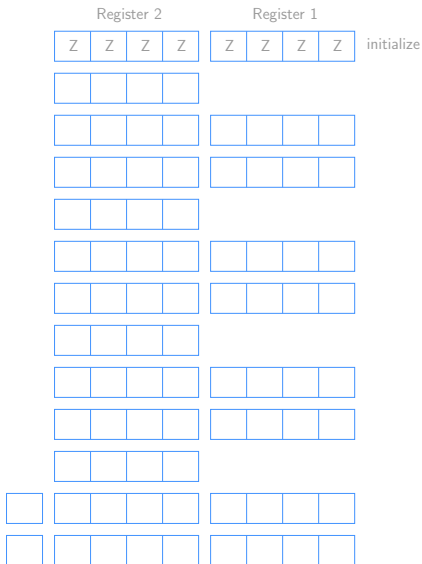
$$\begin{array}{r} 1 \ 0 \ 1 \ 1 \quad A_3 A_2 A_1 A_0 \text{ (decimal 11)} \\ \times 1 \ 1 \ 0 \ 1 \quad B_3 B_2 B_1 B_0 \text{ (decimal 13)} \\ \hline + 1 \ 0 \ 1 \ 1 \quad \text{since } B_0 = 1 \\ 0 \ 0 \ 0 \ 0 \ Z \quad \text{since } B_1 = 0 \\ \hline + 0 \ 1 \ 0 \ 1 \ 1 \quad \text{addition} \\ 1 \ 0 \ 1 \ 1 \ Z \ Z \quad \text{since } B_2 = 1 \\ \hline + 1 \ 1 \ 0 \ 1 \ 1 \ 1 \quad \text{addition} \\ 1 \ 0 \ 1 \ 1 \ Z \ Z \ Z \quad \text{since } B_3 = 1 \\ \hline 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \quad \text{addition (decimal 143)} \end{array}$$

Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.

Multiplication using shift and add

	1	0	1	1		$A_3A_2A_1A_0$	(decimal 11)
	×	1	1	0	1	$B_3B_2B_1B_0$	(decimal 13)
	<hr/>						
+		1	0	1	1	since $B_0 = 1$	
		0	0	0	0	since $B_1 = 0$	
	<hr/>						
+		0	1	0	1	addition	
	1	0	1	1	Z	since $B_2 = 1$	
	<hr/>						
+		1	1	0	1	addition	
	1	0	1	1	Z	since $B_3 = 1$	
	<hr/>						
	1	0	0	0	1	addition	(decimal 143)

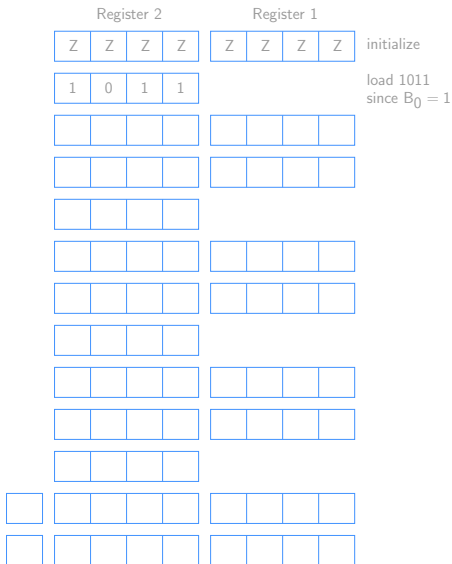
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1	0	1	1	$A_3A_2A_1A_0$ (decimal 11)				
	×	1	1	0	1	$B_3B_2B_1B_0$ (decimal 13)			
<hr/>									
+		1	0	1	1	since $B_0 = 1$			
		0	0	0	0	since $B_1 = 0$			
<hr/>									
+		0	1	0	1	1	addition		
	1	0	1	1	Z	Z	since $B_2 = 1$		
<hr/>									
+		1	1	0	1	1	1	addition	
	1	0	1	1	Z	Z	Z	since $B_3 = 1$	
<hr/>									
	1	0	0	0	1	1	1	1	addition (decimal 143)

Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.

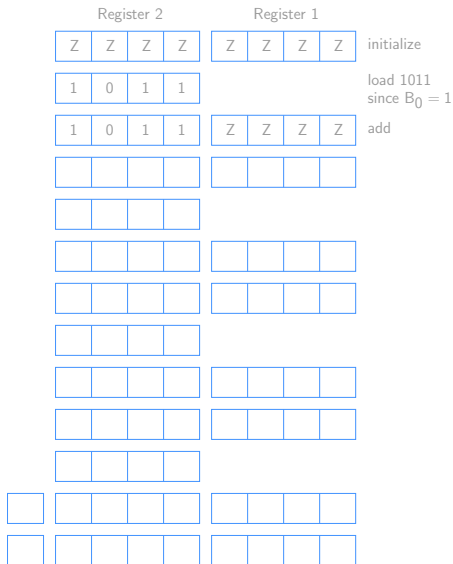


Multiplication using shift and add

$$\begin{array}{r}
 \begin{array}{cccc} 1 & 0 & 1 & 1 \\ \times & 1 & 1 & 0 & 1 \end{array} \\
 \hline
 \begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & Z \end{array} \\
 + \\
 \begin{array}{cccc} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & Z & Z \end{array} \\
 + \\
 \begin{array}{cccc} 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & Z & Z & Z \end{array} \\
 + \\
 \begin{array}{cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}
 \end{array}$$

$A_3A_2A_1A_0$ (decimal 11)
 $B_3B_2B_1B_0$ (decimal 13)
 since $B_0 = 1$
 since $B_1 = 0$
 addition
 since $B_2 = 1$
 addition
 since $B_3 = 1$
 addition (decimal 143)

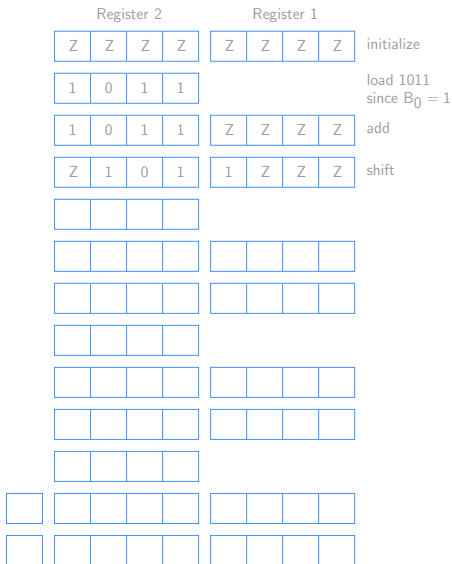
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
	× 1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
	1 0 1 1	
+	0 0 0 0 Z	since $B_0 = 1$ since $B_1 = 0$
	0 1 0 1 1	
+	1 0 1 1 Z Z	addition since $B_2 = 1$
	1 1 0 1 1 1	
+	1 0 1 1 Z Z Z	addition since $B_3 = 1$
	1 0 0 0 1 1 1 1	
		addition (decimal 143)

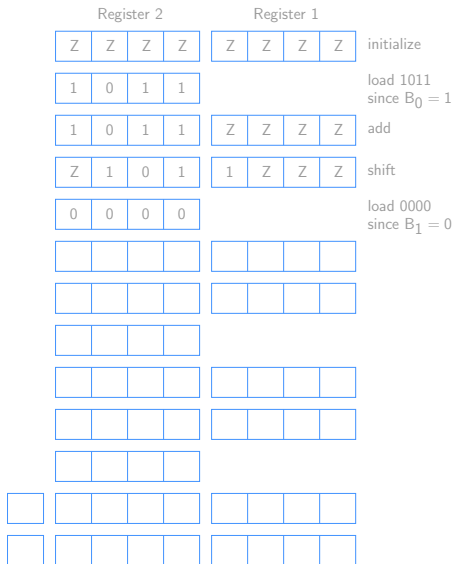
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
\times	1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
<hr/>		
+	1 0 1 1	since $B_0 = 1$
	0 0 0 0 Z	since $B_1 = 0$
<hr/>		
+	0 1 0 1 1	addition
	1 0 1 1 Z Z	since $B_2 = 1$
<hr/>		
+	1 1 0 1 1 1	addition
	1 0 1 1 Z Z Z	since $B_3 = 1$
<hr/>		
	1 0 0 0 1 1 1 1	addition (decimal 143)

Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.

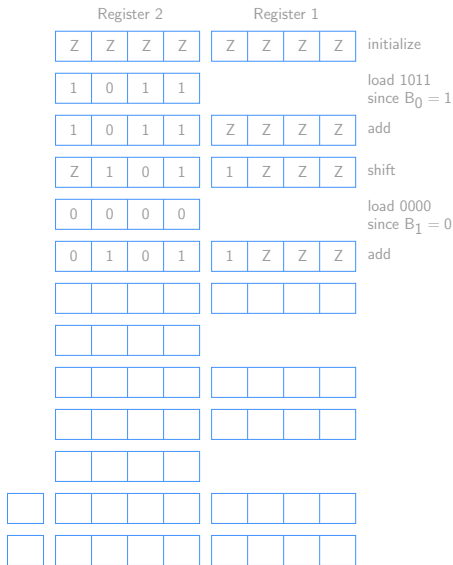


Multiplication using shift and add

$$\begin{array}{r}
 \begin{array}{cccc} 1 & 0 & 1 & 1 \\ \times & 1 & 1 & 0 & 1 \end{array} \\
 \hline
 \begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & Z \end{array} \\
 \hline
 \begin{array}{cccc} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & Z & Z \end{array} \\
 \hline
 \begin{array}{cccc} 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & Z & Z & Z \end{array} \\
 \hline
 \begin{array}{cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}
 \end{array}$$

$A_3A_2A_1A_0$ (decimal 11)
 $B_3B_2B_1B_0$ (decimal 13)
 since $B_0 = 1$
 since $B_1 = 0$
 addition
 since $B_2 = 1$
 addition
 since $B_3 = 1$
 addition (decimal 143)

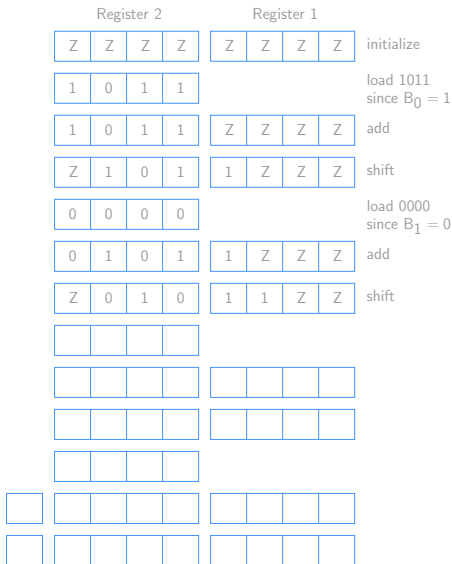
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
	\times 1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
	1 0 1 1	
+	0 0 0 0 Z	since $B_0 = 1$ since $B_1 = 0$
	0 1 0 1 1	
+	1 0 1 1 Z Z	addition since $B_2 = 1$
	1 1 0 1 1 1	
+	1 0 1 1 Z Z Z	addition since $B_3 = 1$
	1 0 0 0 1 1 1 1	
		addition (decimal 143)

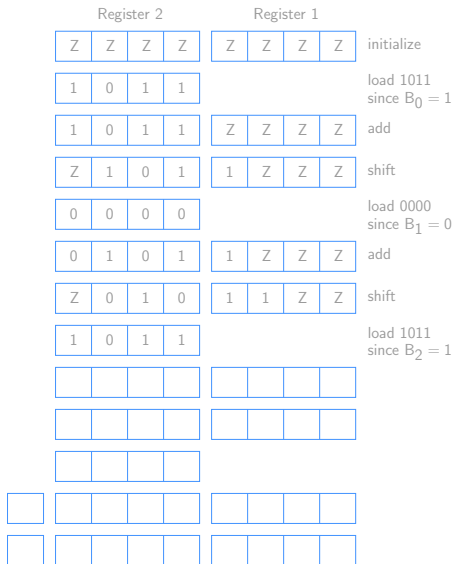
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
\times	1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
<hr/>		
+	1 0 1 1	since $B_0 = 1$
	0 0 0 0 Z	since $B_1 = 0$
<hr/>		
	0 1 0 1 1	addition
+	1 0 1 1 Z Z	since $B_2 = 1$
<hr/>		
	1 1 0 1 1 1	addition
+	1 0 1 1 Z Z Z	since $B_3 = 1$
<hr/>		
	1 0 0 0 1 1 1 1	addition (decimal 143)

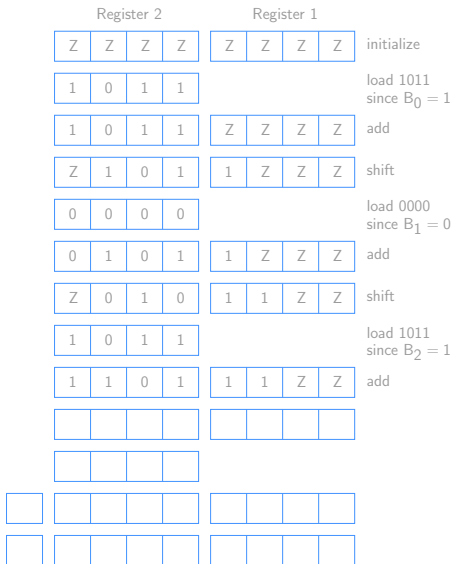
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
	× 1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
	1 0 1 1	
+	0 0 0 0 Z	since $B_0 = 1$ since $B_1 = 0$
	0 1 0 1 1	
+	1 0 1 1 Z Z	addition since $B_2 = 1$
	1 1 0 1 1 1	
+	1 0 1 1 Z Z Z	addition since $B_3 = 1$
	1 0 0 0 1 1 1 1	
		addition (decimal 143)

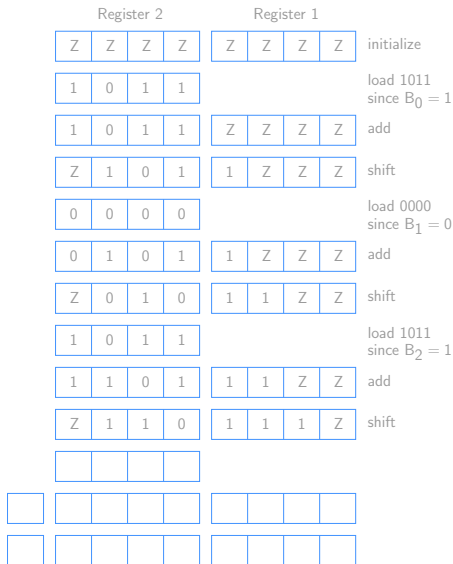
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
×	1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
<hr/>		
+	1 0 1 1	since $B_0 = 1$
	0 0 0 0 Z	since $B_1 = 0$
<hr/>		
+	0 1 0 1 1	addition
	1 0 1 1 Z Z	since $B_2 = 1$
<hr/>		
+	1 1 0 1 1 1	addition
	1 0 1 1 Z Z Z	since $B_3 = 1$
<hr/>		
	1 0 0 0 1 1 1 1	addition (decimal 143)

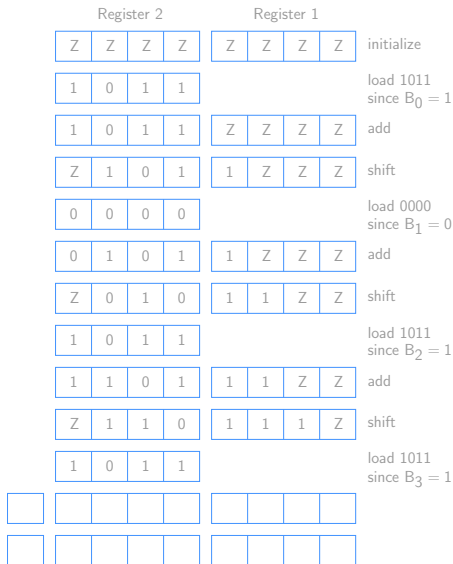
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
×	1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
<hr/>		
+	1 0 1 1	since $B_0 = 1$
	0 0 0 0 Z	since $B_1 = 0$
<hr/>		
+	0 1 0 1 1	addition
	1 0 1 1 Z Z	since $B_2 = 1$
<hr/>		
+	1 1 0 1 1 1	addition
	1 0 1 1 Z Z Z	since $B_3 = 1$
<hr/>		
	1 0 0 0 1 1 1 1	addition (decimal 143)

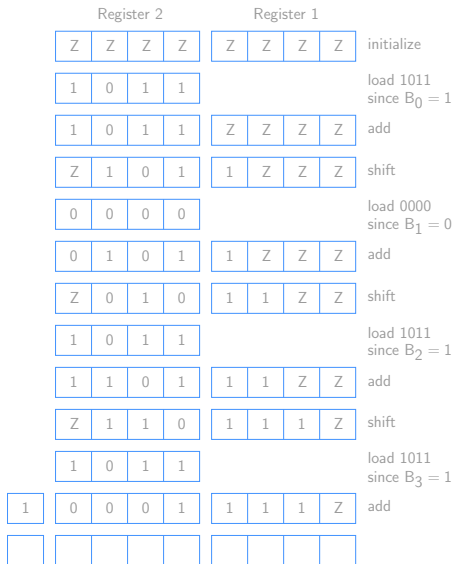
Note that $Z = 0$. We use Z to denote 0s which are independent of the numbers being multiplied.



Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
×	1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
<hr/>		
+	1 0 1 1	since $B_0 = 1$
	0 0 0 0 Z	since $B_1 = 0$
<hr/>		
+	0 1 0 1 1	addition
	1 0 1 1 Z Z	since $B_2 = 1$
<hr/>		
+	1 1 0 1 1 1	addition
	1 0 1 1 Z Z Z	since $B_3 = 1$
<hr/>		
	1 0 0 0 1 1 1 1	addition (decimal 143)

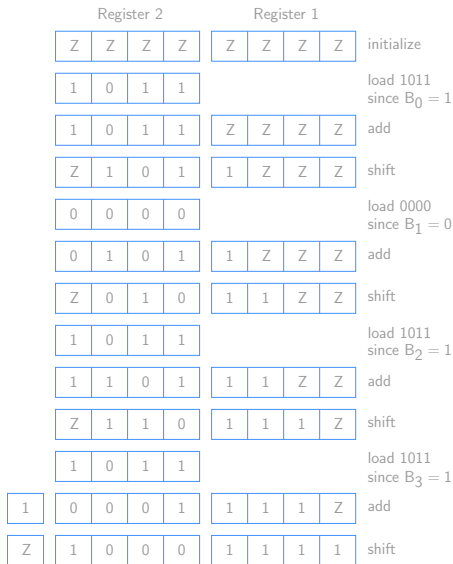
Note that Z = 0. We use Z to denote 0s which are independent of the numbers being multiplied.



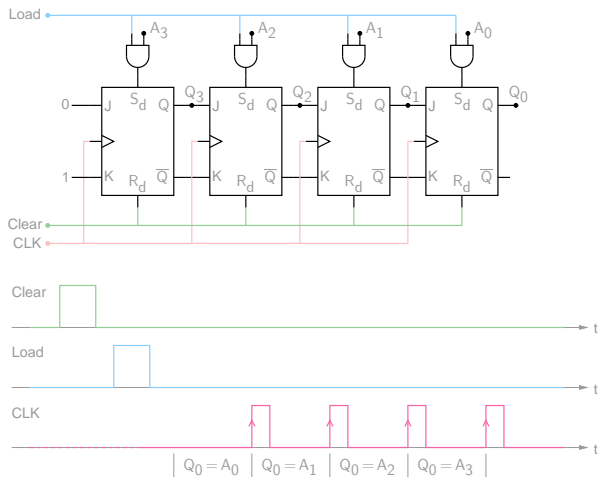
Multiplication using shift and add

	1 0 1 1	$A_3A_2A_1A_0$ (decimal 11)
\times	1 1 0 1	$B_3B_2B_1B_0$ (decimal 13)
<hr/>		
+	1 0 1 1	since $B_0 = 1$
	0 0 0 0 Z	since $B_1 = 0$
<hr/>		
+	0 1 0 1 1	addition
	1 0 1 1 Z Z	since $B_2 = 1$
<hr/>		
+	1 1 0 1 1 1	addition
	1 0 1 1 Z Z Z	since $B_3 = 1$
<hr/>		
	1 0 0 0 1 1 1 1	addition (decimal 143)

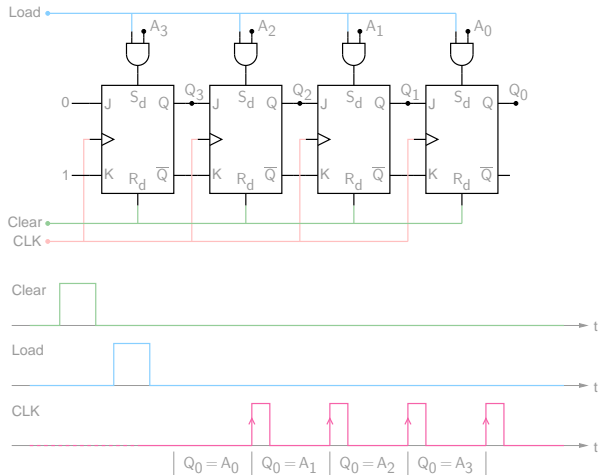
Note that Z = 0. We use Z to denote 0s which are independent of the numbers being multiplied.



Parallel in-serial out data movement

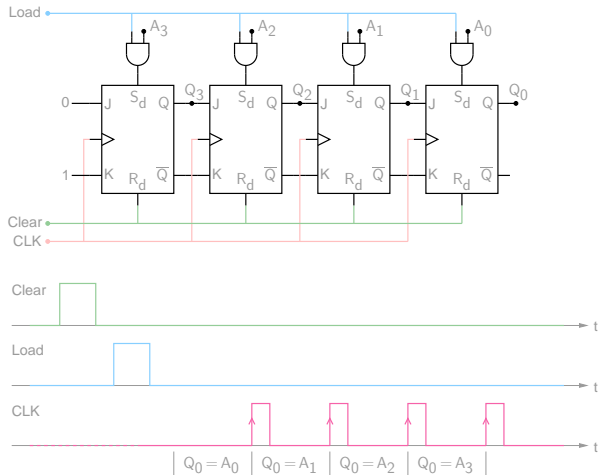


Parallel in-serial out data movement



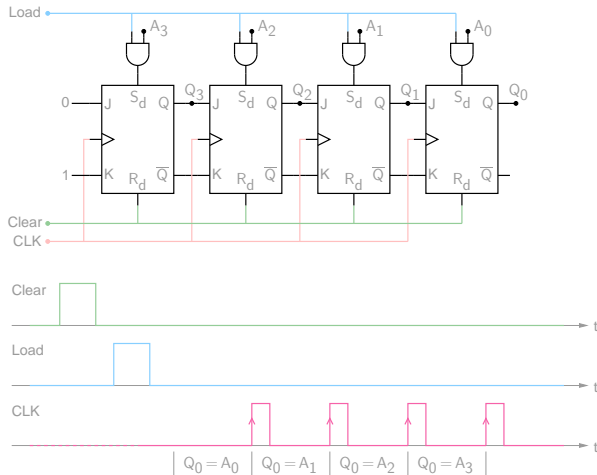
* All flip-flops are cleared in the beginning (with $R_d = \text{Clear} = 1$, $S_d = 0$).

Parallel in-serial out data movement



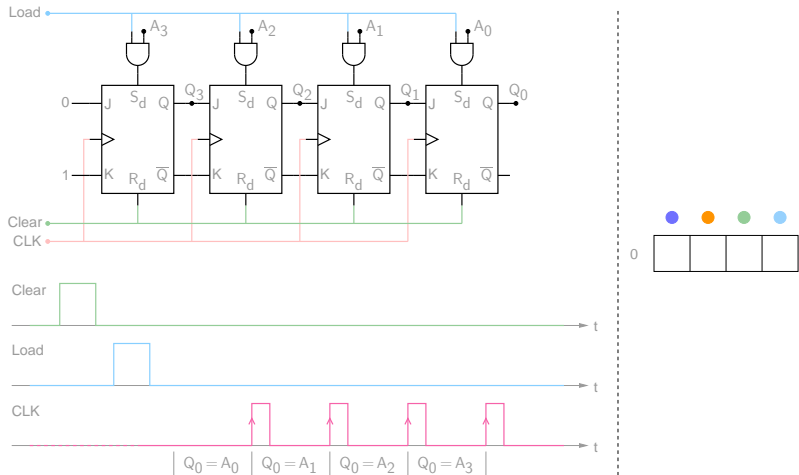
- * All flip-flops are cleared in the beginning (with $R_d = \text{Clear} = 1$, $S_d = 0$).
- * When $\text{Load} = 1$, $S_d = A_i$, $R_d = 0 \rightarrow A_i$ gets loaded into the i^{th} flip-flop. (We will assume that CLK has been made 0 in this initial phase.)

Parallel in-serial out data movement



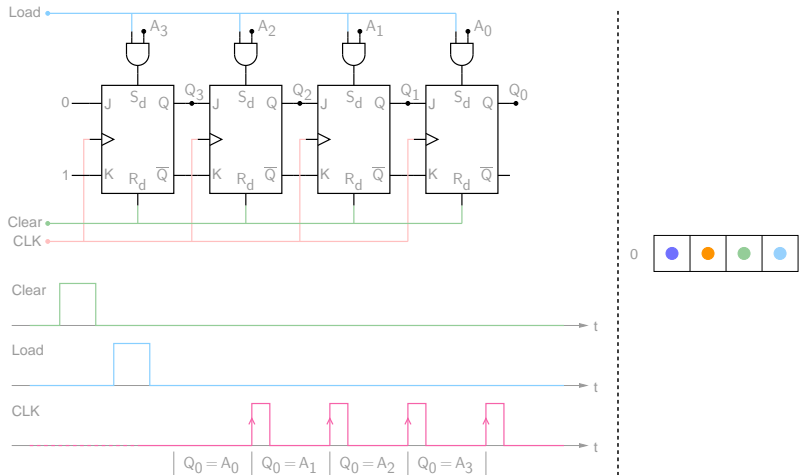
- * All flip-flops are cleared in the beginning (with $R_d = \text{Clear} = 1$, $S_d = 0$).
- * When $\text{Load} = 1$, $S_d = A_i$, $R_d = 0 \rightarrow A_i$ gets loaded into the i^{th} flip-flop. (We will assume that CLK has been made 0 in this initial phase.)
- * Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output Q_0 .

Parallel in-serial out data movement



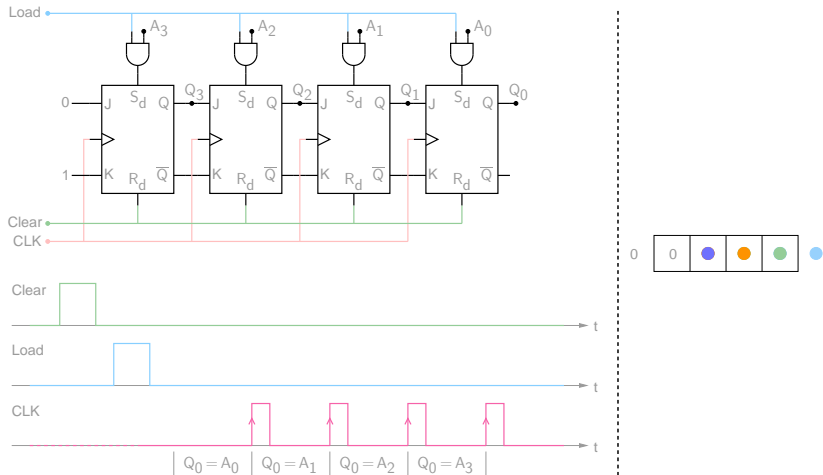
- * All flip-flops are cleared in the beginning (with $R_d = \text{Clear} = 1$, $S_d = 0$).
- * When $\text{Load} = 1$, $S_d = A_i$, $R_d = 0 \rightarrow A_i$ gets loaded into the i^{th} flip-flop. (We will assume that CLK has been made 0 in this initial phase.)
- * Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output Q_0 .

Parallel in-serial out data movement



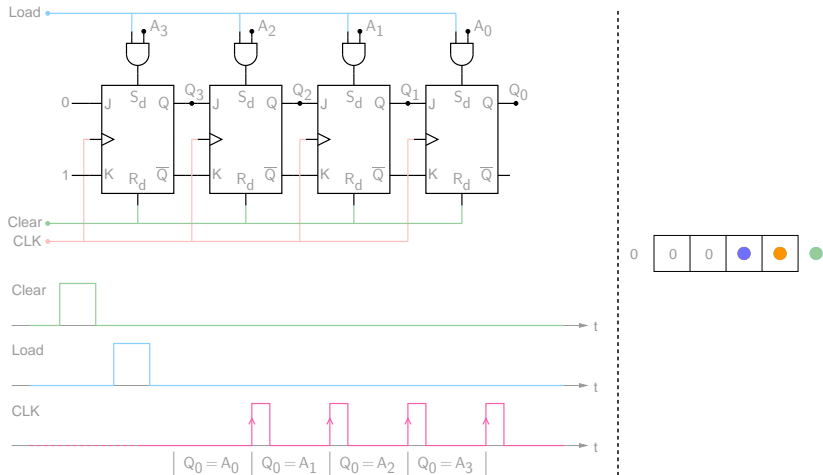
- * All flip-flops are cleared in the beginning (with $R_d = \text{Clear} = 1$, $S_d = 0$).
- * When $\text{Load} = 1$, $S_d = A_i$, $R_d = 0 \rightarrow A_i$ gets loaded into the i^{th} flip-flop. (We will assume that CLK has been made 0 in this initial phase.)
- * Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output Q_0 .

Parallel in-serial out data movement



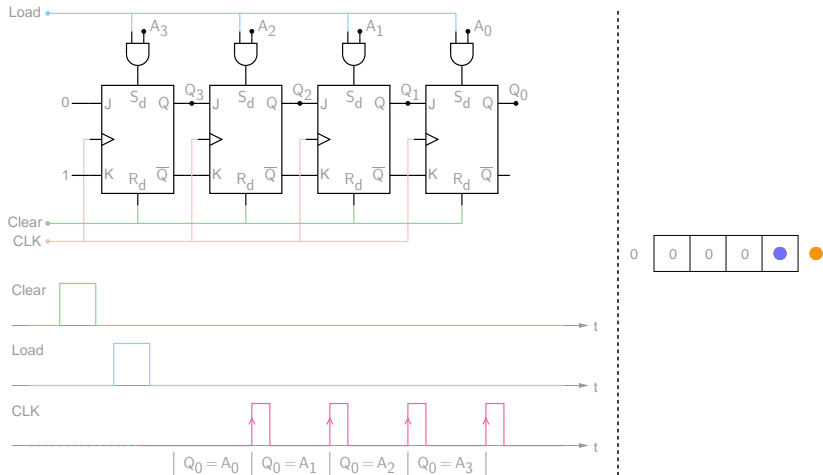
- * All flip-flops are cleared in the beginning (with $R_d = Clear = 1$, $S_d = 0$).
- * When $Load = 1$, $S_d = A_i$, $R_d = 0 \rightarrow A_i$ gets loaded into the i^{th} flip-flop. (We will assume that CLK has been made 0 in this initial phase.)
- * Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output Q_0 .

Parallel in-serial out data movement



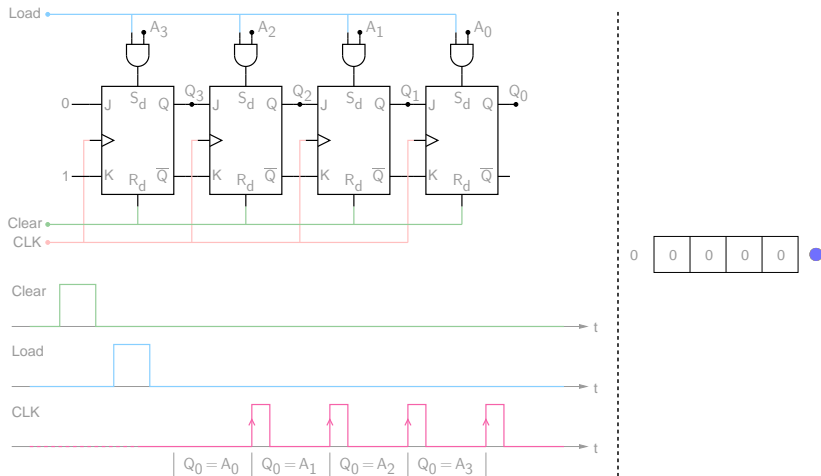
- * All flip-flops are cleared in the beginning (with $R_d = \text{Clear} = 1$, $S_d = 0$).
- * When Load = 1, $S_d = A_i$, $R_d = 0 \rightarrow A_i$ gets loaded into the i^{th} flip-flop. (We will assume that CLK has been made 0 in this initial phase.)
- * Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output Q_0 .

Parallel in-serial out data movement



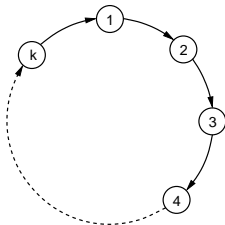
- * All flip-flops are cleared in the beginning (with $R_d = \text{Clear} = 1$, $S_d = 0$).
- * When Load = 1, $S_d = A_i$, $R_d = 0 \rightarrow A_i$ gets loaded into the i^{th} flip-flop. (We will assume that CLK has been made 0 in this initial phase.)
- * Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output Q_0 .

Parallel in-serial out data movement

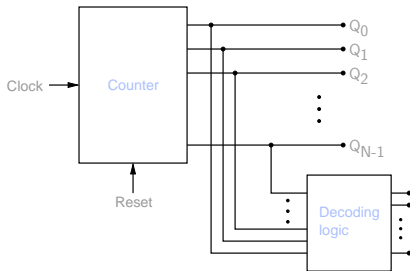


- * All flip-flops are cleared in the beginning (with $R_d = \text{Clear} = 1$, $S_d = 0$).
- * When $\text{Load} = 1$, $S_d = A_i$, $R_d = 0 \rightarrow A_i$ gets loaded into the i^{th} flip-flop. (We will assume that CLK has been made 0 in this initial phase.)
- * Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output Q_0 .

Counters

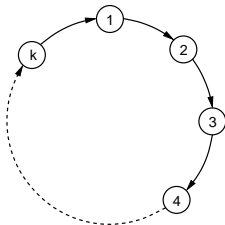


State transition diagram

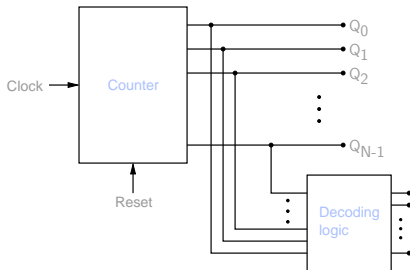


General configuration

Counters



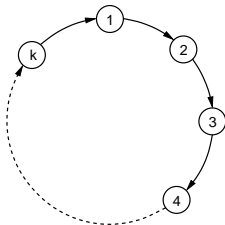
State transition diagram



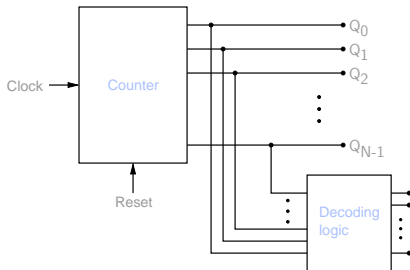
General configuration

- * A counter with k states is called a modulo- k (mod- k) counter.

Counters



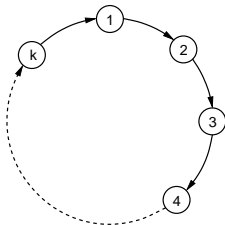
State transition diagram



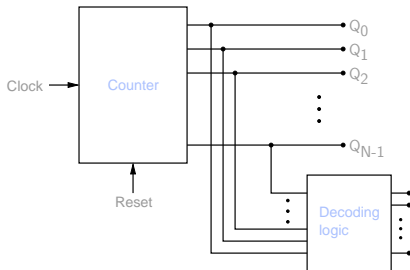
General configuration

- * A counter with k states is called a modulo- k (mod- k) counter.
- * A counter can be made with flip-flops, each flip-flop serving as a memory element with two states (0 or 1).

Counters



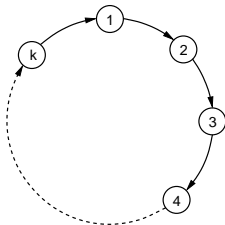
State transition diagram



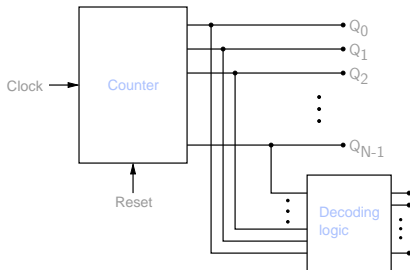
General configuration

- * A counter with k states is called a modulo- k (mod- k) counter.
- * A counter can be made with flip-flops, each flip-flop serving as a memory element with two states (0 or 1).
- * If there are N flip-flops in a counter, there are 2^N possible states (since each flip-flop can have $Q = 0$ or $Q = 1$). It is possible to exclude some of these states.
→ N flip-flops can be used to make a mod- k counter with $k \leq 2^N$.

Counters



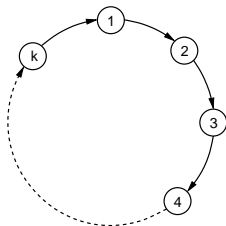
State transition diagram



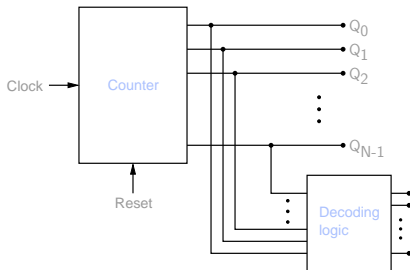
General configuration

- * A counter with k states is called a modulo- k (mod- k) counter.
- * A counter can be made with flip-flops, each flip-flop serving as a memory element with two states (0 or 1).
- * If there are N flip-flops in a counter, there are 2^N possible states (since each flip-flop can have $Q = 0$ or $Q = 1$). It is possible to exclude some of these states.
→ N flip-flops can be used to make a mod- k counter with $k \leq 2^N$.
- * Typically, a reset facility is also provided, which can be used to force a certain state to initialize the counter.

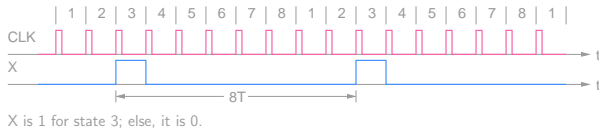
Counters



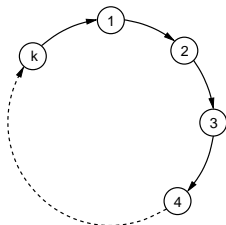
State transition diagram



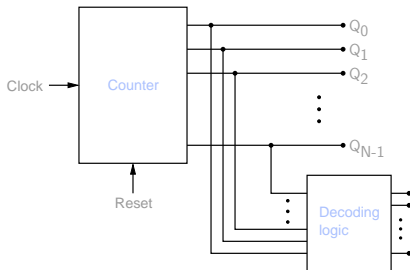
General configuration



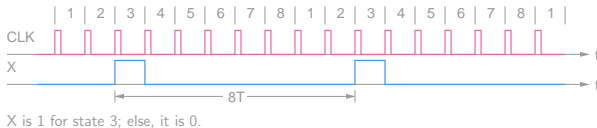
Counters



State transition diagram

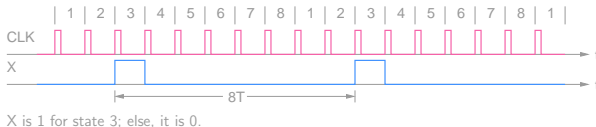
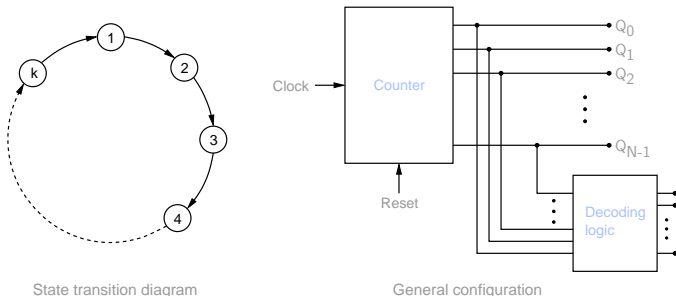


General configuration



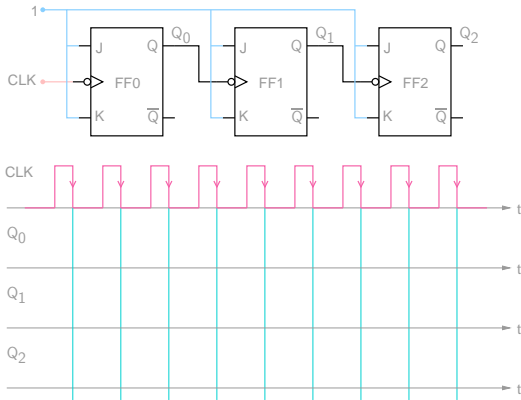
- * The counter outputs (i.e., the flip-flop outputs, Q_0 , Q_1 , \dots Q_{N-1}) can be decoded using appropriate logic.

Counters

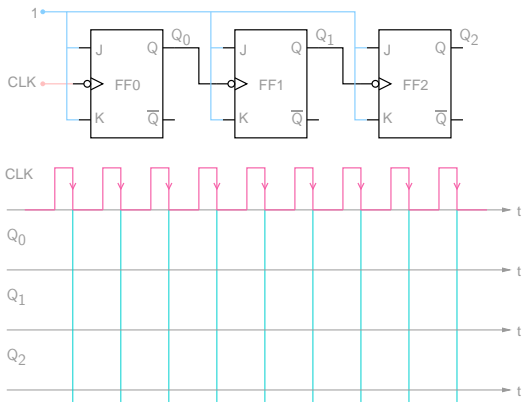


- * The counter outputs (i.e., the flip-flop outputs, Q_0, Q_1, \dots, Q_{N-1}) can be decoded using appropriate logic.
- * In particular, it is possible to have a decoder output (say, X) which is 1 only for state i , and 0 otherwise.
 → For k clock pulses, we get a single pulse at X , i.e., the clock frequency has been divided by k . For this reason, a mod- k counter is also called a divide-by- k counter.

A binary ripple counter

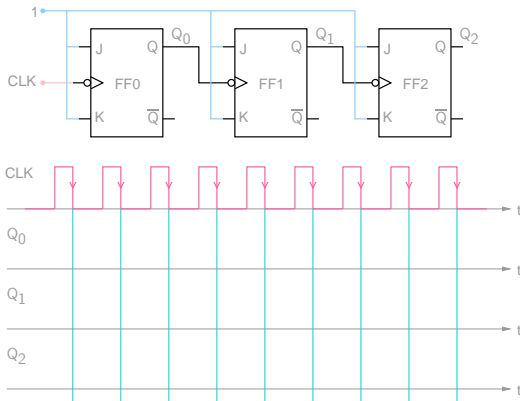


A binary ripple counter



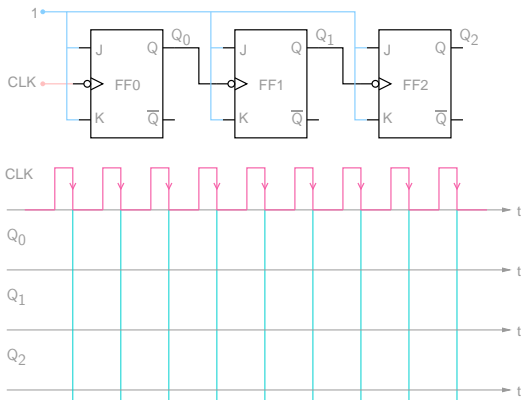
* $J = K = 1$ for all flip-flops. Let $Q_0 = Q_1 = Q_2 = 0$ initially.

A binary ripple counter



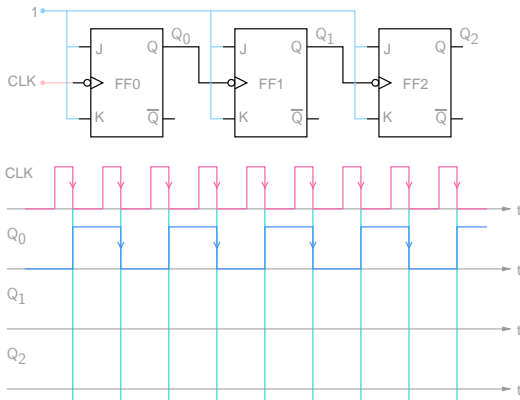
- * $J = K = 1$ for all flip-flops. Let $Q_0 = Q_1 = Q_2 = 0$ initially.
- * Since $J = K = 1$, each flip-flop will toggle when an active (in this case, negative) clock edge arrives.

A binary ripple counter



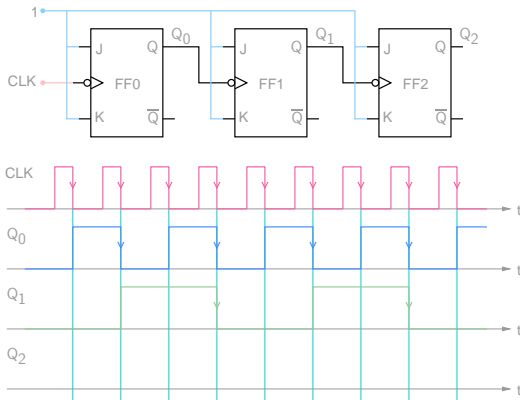
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- * Since $J = K = 1$, each flip-flop will toggle when an active (in this case, negative) clock edge arrives.
- * For FF1 and FF2, Q_0 and Q_1 , respectively, provide the clock.

A binary ripple counter



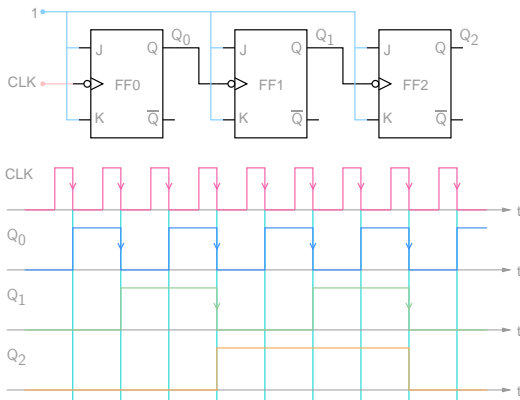
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A binary ripple counter



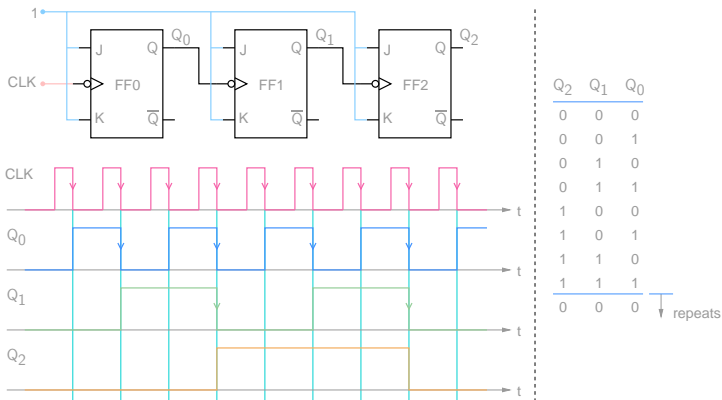
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A binary ripple counter



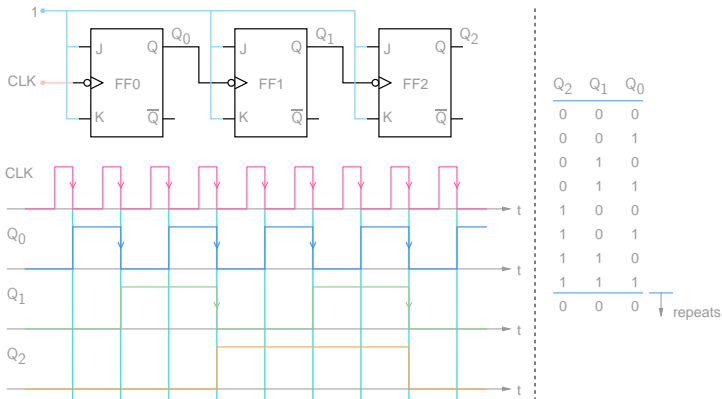
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A binary ripple counter



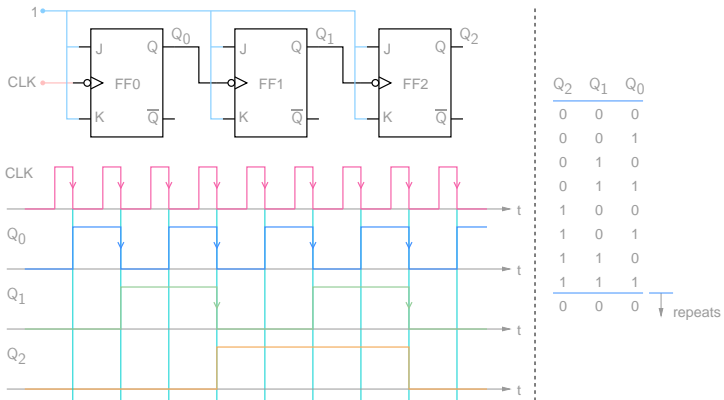
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A binary ripple counter

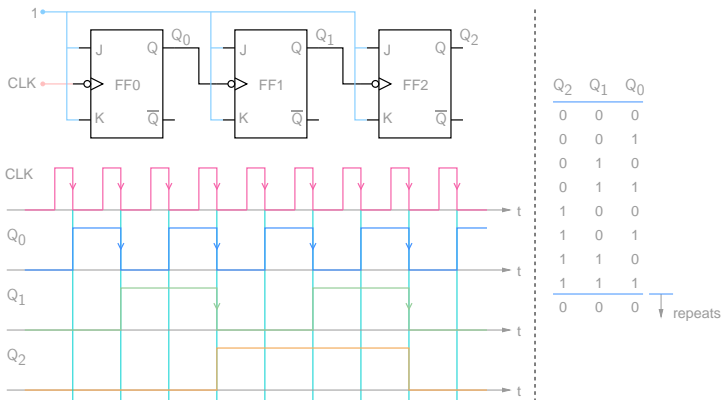


- * $J = K = 1$ for all flip-flops. Let $Q_0 = Q_1 = Q_2 = 0$ initially.
- * Since $J = K = 1$, each flip-flop will toggle when an active (in this case, negative) clock edge arrives.
- * For FF1 and FF2, Q_0 and Q_1 , respectively, provide the clock.
- * Note that the direct inputs S_d and R_d (not shown) are assumed to be $S_d = R_d = 0$ for all flip-flops, allowing normal flip-flop operation.

A binary ripple counter

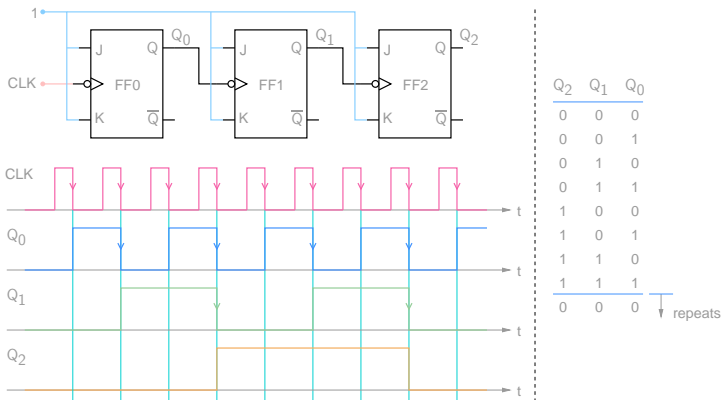


A binary ripple counter



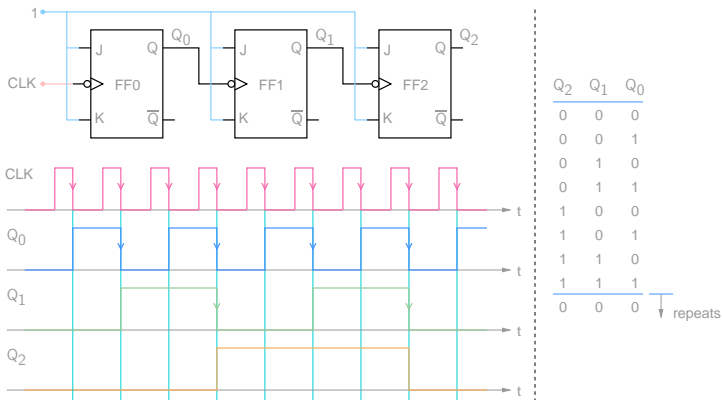
- * The counter has 8 states, $Q_2 Q_1 Q_0 = 000, 001, 010, 011, 100, 101, 110, 111$.
→ it is a *mod-8* counter. In particular, it is a *binary, mod-8, up* counter (since it counts *up* from 000 to 111).

A binary ripple counter



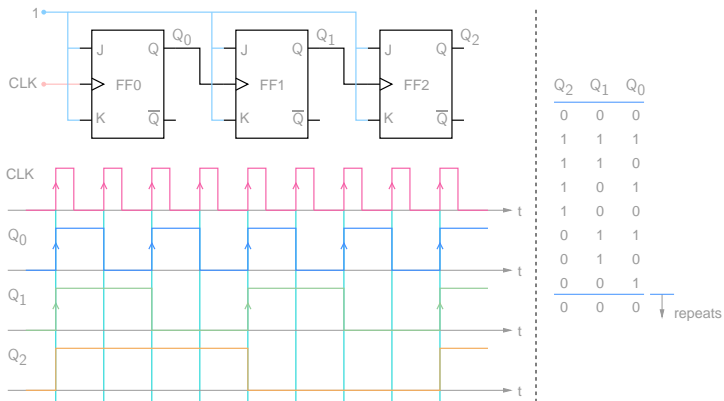
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→ it is a *mod-8* counter. In particular, it is a *binary, mod-8, up* counter (since it counts *up* from 000 to 111).
- * If the clock frequency is f_c , the frequency at the Q_0, Q_1, Q_2 outputs is $f_c/2, f_c/4, f_c/8$, respectively. For this counter, therefore, div-by-2, div-by-4, div-by-8 outputs are already available, without requiring decoding logic.

A binary ripple counter

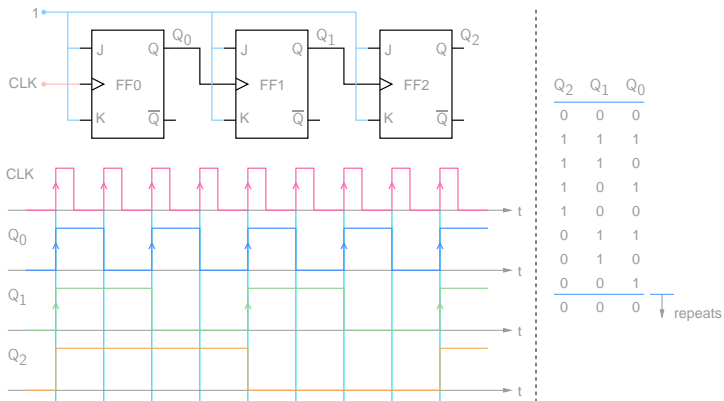


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- * If the clock frequency is f_c , the frequency at the Q_0, Q_1, Q_2 outputs is $f_c/2, f_c/4, f_c/8$, respectively. For this counter, therefore, div-by-2, div-by-4, div-by-8 outputs are already available, without requiring decoding logic.
- * This type of counter is called a "ripple" counter since the clock transitions *ripple* through the flip-flops.

A binary ripple counter

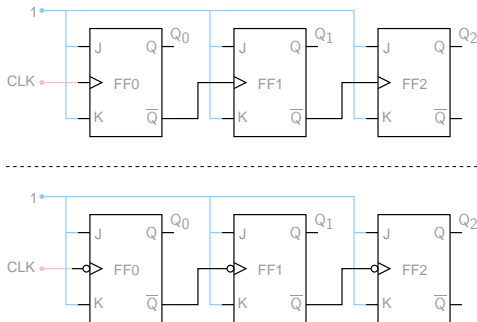


A binary ripple counter



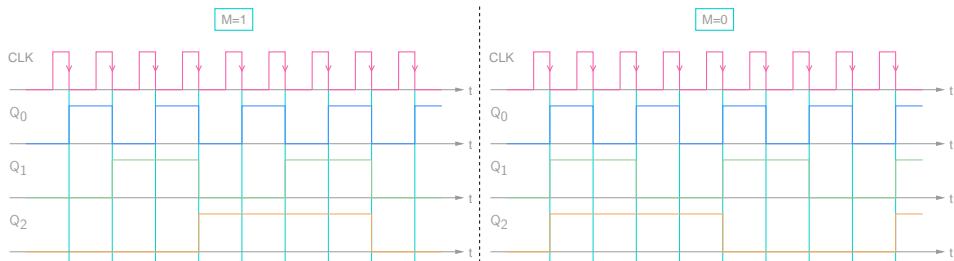
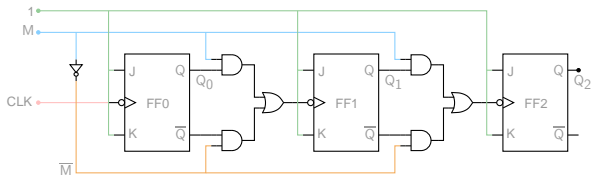
- * If positive edge-triggered flip-flops are used, we get a binary *down* counter (counting down from 1111 to 0000).

Binary ripple counters

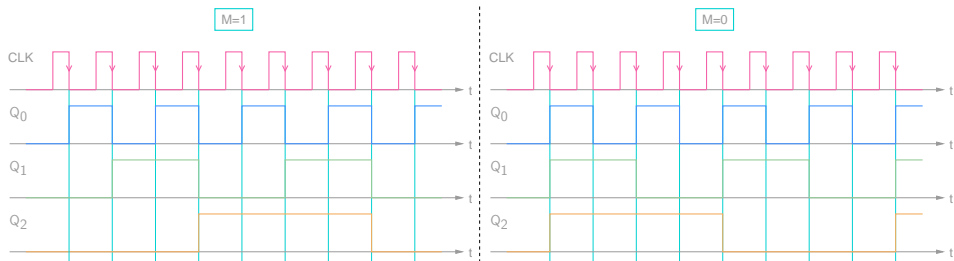
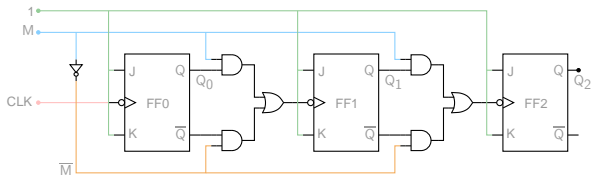


- * Home work: Sketch the waveforms (CLK, Q_0 , Q_1 , Q_2), and tabulate the counter states in each case.

Up-down binary ripple counters

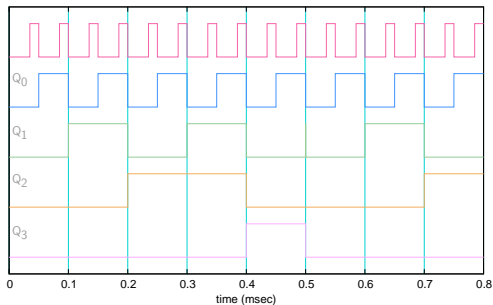
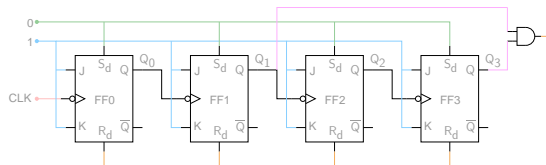


Up-down binary ripple counters



- * When Mode (M) = 1, the counter counts up; else, it counts down.
(SEQUEL file: ee101_counter_3.sqproj)

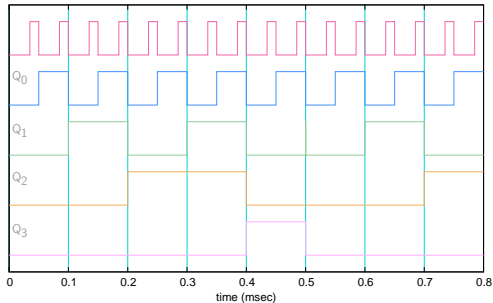
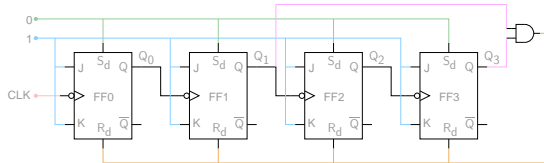
Decade counter using direct inputs



Q ₃	Q ₂	Q ₁	Q ₀
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
0	0	0	0

↓ repeats

Decade counter using direct inputs

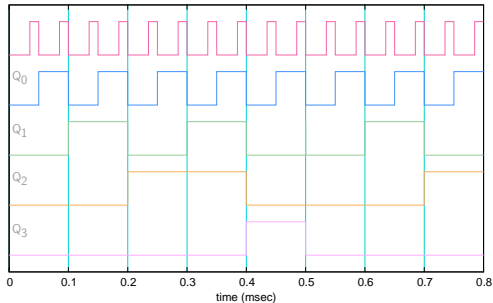
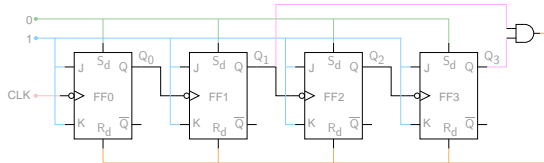


Q ₃	Q ₂	Q ₁	Q ₀
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
0	0	0	0

↓ repeats

- * When the counter reaches $Q_3 Q_2 Q_1 Q_0 = 1010$ (i.e., decimal 10), $Q_3 Q_1 = 1$, and the flip-flops are cleared to $Q_3 Q_2 Q_1 Q_0 = 0000$.

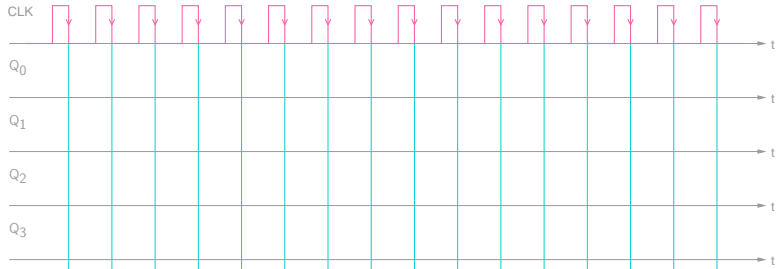
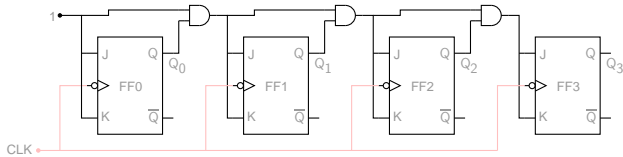
Decade counter using direct inputs



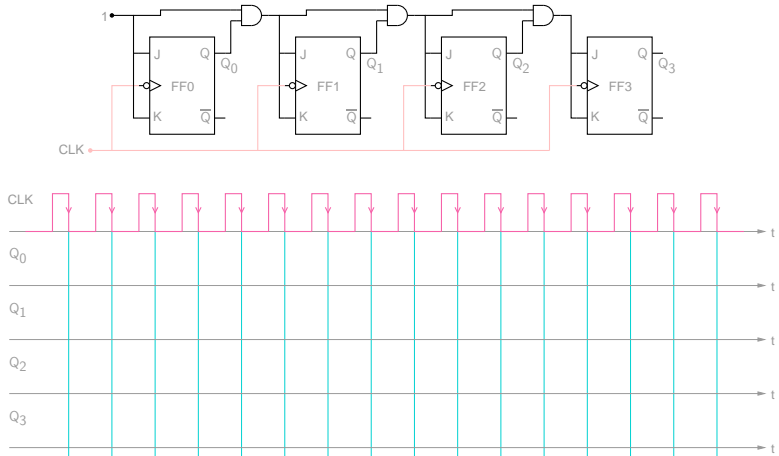
Q ₃	Q ₂	Q ₁	Q ₀
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
0	0	0	0
↓ repeats			

- * When the counter reaches $Q_3 Q_2 Q_1 Q_0 = 1010$ (i.e., decimal 10), $Q_3 Q_1 = 1$, and the flip-flops are cleared to $Q_3 Q_2 Q_1 Q_0 = 0000$.
- * The counter counts from 0000 (decimal 0) to 1001 (decimal 9) \rightarrow "decade counter."
(SEQUEL file: ee101_counter_5.sqproj)

A synchronous counter

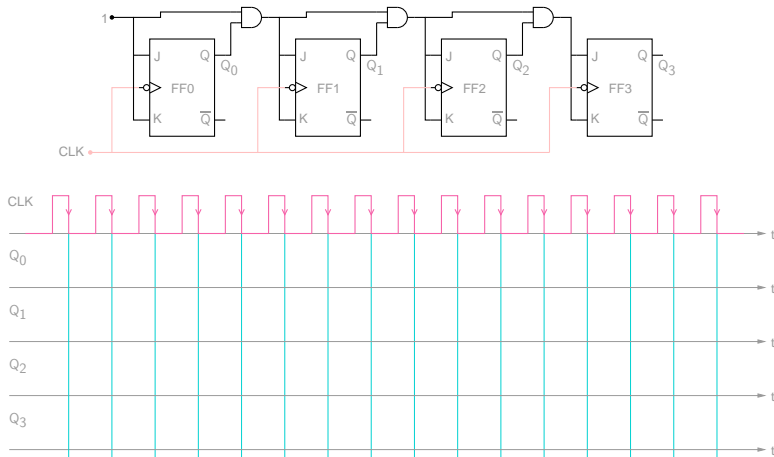


A synchronous counter



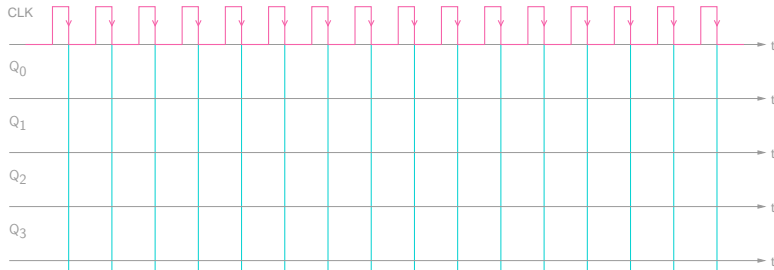
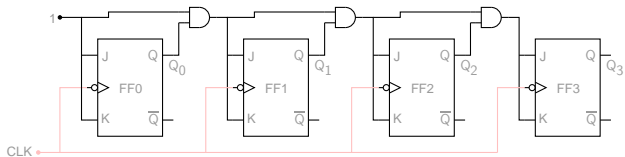
- * Since all flip-flops are driven by the same clock, the counter is called a “synchronous” counter.

A synchronous counter



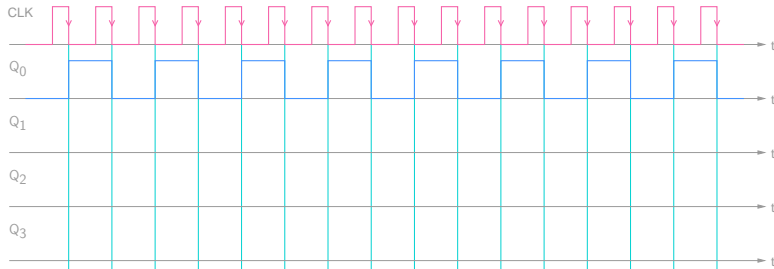
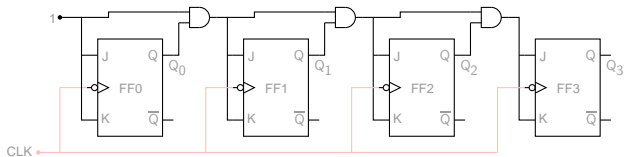
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- * $J_0 = K_0 = 1$, $J_1 = K_1 = Q_0$, $J_2 = K_2 = Q_1 Q_0$, $J_3 = K_3 = Q_2 Q_1 Q_0$.

A synchronous counter



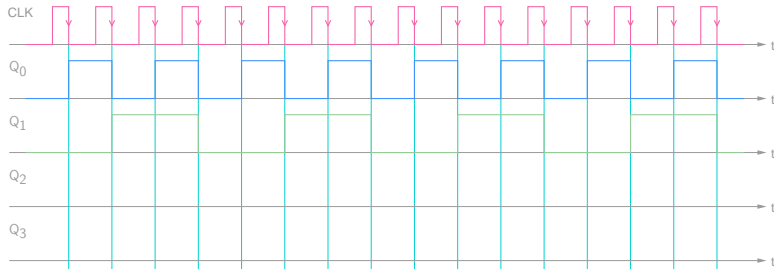
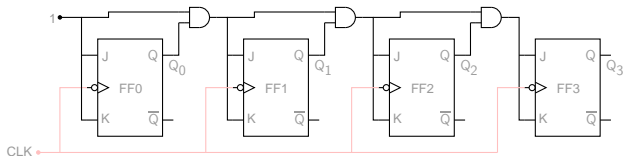
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- * FF0 toggles at every active edge.
FF1 toggles if $Q_0 = 1$ (just before the active clock edge); else, it retains its previous state.
Similar comments apply to FF2 and FF3.

A synchronous counter



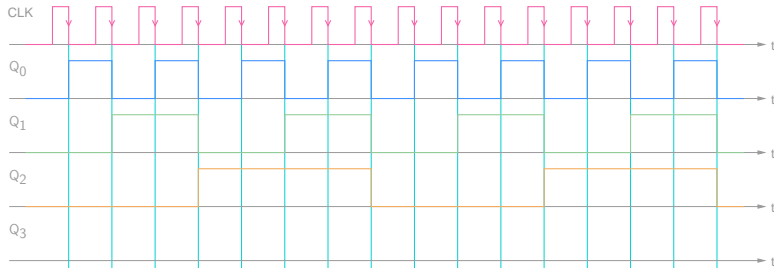
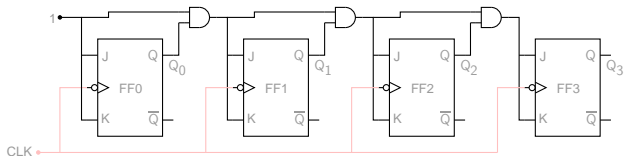
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A synchronous counter



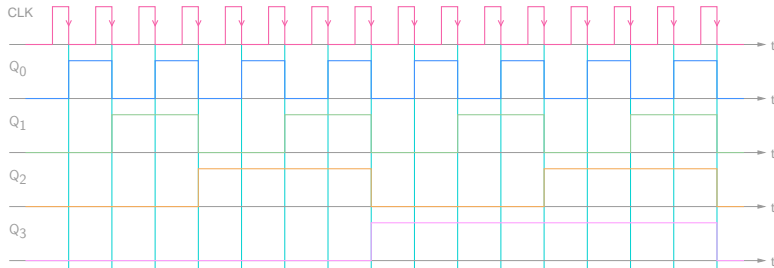
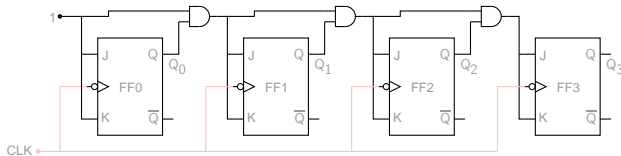
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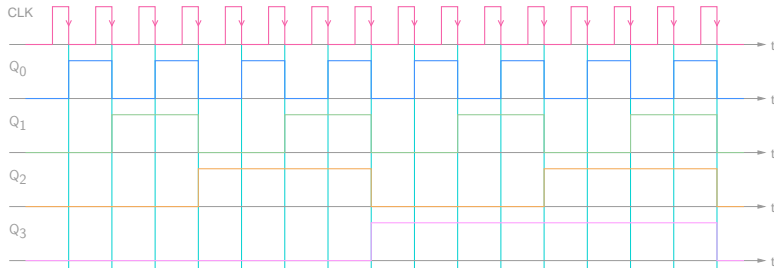
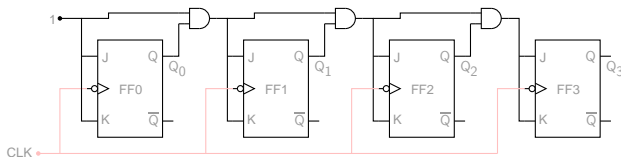
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A synchronous counter



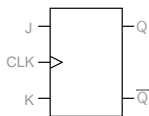
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Similar comments apply to FF2 and FF3.

A synchronous counter



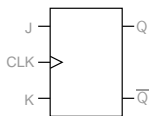
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- * FF0 toggles at every active edge.
FF1 toggles if $Q_0 = 1$ (just before the active clock edge); else, it retains its previous state. Similar comments apply to FF2 and FF3.
- * From the waveforms, we see that it is a binary up counter.

Design of synchronous counters



CLK	J	K	Q_{n+1}
↑	0	0	Q_n
↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

Design of synchronous counters

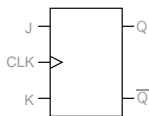


CLK	J	K	Q_{n+1}
↑	0	0	Q_n
↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

CLK	Q_n	Q_{n+1}	J	K

- * Consider the *reverse* problem: We are given Q_n and the next desired state (Q_{n+1}). What should J and K be in order to make that happen?

Design of synchronous counters

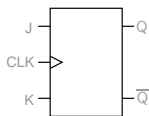


CLK	J	K	Q_{n+1}
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↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

CLK	Q_n	Q_{n+1}	J	K

- * Consider the *reverse* problem: We are given Q_n and the next desired state (Q_{n+1}). What should J and K be in order to make that happen?
- * $Q_n = 0, Q_{n+1} = 0$: We can either force $Q_{n+1} = 0$ with $J = 0, K = 1$, or let $Q_{n+1} = Q_n = 0$ by making $J = 0, K = 0$.

Design of synchronous counters

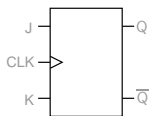


CLK	J	K	Q_{n+1}
↑	0	0	Q_n
↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

CLK	Q_n	Q_{n+1}	J	K

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 → $J = 0, K = X$ (i.e., K can be 0 or 1).

Design of synchronous counters

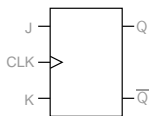


CLK	J	K	Q_{n+1}
↑	0	0	Q_n
↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X

- * Consider the *reverse* problem: We are given Q_n and the next desired state (Q_{n+1}). What should J and K be in order to make that happen?
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Design of synchronous counters

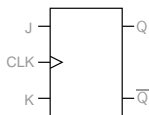


CLK	J	K	Q_{n+1}
↑	0	0	Q_n
↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X

- * Consider the *reverse* problem: We are given Q_n and the next desired state (Q_{n+1}). What should J and K be in order to make that happen?
- * $Q_n = 0, Q_{n+1} = 0$: We can either force $Q_{n+1} = 0$ with $J = 0, K = 1$, or let $Q_{n+1} = Q_n = 0$ by making $J = 0, K = 0$.
→ $J = 0, K = X$ (i.e., K can be 0 or 1).
- * Similarly, work out the other entries in the table.

Design of synchronous counters

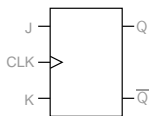


CLK	J	K	Q_{n+1}
↑	0	0	Q_n
↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

- * Consider the *reverse* problem: We are given Q_n and the next desired state (Q_{n+1}). What should J and K be in order to make that happen?
- * $Q_n = 0, Q_{n+1} = 0$: We can either force $Q_{n+1} = 0$ with $J = 0, K = 1$, or let $Q_{n+1} = Q_n = 0$ by making $J = 0, K = 0$.
→ $J = 0, K = X$ (i.e., K can be 0 or 1).
- * Similarly, work out the other entries in the table.

Design of synchronous counters



CLK	J	K	Q_{n+1}
↑	0	0	Q_n
↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

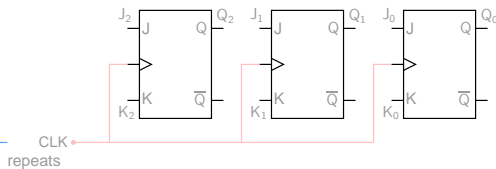
CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

- * Consider the *reverse* problem: We are given Q_n and the next desired state (Q_{n+1}). What should J and K be in order to make that happen?
- * $Q_n = 0, Q_{n+1} = 0$: We can either force $Q_{n+1} = 0$ with $J = 0, K = 1$, or let $Q_{n+1} = Q_n = 0$ by making $J = 0, K = 0$.
→ $J = 0, K = X$ (i.e., K can be 0 or 1).
- * Similarly, work out the other entries in the table.
- * The table for a negative edge-triggered flip-flop would be identical except for the active edge.

Design of synchronous counters

state	Q_2	Q_1	Q_0
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
1	0	0	0

↓ repeats



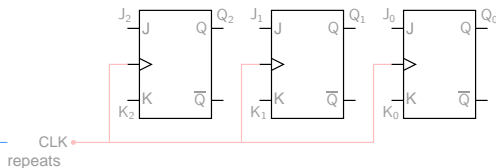
CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design a synchronous mod-5 counter with the given state transition table.

Design of synchronous counters

state	Q_2	Q_1	Q_0
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
1	0	0	0

↓ repeats



CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

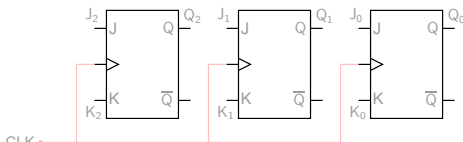
Design a synchronous mod-5 counter with the given state transition table.

Outline of method:

Design of synchronous counters

state	Q_2	Q_1	Q_0
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
1	0	0	0

↓ repeats



CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design a synchronous mod-5 counter with the given state transition table.

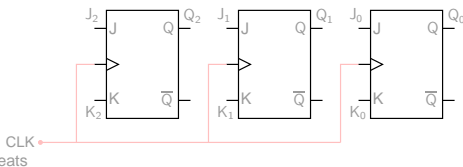
Outline of method:

- * State 1 \rightarrow State 2 means
 $Q_2: 0 \rightarrow 0,$
 $Q_1: 0 \rightarrow 0,$
 $Q_0: 0 \rightarrow 1.$

Design of synchronous counters

state	Q_2	Q_1	Q_0
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
1	0	0	0

↓ repeats



CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design a synchronous mod-5 counter with the given state transition table.

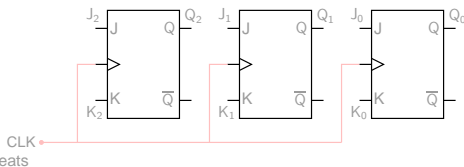
Outline of method:

- * State 1 \rightarrow State 2 means
 $Q_2: 0 \rightarrow 0$,
 $Q_1: 0 \rightarrow 0$,
 $Q_0: 0 \rightarrow 1$.
- * Refer to the right table. For $Q_2: 0 \rightarrow 0$, we must have $J_2 = 0$, $K_2 = X$, and so on.

Design of synchronous counters

state	Q_2	Q_1	Q_0
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
1	0	0	0

↓ repeats



CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design a synchronous mod-5 counter with the given state transition table.

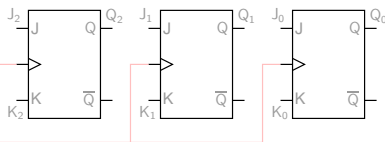
Outline of method:

- * State 1 \rightarrow State 2 means
 $Q_2: 0 \rightarrow 0$,
 $Q_1: 0 \rightarrow 0$,
 $Q_0: 0 \rightarrow 1$.
- * Refer to the right table. For $Q_2: 0 \rightarrow 0$, we must have $J_2 = 0$, $K_2 = X$, and so on.
- * When we cover all transitions in the left table, we have the truth tables for J_0 , K_0 , J_1 , K_1 , J_2 , K_2 in terms of Q_1 , Q_2 , Q_3 .

Design of synchronous counters

state	Q_2	Q_1	Q_0
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
1	0	0	0

CLK
↓ repeats



CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design a synchronous mod-5 counter with the given state transition table.

Outline of method:

- * State 1 \rightarrow State 2 means
 Q_2 : 0 \rightarrow 0,
 Q_1 : 0 \rightarrow 0,
 Q_0 : 0 \rightarrow 1.
- * Refer to the right table. For Q_2 : 0 \rightarrow 0, we must have $J_2 = 0$, $K_2 = X$, and so on.
- * When we cover all transitions in the left table, we have the truth tables for J_0 , K_0 , J_1 , K_1 , J_2 , K_2 in terms of Q_1 , Q_2 , Q_3 .
- * The last step is to come up with suitable functions for J_0 , K_0 , J_1 , K_1 , J_2 , K_2 in terms of Q_1 , Q_2 , Q_3 . This can be done with K-maps. (If the number of flip-flops is more than 4, other techniques can be employed.)

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0						
2	0	0	1						
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0						
2	0	0	1						
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X				
2	0	0	1						
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X		
2	0	0	1						
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1						
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1						
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X				
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X		
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0						
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X				
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0		
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1						
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X				
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1		
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0						
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1				
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1	0	X		
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1	0	X	0	X
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1	0	X	0	X
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

- * We now have the truth tables for J_0 , K_0 , J_1 , K_1 , J_2 , K_2 in terms of Q_0 , Q_1 , Q_2 . The next step is to find logical functions for each of them.

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1	0	X	0	X
1	0	0	0						

CLK	Q_n	Q_{n+1}	J	K
↑	0	0	0	X
↑	0	1	1	X
↑	1	0	X	1
↑	1	1	X	0

- * We now have the truth tables for J_0 , K_0 , J_1 , K_1 , J_2 , K_2 in terms of Q_0 , Q_1 , Q_2 . The next step is to find logical functions for each of them.
- * Note that we have not tabulated the J and K values for those combinations of Q_0 , Q_1 , Q_2 which do not occur in the state transition table (such as $Q_2 Q_1 Q_0 = 110$). We treat these as don't care conditions (next slide).

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1	0	X	0	X
1	0	0	0						

J_2

Q_2Q_1	00	01	11	10
Q_0	0	0	0	X
1	0	1	X	X

J_1

Q_2Q_1	00	01	11	10
Q_0	0	0	X	X
1	1	X	X	X

J_0

Q_2Q_1	00	01	11	10
Q_0	0	1	1	0
1	X	X	X	X

K_2

Q_2Q_1	00	01	11	10
Q_0	0	X	X	1
1	X	X	X	X

K_1

Q_2Q_1	00	01	11	10
Q_0	0	X	0	X
1	X	1	X	X

K_0

Q_2Q_1	00	01	11	10
Q_0	0	X	X	X
1	1	1	X	X

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1	0	X	0	X
1	0	0	0						

J_2

$Q_2 Q_1$	00	01	11	10
Q_0	0	0	0	X
1	0	1	X	X

J_1

$Q_2 Q_1$	00	01	11	10
Q_0	0	0	X	X
1	1	X	X	X

J_0

$Q_2 Q_1$	00	01	11	10
Q_0	0	1	1	0
1	X	X	X	X

K_2

$Q_2 Q_1$	00	01	11	10
Q_0	0	X	X	1
1	X	X	X	X

K_1

$Q_2 Q_1$	00	01	11	10
Q_0	0	X	0	X
1	X	1	X	X

K_0

$Q_2 Q_1$	00	01	11	10
Q_0	0	X	X	X
1	1	1	X	X

- * We treat the unused states ($Q_2 Q_1 Q_0 = 101, 110, 111$) as (additional) don't care conditions. Since these are different from the don't care conditions arising from the state transition table, we mark them with a different colour.

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1	0	X	0	X
1	0	0	0						

J_2

Q_2Q_1	00	01	11	10
Q_0	0	0	0	X
1	0	1	X	X

J_1

Q_2Q_1	00	01	11	10
Q_0	0	0	X	X
1	1	X	X	X

J_0

Q_2Q_1	00	01	11	10
Q_0	0	1	1	0
1	X	X	X	X

K_2

Q_2Q_1	00	01	11	10
Q_0	0	X	X	1
1	X	X	X	X

K_1

Q_2Q_1	00	01	11	10
Q_0	0	X	0	X
1	X	1	X	X

K_0

Q_2Q_1	00	01	11	10
Q_0	0	X	X	X
1	1	1	X	X

- * We treat the unused states ($Q_2Q_1Q_0 = 101, 110, 111$) as (additional) don't care conditions. Since these are different from the don't care conditions arising from the state transition table, we mark them with a different colour.
- * We will assume that a suitable initialization facility is provided to ensure that the counter starts up in one of the five allowed states (say, $Q_2Q_1Q_0 = 000$).

Design of synchronous counters

state	Q_2	Q_1	Q_0	J_2	K_2	J_1	K_1	J_0	K_0
1	0	0	0	0	X	0	X	1	X
2	0	0	1	0	X	1	X	X	1
3	0	1	0	0	X	X	0	1	X
4	0	1	1	1	X	X	1	X	1
5	1	0	0	X	1	0	X	0	X
1	0	0	0						

J_2

Q_2Q_1	00	01	11	10
Q_0	0	0	0	X
1	0	1	X	X

J_1

Q_2Q_1	00	01	11	10
Q_0	0	0	X	X
1	1	X	X	X

J_0

Q_2Q_1	00	01	11	10
Q_0	0	1	1	0
1	X	X	X	X

K_2

Q_2Q_1	00	01	11	10
Q_0	0	X	X	1
1	X	X	X	X

K_1

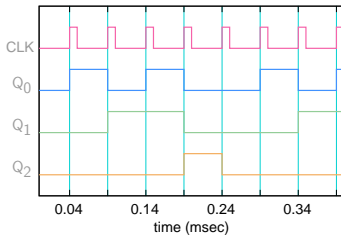
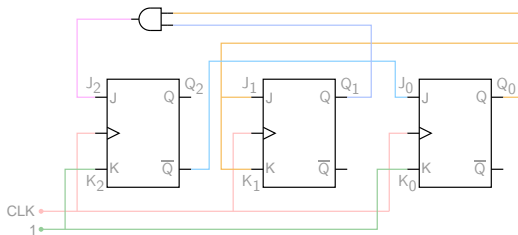
Q_2Q_1	00	01	11	10
Q_0	0	X	0	X
1	X	1	X	X

K_0

Q_2Q_1	00	01	11	10
Q_0	0	X	X	X
1	1	1	X	X

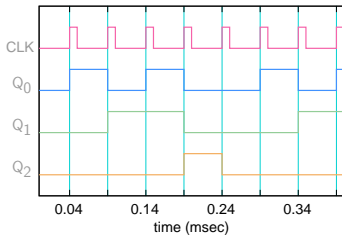
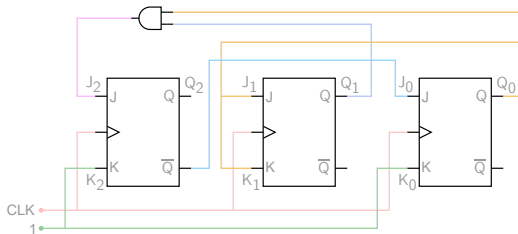
- * We treat the unused states ($Q_2Q_1Q_0 = 101, 110, 111$) as (additional) don't care conditions. Since these are different from the don't care conditions arising from the state transition table, we mark them with a different colour.
- * We will assume that a suitable initialization facility is provided to ensure that the counter starts up in one of the five allowed states (say, $Q_2Q_1Q_0 = 000$).
- * From the K-maps, $J_2 = Q_1Q_0$, $K_2 = 1$, $J_1 = Q_0$, $K_1 = Q_0$, $J_0 = \overline{Q_2}$, $K_0 = 1$.

Design of synchronous counters: verification



(SEQUEL file: ee101_counter_6.sqproj)

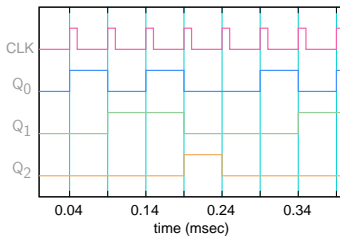
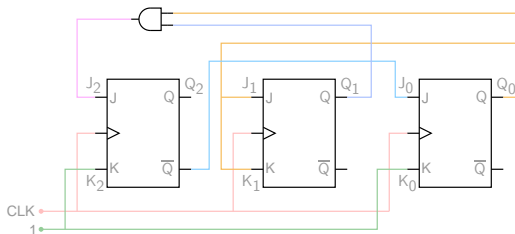
Design of synchronous counters: verification



(SEQUEL file: ee101_counter_6.sqproj)

* $J_2 = Q_1 Q_0$, $K_2 = 1$, $J_1 = Q_0$, $K_1 = Q_0$, $J_0 = \overline{Q_2}$, $K_0 = 1$.

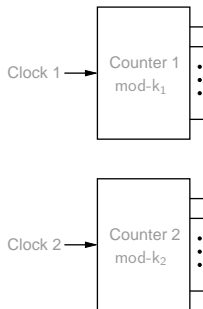
Design of synchronous counters: verification



(SEQUEL file: ee101_counter_6.sqproj)

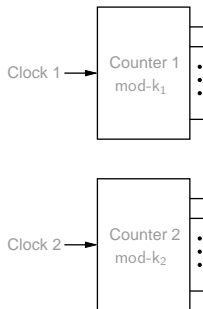
- * $J_2 = Q_1 Q_0$, $K_2 = 1$, $J_1 = Q_0$, $K_1 = Q_0$, $J_0 = \overline{Q_2}$, $K_0 = 1$.
- * Note that the design is independent of whether positive or negative edge-triggered flip-flops are used.

Combination of counters



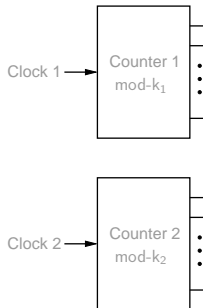
- * Consider two counters, Counter 1 (mod- k_1) and Counter 2 (mod- k_2).
(Each of them can be ripple or synchronous type.)

Combination of counters



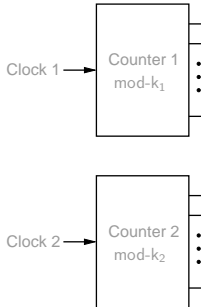
- * Consider two counters, Counter 1 (mod- k_1) and Counter 2 (mod- k_2). (Each of them can be ripple or synchronous type.)
- * Since Counter 1 has k_1 states and Counter 2 has k_2 states, we can get a new counter with $k_1 k_2$ states if appropriate *synchronisation* is provided between the two clocks.

Combination of counters



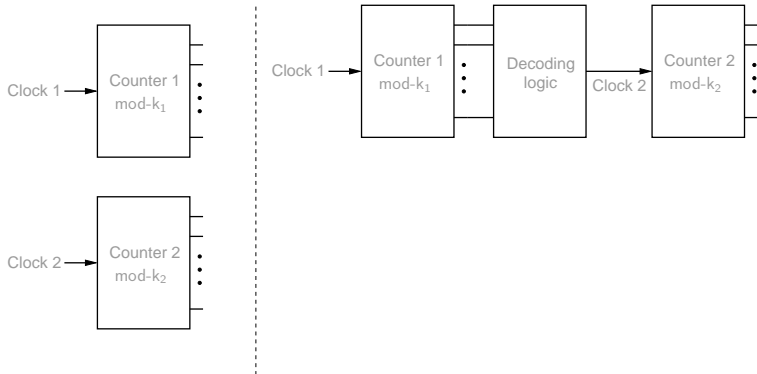
- * Consider two counters, Counter 1 (mod- k_1) and Counter 2 (mod- k_2). (Each of them can be ripple or synchronous type.)
- * Since Counter 1 has k_1 states and Counter 2 has k_2 states, we can get a new counter with $k_1 k_2$ states if appropriate *synchronisation* is provided between the two clocks.
- * There are two ways of providing synchronisation:

Combination of counters



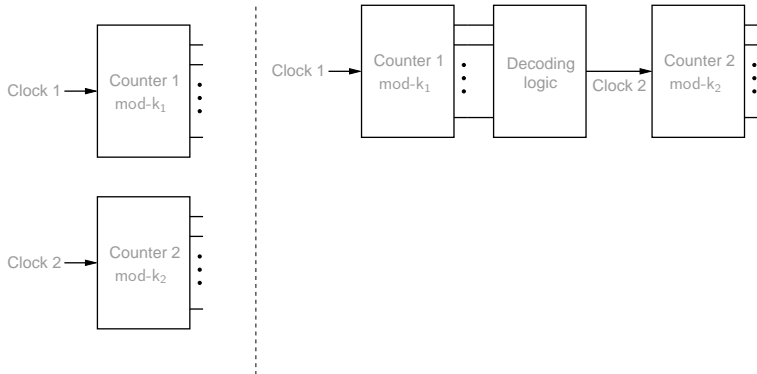
- * Consider two counters, Counter 1 (mod- k_1) and Counter 2 (mod- k_2). (Each of them can be ripple or synchronous type.)
- * Since Counter 1 has k_1 states and Counter 2 has k_2 states, we can get a new counter with $k_1 k_2$ states if appropriate *synchronisation* is provided between the two clocks.
- * There are two ways of providing synchronisation:
 - derive Clock 2 from Clock 1 (using some decoding logic, if necessary)

Combination of counters



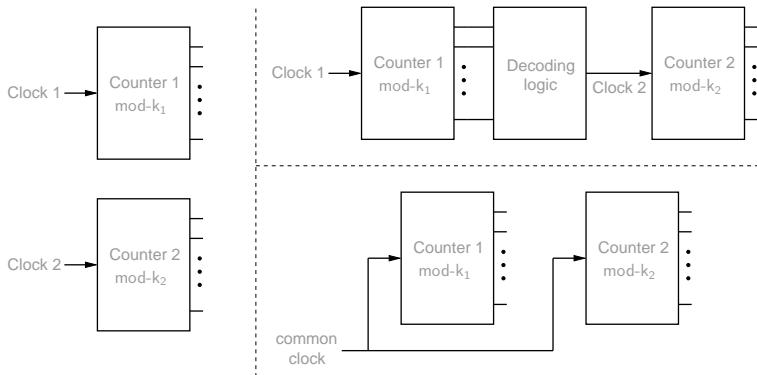
- * Consider two counters, Counter 1 (mod- k_1) and Counter 2 (mod- k_2). (Each of them can be ripple or synchronous type.)
- * Since Counter 1 has k_1 states and Counter 2 has k_2 states, we can get a new counter with $k_1 k_2$ states if appropriate *synchronisation* is provided between the two clocks.
- * There are two ways of providing synchronisation:
 - derive Clock 2 from Clock 1 (using some decoding logic, if necessary)

Combination of counters



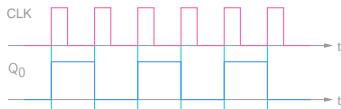
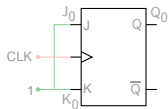
- * Consider two counters, Counter 1 (mod- k_1) and Counter 2 (mod- k_2). (Each of them can be ripple or synchronous type.)
- * Since Counter 1 has k_1 states and Counter 2 has k_2 states, we can get a new counter with $k_1 k_2$ states if appropriate *synchronisation* is provided between the two clocks.
- * There are two ways of providing synchronisation:
 - derive Clock 2 from Clock 1 (using some decoding logic, if necessary)
 - drive the two counters with the same clock

Combination of counters

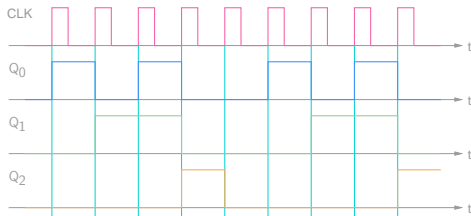
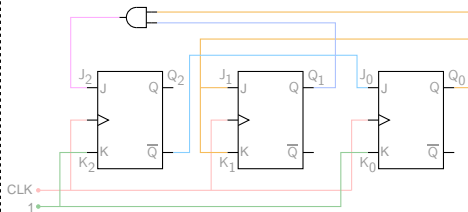


- * Consider two counters, Counter 1 (mod- k_1) and Counter 2 (mod- k_2). (Each of them can be ripple or synchronous type.)
- * Since Counter 1 has k_1 states and Counter 2 has k_2 states, we can get a new counter with $k_1 k_2$ states if appropriate *synchronisation* is provided between the two clocks.
- * There are two ways of providing synchronisation:
 - derive Clock 2 from Clock 1 (using some decoding logic, if necessary)
 - drive the two counters with the same clock

Combination of counters

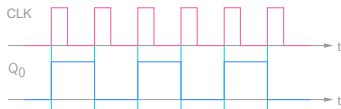
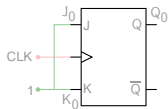


mod-2 counter

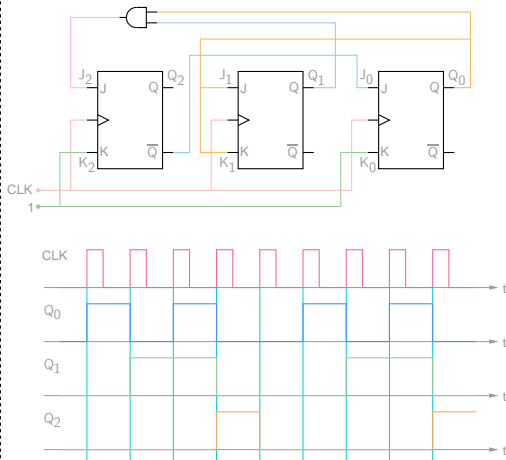


mod-5 counter

Combination of counters



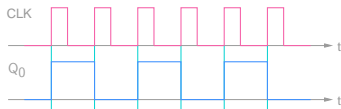
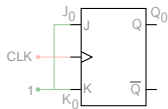
mod-2 counter



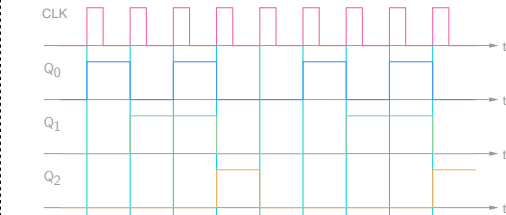
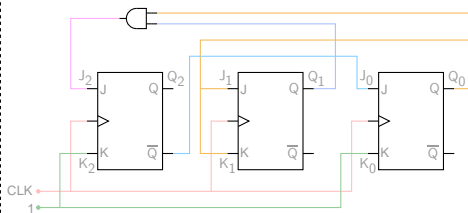
mod-5 counter

- * Let us combine the mod-2 and mod-5 counters to make a mod-10 counter.

Combination of counters



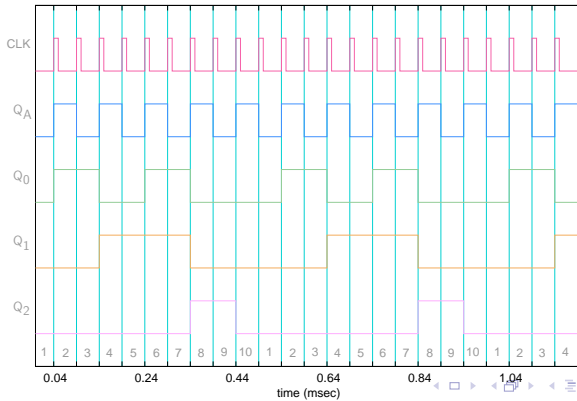
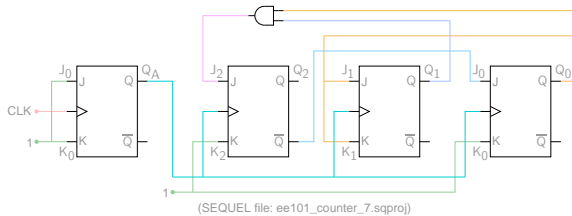
mod-2 counter



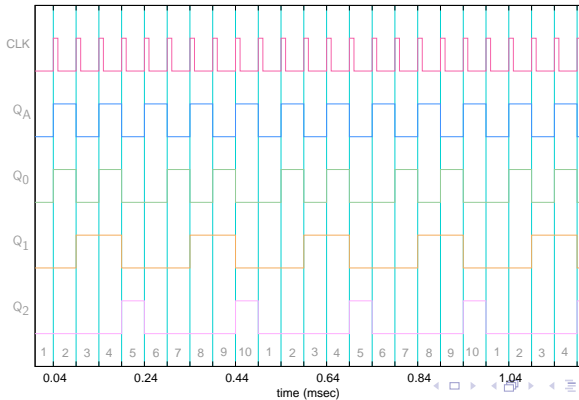
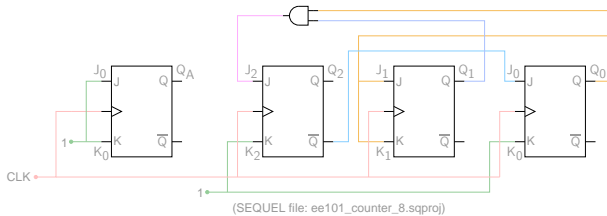
mod-5 counter

- * Let us combine the mod-2 and mod-5 counters to make a mod-10 counter.
- * We will follow two approaches (as described earlier):
 - A: The clock for the second (mod-5) counter is derived from the first (mod-2) counter.
 - B: A common clock is used to drive the mod-2 and mod-5 counters.

Approach A



Approach B



Combination of counters

- * Show that, by connecting the \overline{Q} output of the mod-2 counter (instead of the Q output) to the clock input of the mod-5 counter in the ripple connection (“Approach A”) circuit, we get a decade counter, counting up from 0000 to 1001.

Combination of counters

- * Show that, by connecting the \overline{Q} output of the mod-2 counter (instead of the Q output) to the clock input of the mod-5 counter in the ripple connection (“Approach A”) circuit, we get a decade counter, counting up from 0000 to 1001.
- * Derive appropriate decoding logic for each of the ten counters states (i.e., the output should be 1 for only that particular state and 0 otherwise).

Combination of counters

- * Show that, by connecting the \overline{Q} output of the mod-2 counter (instead of the Q output) to the clock input of the mod-5 counter in the ripple connection (“Approach A”) circuit, we get a decade counter, counting up from 0000 to 1001.
- * Derive appropriate decoding logic for each of the ten counters states (i.e., the output should be 1 for only that particular state and 0 otherwise).
- * Derive appropriate decoding logic which will give a symmetrical square wave (i.e., a duty cycle of 50 %) with a frequency of $f_c/10$, where f_c is the clock frequency.

Combination of counters

- * Show that, by connecting the \overline{Q} output of the mod-2 counter (instead of the Q output) to the clock input of the mod-5 counter in the ripple connection (“Approach A”) circuit, we get a decade counter, counting up from 0000 to 1001.
- * Derive appropriate decoding logic for each of the ten counters states (i.e., the output should be 1 for only that particular state and 0 otherwise).
- * Derive appropriate decoding logic which will give a symmetrical square wave (i.e., a duty cycle of 50 %) with a frequency of $f_c/10$, where f_c is the clock frequency.
- * Verify your design by simulation.