

EE101: Basics

KCL, KVL, power, Thevenin's theorem

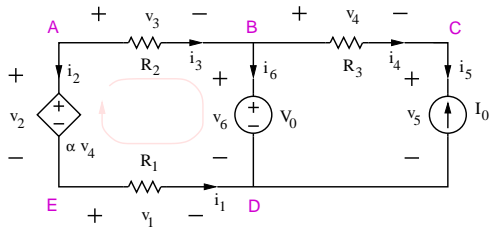


M. B. Patil

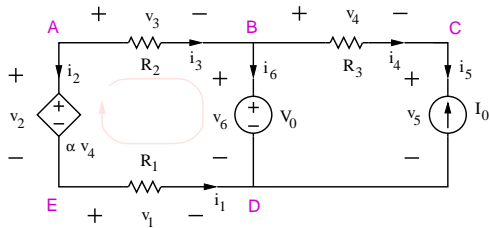
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Department of Electrical Engineering
Indian Institute of Technology Bombay

Kirchhoff's laws



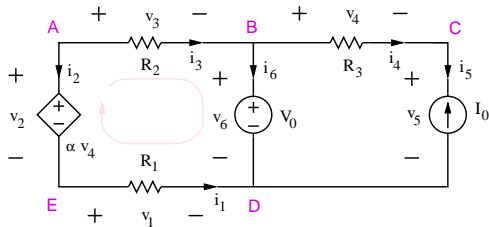
Kirchhoff's laws



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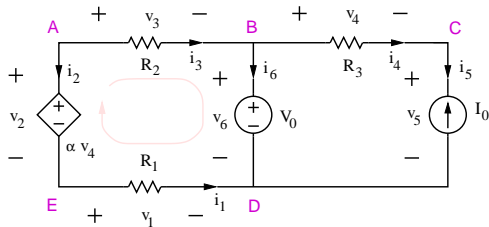
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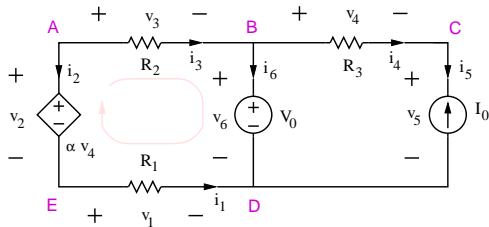
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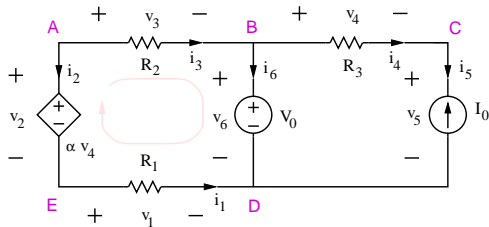
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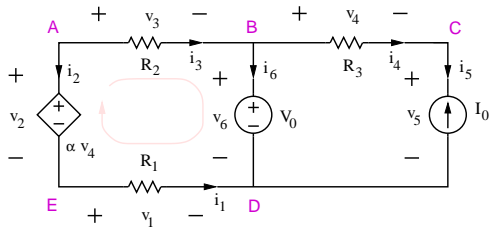
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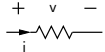
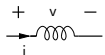
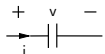
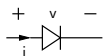
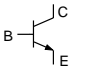
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





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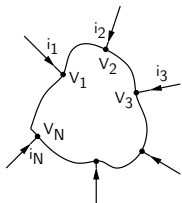
(We have followed the convention that voltage *drop* across a branch is positive.)

Element	Symbol	Equation
Resistor		$v = R i$
Inductor		$v = L \frac{di}{dt}$
Capacitor		$i = C \frac{dv}{dt}$
Diode		to be discussed
BJT		to be discussed

	Element	Symbol	Equation
Independent	Voltage source		$v(t) = v_s(t)$
	Current source		$i(t) = i_s(t)$
Dependent	VCVS		$v(t) = \alpha v_c(t)$
	VCCS		$i(t) = g v_c(t)$
	CCVS		$v(t) = r i_c(t)$
	CCCS		$i(t) = \beta i_c(t)$

- * α, β : dimensionless, r : Ω , g : Ω^{-1} or \mathcal{U} ("mho")
- * The subscript 'c' denotes the controlling voltage or current.

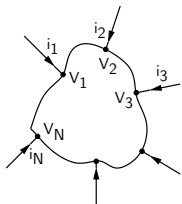
Instantaneous power absorbed by an element



$$P(t) = V_1(t) i_1(t) + V_2(t) i_2(t) + \cdots + V_N(t) i_N(t),$$

where V_1, V_2 , etc. are “node voltages” (measured with respect to a reference node).

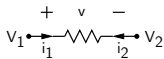
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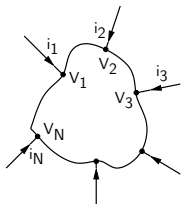
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$$\begin{aligned} P &= V_1 i_1 + V_2 i_2 \\ &= V_1 i_1 + V_2 (-i_1) \\ &= [V_1 - V_2] i_1 = v i_1 \end{aligned}$$

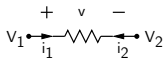
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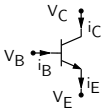
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* three-terminal element:



$$\begin{aligned} P &= V_B i_B + V_C i_C + V_E (-i_E) \\ &= V_B i_B + V_C i_C - V_E (i_B + i_C) \\ &= (V_B - V_E) i_B + (V_C - V_E) i_C \\ &= V_{BE} i_B + V_{CE} i_C \end{aligned}$$

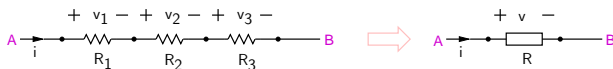
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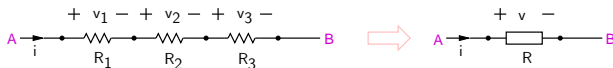
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- * A capacitor can absorb or deliver power. When it is absorbing power, its charge builds up. Similarly, an inductor can store energy (in the form of magnetic flux).

Resistors in series

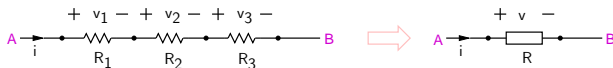


Resistors in series



$$v_1 = i R_1, v_2 = i R_2, v_3 = i R_3, \Rightarrow v = v_1 + v_2 + v_3 = i(R_1 + R_2 + R_3)$$

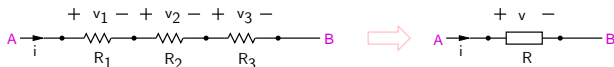
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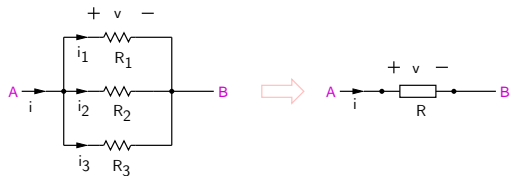
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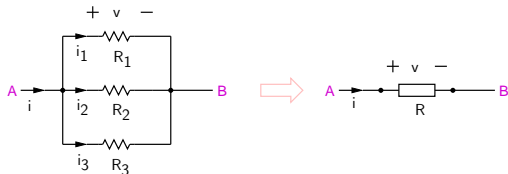
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- * The voltage drop across R_k is $v \times \frac{R_k}{R_{eq}}$.

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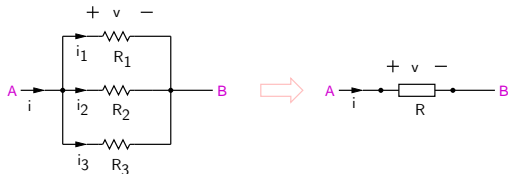
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$i_1 = G_1 v$, $i_2 = G_2 v$, $i_3 = G_3 v$, where $G_1 = 1/R_1$, etc.

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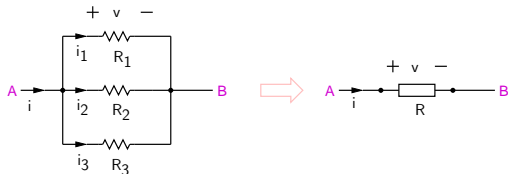


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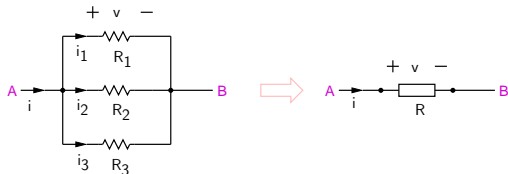


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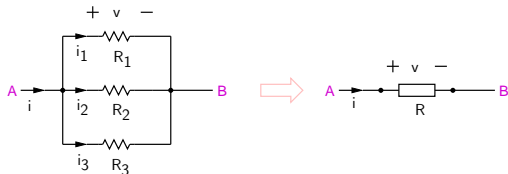


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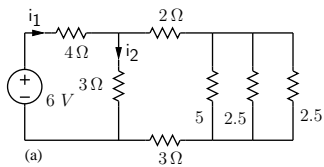
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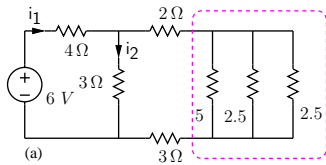
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- * If $R_k = 0$, all of the current will go through R_k .

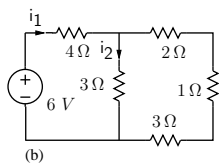
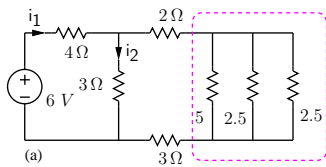
Example



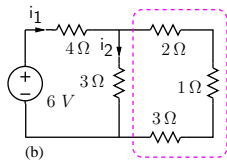
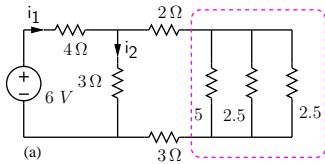
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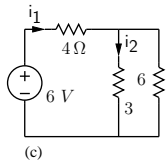
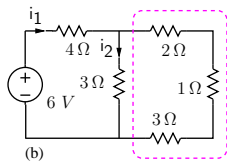
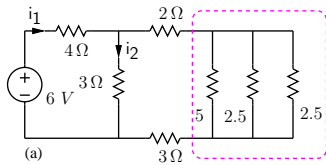
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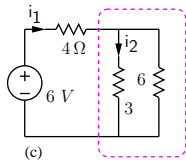
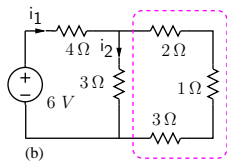
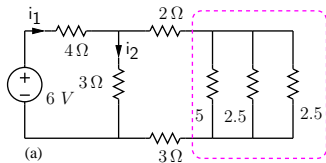
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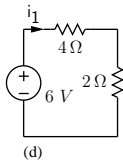
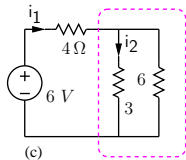
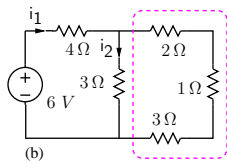
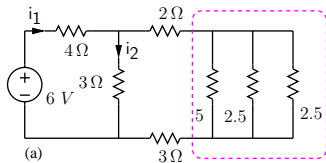
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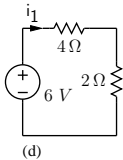
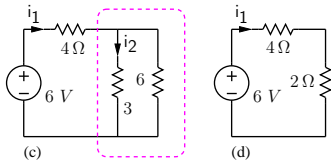
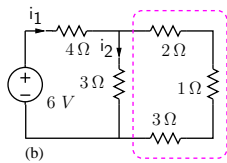
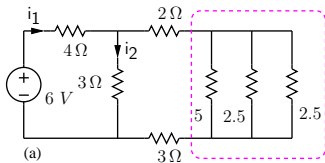
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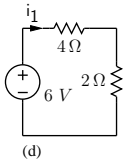
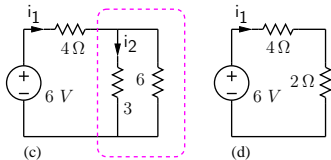
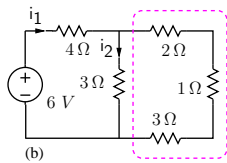
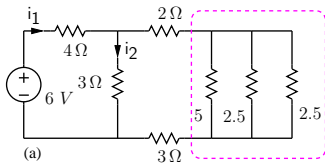


Example



$$i_1 = \frac{6V}{4\Omega + 2\Omega} = 1A.$$

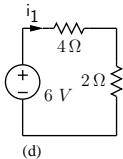
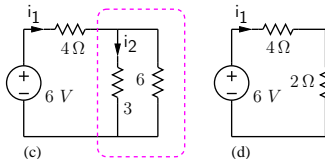
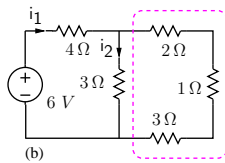
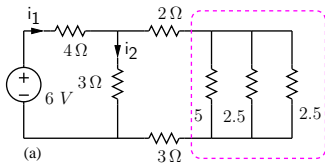
Example



$$i_1 = \frac{6V}{4\Omega + 2\Omega} = 1A.$$

$$i_2 = i_1 \times \frac{6\Omega}{6\Omega + 3\Omega} = \frac{2}{3}A.$$

Example



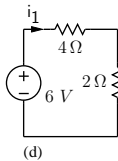
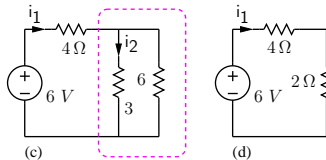
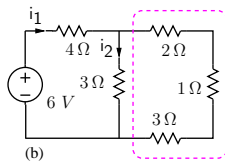
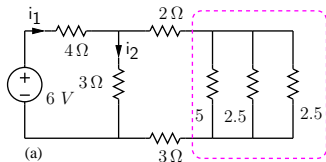
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Home work:

- * Verify that KCL and KVL are satisfied for each node/loop.

Example



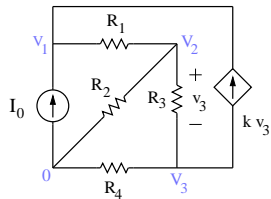
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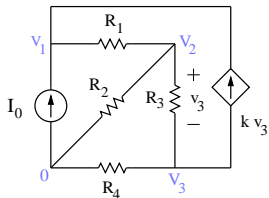
- * Verify that KCL and KVL are satisfied for each node/loop.
- * Verify that the total power absorbed by the resistors is equal to the power supplied by the source.

Nodal analysis

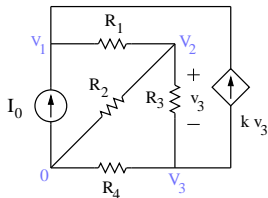


Nodal analysis

- * Take some node as the “reference node” and denote the node voltages of the remaining nodes by V_1 , V_2 , etc.

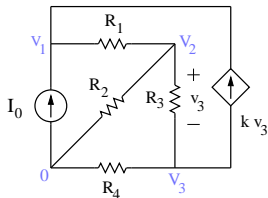


Nodal analysis



- * Take some node as the “reference node” and denote the node voltages of the remaining nodes by V_1 , V_2 , etc.
- * Write KCL at each node in terms of the node voltages. Follow a fixed convention, e.g., current *leaving* a node is *positive*.

Nodal analysis



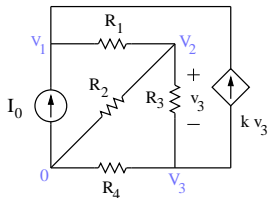
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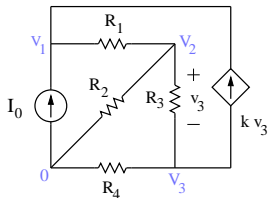
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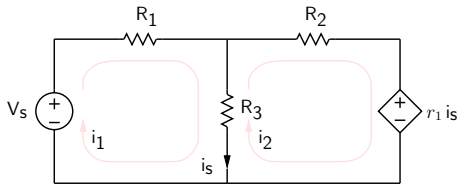
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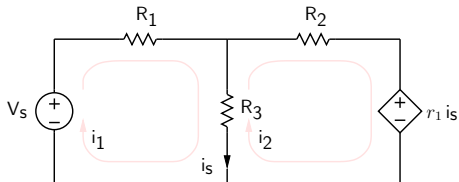
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- * Remark: Nodal analysis needs to be modified if there are voltage sources.

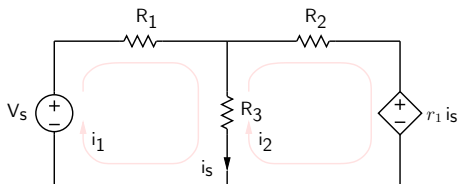
Mesh analysis



Mesh analysis

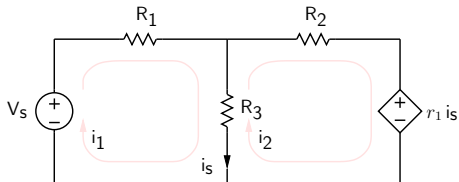


- * Write KVL for each loop in terms of the “mesh currents” i_1 and i_2 . Use a fixed convention, e.g., voltage drop is positive. (Note that $i_s = i_1 - i_2$.)



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- * Solve for i_1 and i_2 \rightarrow compute other quantities of interest (branch currents and branch voltages).

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- * Caution: Superposition cannot be applied to *dependent* sources.

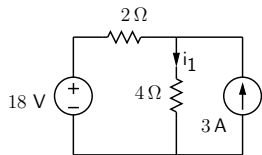
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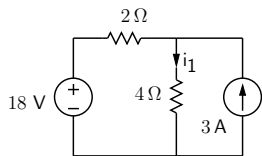
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- * Deactivating a current source $\Rightarrow i_s = 0$, i.e., replace the current source with an open circuit.
- * Deactivating a voltage source $\Rightarrow v_s = 0$, i.e., replace the voltage source with a short circuit.

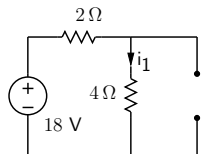
Example



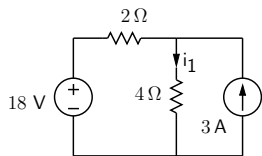
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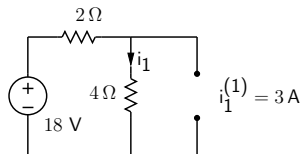
Case 1: Keep V_s , deactivate I_s .



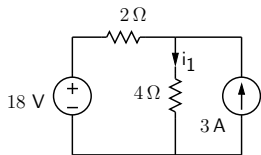
Example



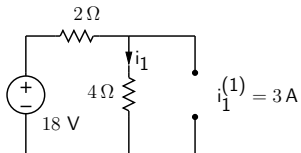
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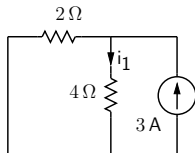
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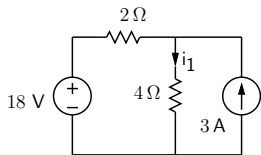
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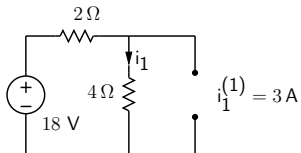
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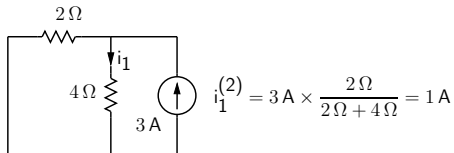
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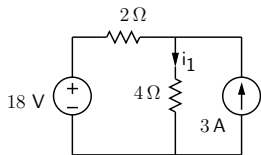
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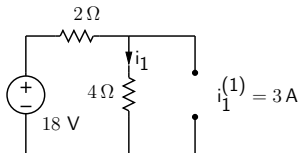


Example

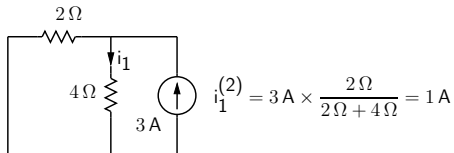


$$i_1^{\text{net}} = i_1^{(1)} + i_1^{(2)} = 3 + 1 = 4 \text{ A}$$

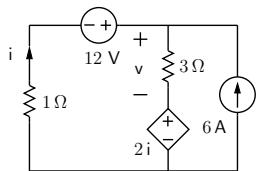
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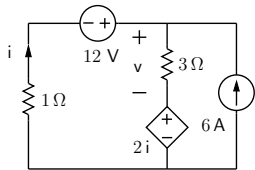
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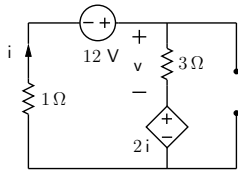
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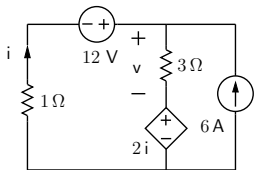
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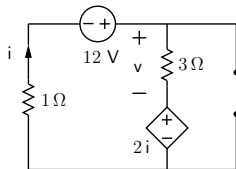
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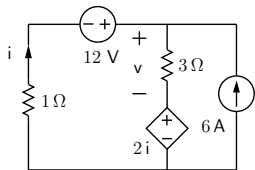
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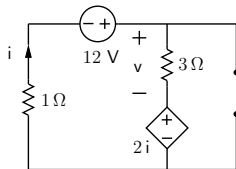
$$\text{KVL: } -12 + 3i + 2i + i = 0$$

$$\Rightarrow i = 2\text{ A}, v^{(1)} = 6\text{ V}.$$

Example

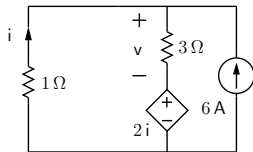


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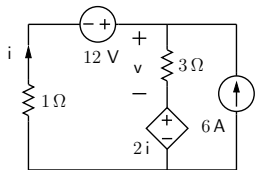


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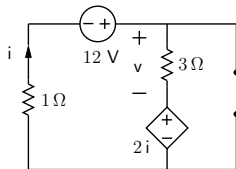
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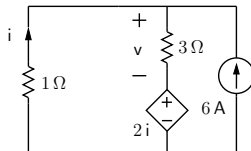
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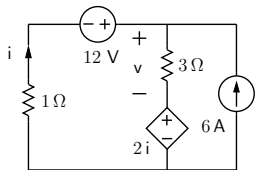
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$$\text{KVL: } i + (6 + i)3 + 2i = 0$$

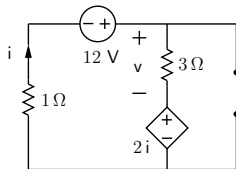
$$\Rightarrow i = -3\text{ A}, v^{(2)} = (-3 + 6) \times 3 = 9\text{ V}.$$

Example



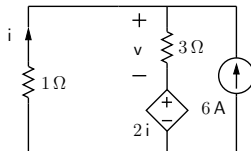
$$v_{\text{net}} = v(1) + v(2) = 6 + 9 = 15 \text{ V}$$

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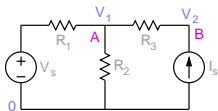
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Case 2: Keep I_S , deactivate V_S .

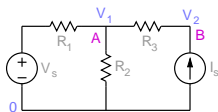


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Superposition: Why does it work?



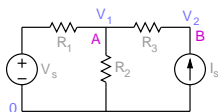
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KCL at nodes A and B:

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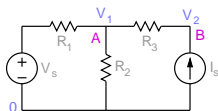
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Writing in a matrix form, we get (using $G_1 = 1/R_1$, etc.),

$$\begin{bmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}$$

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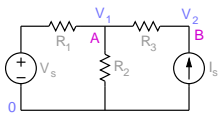
$$\begin{aligned}\frac{1}{R_1}(V_1 - V_s) + \frac{1}{R_2}V_1 + \frac{1}{R_3}(V_1 - V_2) &= 0, \\ -I_s + \frac{1}{R_3}(V_2 - V_1) &= 0.\end{aligned}$$

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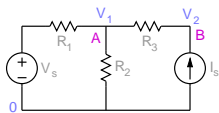
$$\text{i.e., } \mathbf{A} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}.$$

Superposition: Why does it work?



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} .$$

Superposition: Why does it work?

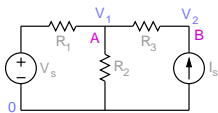


$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}.$$

We are now in a position to see why superposition works.

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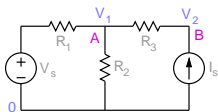
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The first vector is the response due to V_s alone (and I_s deactivated).

The second vector is the response due to I_s alone (and V_s deactivated).

Superposition: Why does it work?



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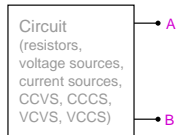
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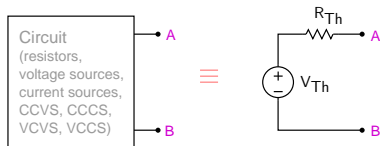
All other currents and voltages are linearly related to V_1 and V_2

\Rightarrow Any voltage (node voltage or branch voltage) or current can also be computed using superposition.

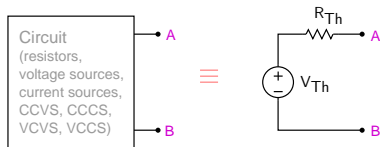
Thevenin's theorem



Thevenin's theorem

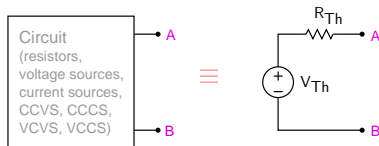


Thevenin's theorem



- * V_{Th} is simply V_{AB} when nothing is connected on the other side, i.e., $V_{Th} = V_{oc}$.

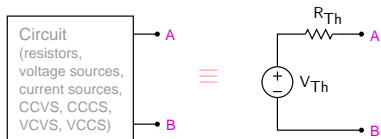
Thevenin's theorem



- * V_{Th} is simply V_{AB} when nothing is connected on the other side, i.e., $V_{Th} = V_{oc}$.
- * R_{Th} can be found by different methods.

Thevenin's theorem: R_{Th}

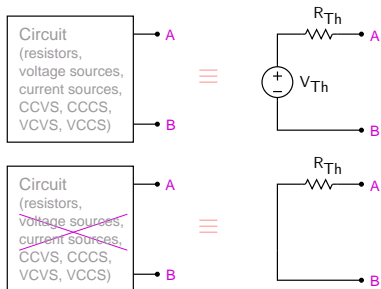
Method 1:



- * Deactivate all *independent* sources.

Thevenin's theorem: R_{Th}

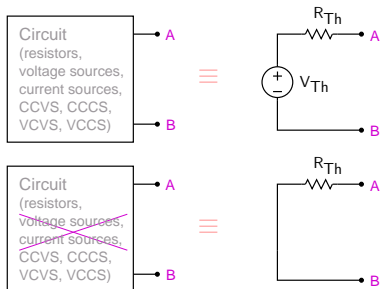
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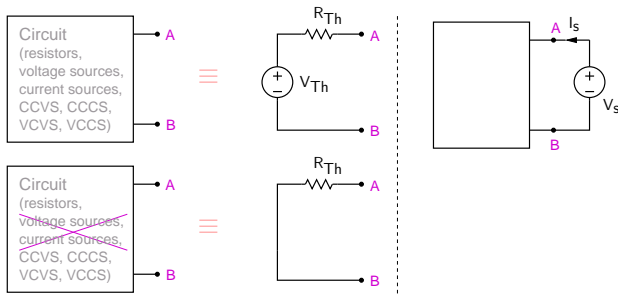
Method 1:



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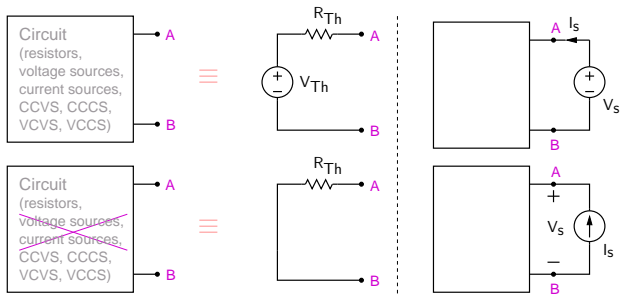
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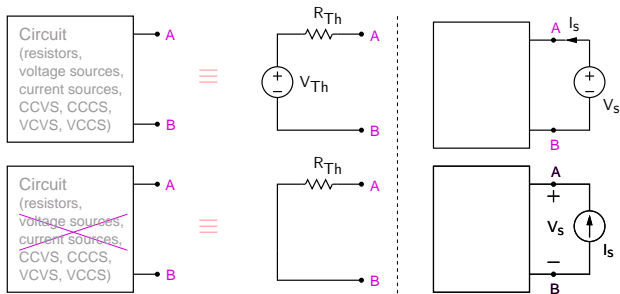
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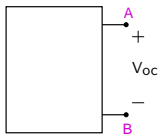
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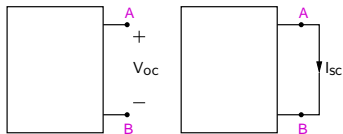
Method 2:



* Find V_{oc} .

Thevenin's theorem: R_{Th}

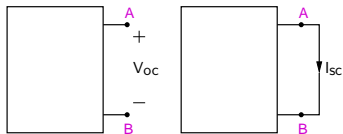
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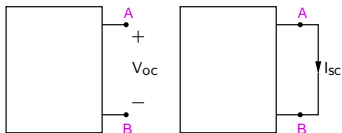
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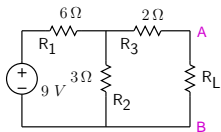
Thevenin's theorem: R_{Th}

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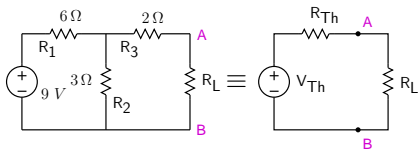


- * Find V_{oc} .
- * Find I_{sc} .
- * $R_{Th} = \frac{V_{oc}}{I_{sc}}$.
- * Note: Sources are *not* deactivated.

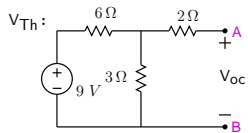
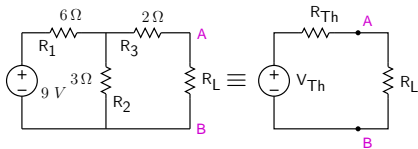
Thevenin's theorem: example



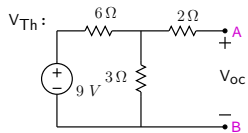
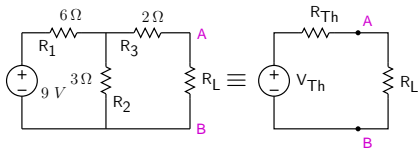
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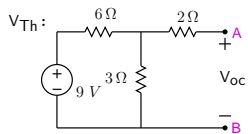
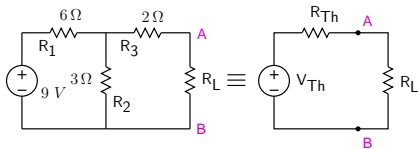


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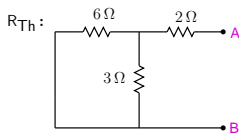


$$\begin{aligned}V_{\text{oc}} &= 9\text{V} \times \frac{3\ \Omega}{6\ \Omega + 3\ \Omega} \\ &= 9\text{V} \times \frac{1}{3} = 3\text{V}\end{aligned}$$

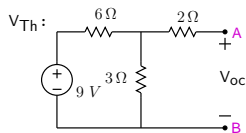
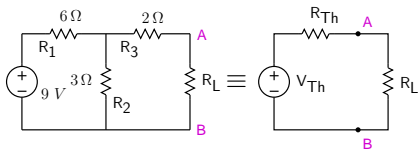
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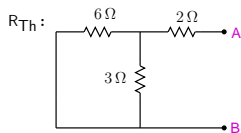
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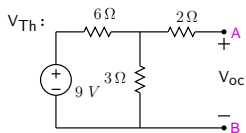
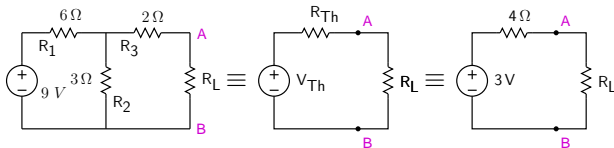


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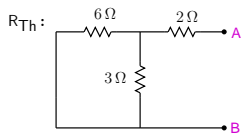


$$\begin{aligned}R_{Th} &= (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2 \\ &= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\Omega\end{aligned}$$

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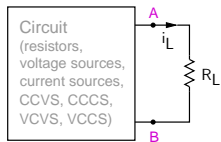


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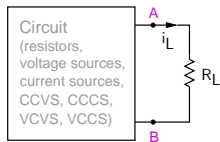


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Maximum power transfer

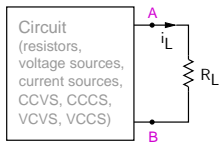


Maximum power transfer



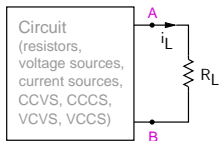
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 $P_L = i_L^2 R_L$.

Maximum power transfer



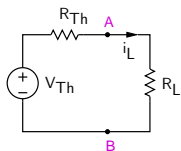
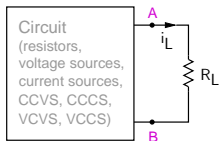
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Maximum power transfer



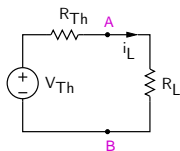
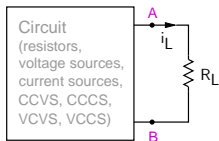
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Maximum power transfer



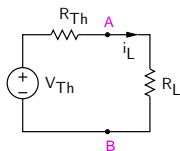
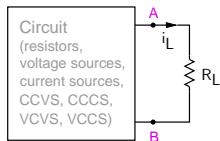
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- * $i_L = \frac{V_{Th}}{R_{Th} + R_L}$,
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Maximum power transfer



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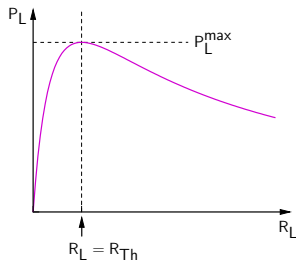
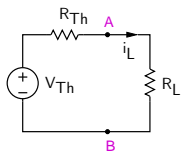
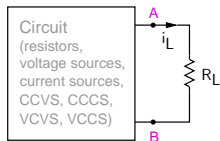
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$$\frac{(R_{Th} + R_L)^2 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} = 0,$$

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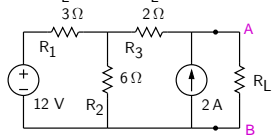
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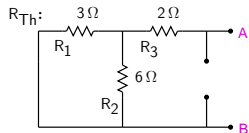
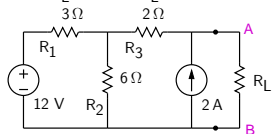
Maximum power transfer: example

Find R_L for which P_L is maximum.



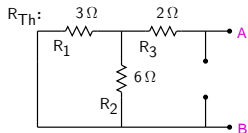
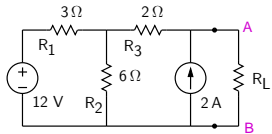
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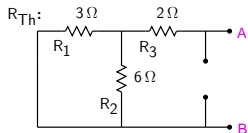
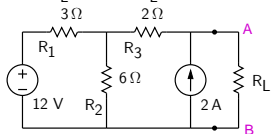


$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2} \right) + 2 = 4\ \Omega$$

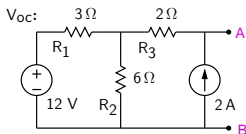
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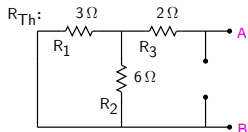
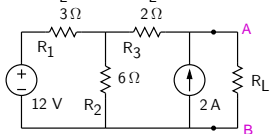
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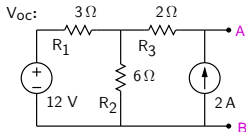


Maximum power transfer: example

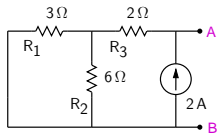
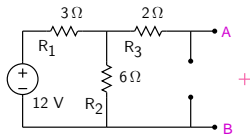
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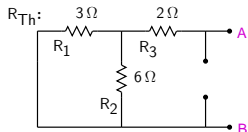
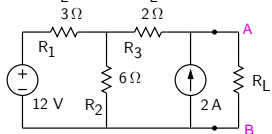


Use superposition to find V_{oc} :

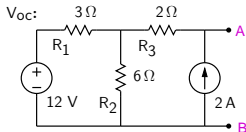


Maximum power transfer: example

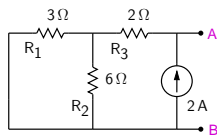
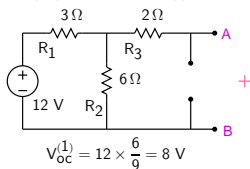
Find R_L for which P_L is maximum.



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$
$$= 3 \times \left(\frac{1 \times 2}{1 + 2} \right) + 2 = 4 \Omega$$

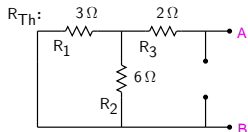
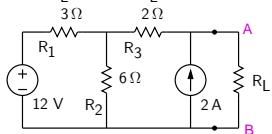


Use superposition to find V_{oc} :

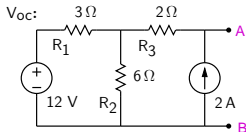


Maximum power transfer: example

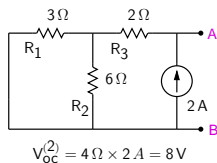
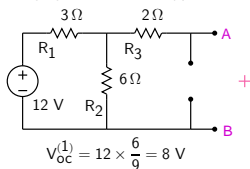
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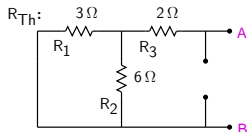
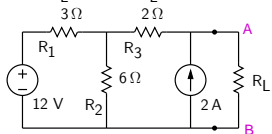


Use superposition to find V_{oc} :

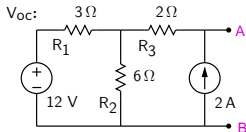


Maximum power transfer: example

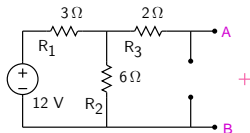
Find R_L for which P_L is maximum.



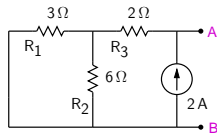
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$
$$= 3 \times \left(\frac{1 \times 2}{1 + 2} \right) + 2 = 4 \Omega$$



Use superposition to find V_{oc} :



$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8 \text{ V}$$

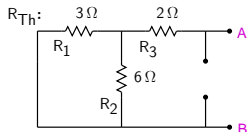
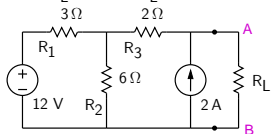


$$V_{oc}^{(2)} = 4 \Omega \times 2 A = 8 \text{ V}$$

$$V_{oc} = V_{oc}^{(1)} + V_{oc}^{(2)} = 8 + 8 = 16 \text{ V}$$

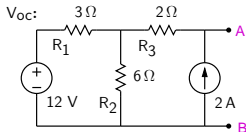
Maximum power transfer: example

Find R_L for which P_L is maximum.

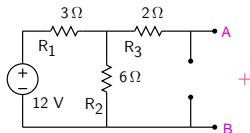


$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

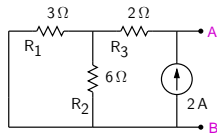
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Use superposition to find V_{oc} :

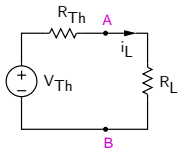


$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8 \text{ V}$$



$$V_{oc}^{(2)} = 4 \Omega \times 2 \text{ A} = 8 \text{ V}$$

$$V_{oc} = V_{oc}^{(1)} + V_{oc}^{(2)} = 8 + 8 = 16 \text{ V}$$

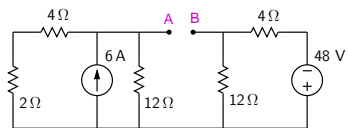


P_L is maximum when $R_L = R_{Th} = 4 \Omega$

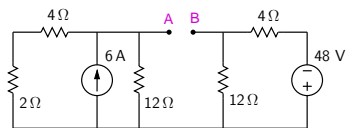
$$\Rightarrow i_L = V_{Th} / (2R_{Th}) = 2 \text{ A}$$

$$P_L^{\max} = 2^2 \times 4 = 16 \text{ W}$$

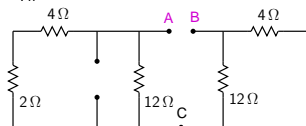
Thevenin's theorem: example



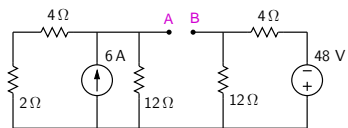
Thevenin's theorem: example



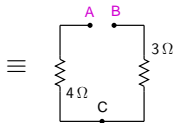
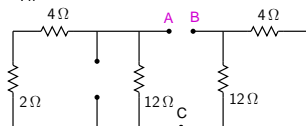
R_{Th} :



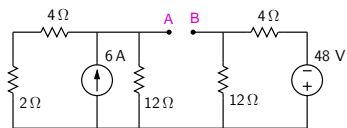
Thevenin's theorem: example



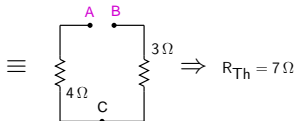
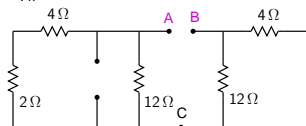
R_{Th} :



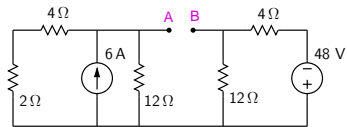
Thevenin's theorem: example



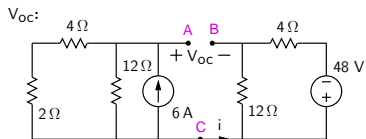
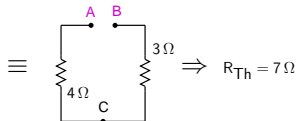
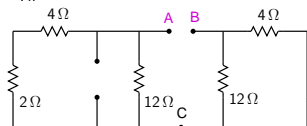
R_{Th} :



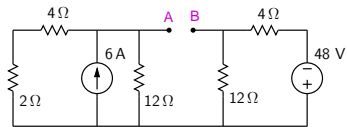
Thevenin's theorem: example



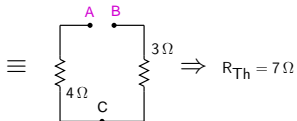
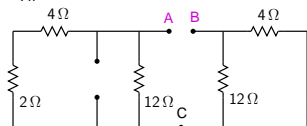
R_{Th} :



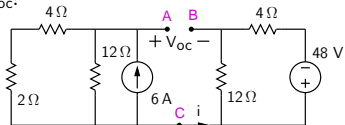
Thevenin's theorem: example



R_{Th} :



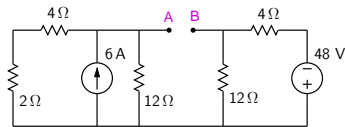
V_{oc} :



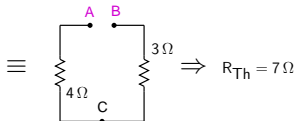
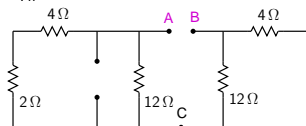
Note: $i = 0$ (since there is no return path).

$$\begin{aligned}V_{AB} &= V_A - V_B \\ &= (V_A - V_C) + (V_C - V_B) \\ &= V_{AC} + V_{CB} \\ &= 24\text{V} + 36\text{V} = 60\text{V}\end{aligned}$$

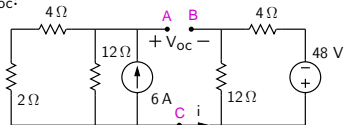
Thevenin's theorem: example



R_{Th} :



V_{oc} :



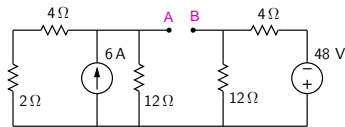
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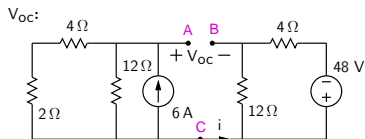
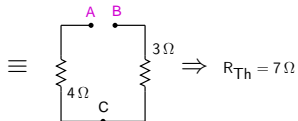
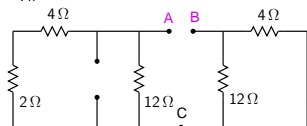
$$V_{Th} = 60\text{ V}$$

$$R_{Th} = 7\ \Omega$$

Thevenin's theorem: example



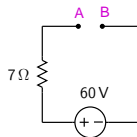
R_{Th} :



Note: $i = 0$ (since there is no return path).

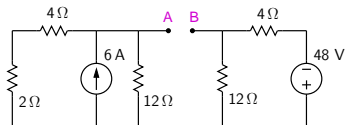
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$$\begin{aligned} V_{Th} &= 60\text{ V} \\ R_{Th} &= 7\ \Omega \end{aligned}$$



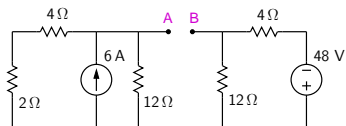
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



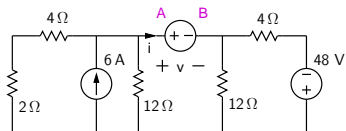
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



Connect a voltage source between A and B.

Plot i versus v .

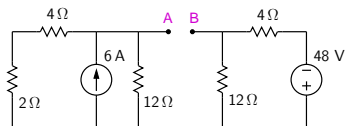


V_{OC} = intercept on the v -axis.

I_{SC} = intercept on the i -axis.

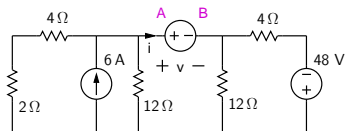
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SEQUEL file: ee101_thevenin_1.sqproj



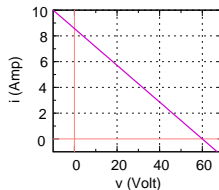
Connect a voltage source between A and B.

Plot i versus v .



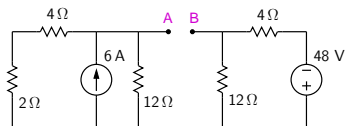
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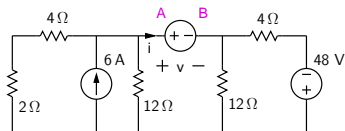
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



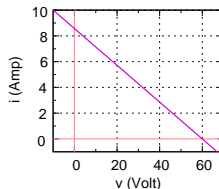
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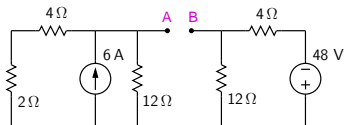


$$V_{OC} = 60\ \text{V}, I_{SC} = 8.5714\ \text{A}$$

$$R_{Th} = V_{SC}/I_{SC} = 7\ \Omega$$

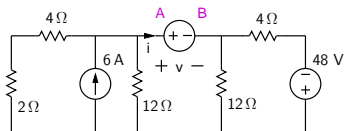
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sproj



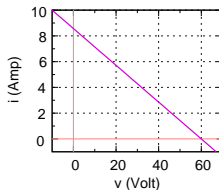
Connect a voltage source between A and B.

Plot i versus v .



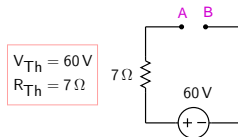
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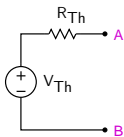


$$V_{OC} = 60 \text{ V}, I_{SC} = 8.5714 \text{ A}$$

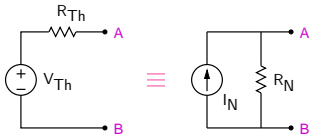
$$R_{Th} = V_{SC}/I_{SC} = 7 \Omega$$



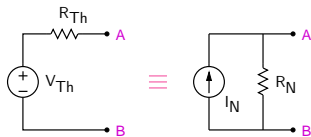
Norton equivalent circuit



Norton equivalent circuit

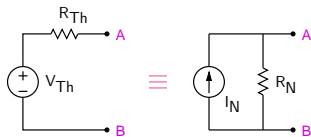


Norton equivalent circuit



* Consider the open circuit case.

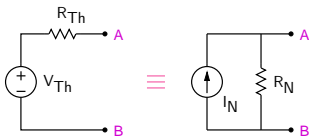
Norton equivalent circuit



- * Consider the open circuit case.

Thevenin circuit: $V_{AB} = V_{Th}$.

Norton equivalent circuit

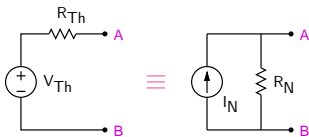


* Consider the open circuit case.

Thevenin circuit: $V_{AB} = V_{Th}$.

Norton circuit: $V_{AB} = I_N R_N$.

Norton equivalent circuit



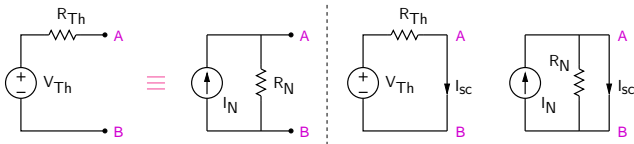
* Consider the open circuit case.

Thevenin circuit: $V_{AB} = V_{Th}$.

Norton circuit: $V_{AB} = I_N R_N$.

$\Rightarrow V_{Th} = I_N R_N$.

Norton equivalent circuit



- * Consider the open circuit case.

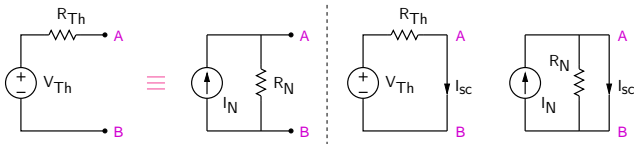
Thevenin circuit: $V_{AB} = V_{Th}$.

Norton circuit: $V_{AB} = I_N R_N$.

$\Rightarrow V_{Th} = I_N R_N$.

- * Consider the short circuit case.

Norton equivalent circuit



- * Consider the open circuit case.

Thevenin circuit: $V_{AB} = V_{Th}$.

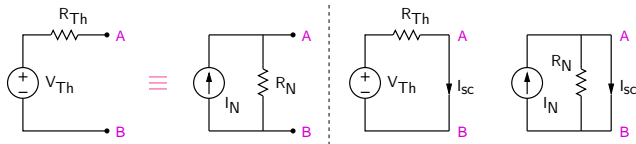
Norton circuit: $V_{AB} = I_N R_N$.

$$\Rightarrow V_{Th} = I_N R_N.$$

- * Consider the short circuit case.

Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.

Norton equivalent circuit



- * Consider the open circuit case.

Thevenin circuit: $V_{AB} = V_{Th}$.

Norton circuit: $V_{AB} = I_N R_N$.

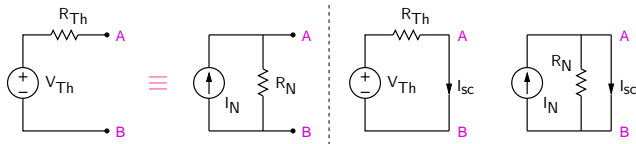
$$\Rightarrow V_{Th} = I_N R_N.$$

- * Consider the short circuit case.

Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.

Norton circuit: $I_{sc} = I_N$.

Norton equivalent circuit



- * Consider the open circuit case.

Thevenin circuit: $V_{AB} = V_{Th}$.

Norton circuit: $V_{AB} = I_N R_N$.

$$\Rightarrow V_{Th} = I_N R_N.$$

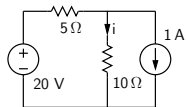
- * Consider the short circuit case.

Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.

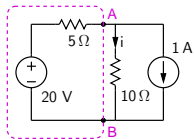
Norton circuit: $I_{sc} = I_N$.

$$\Rightarrow R_{Th} = R_N.$$

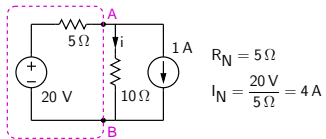
Example



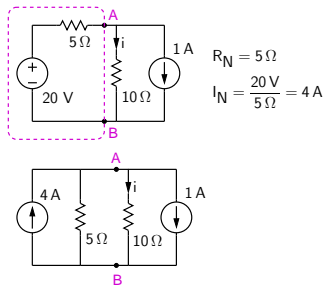
Example



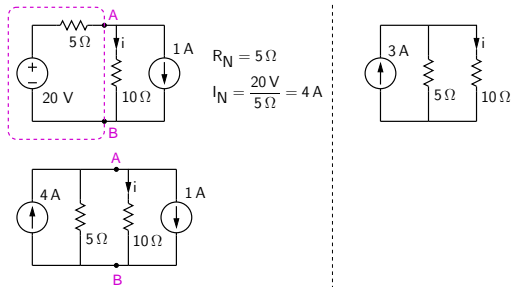
Example



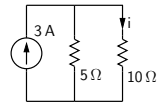
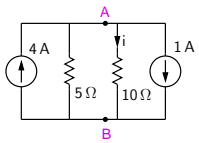
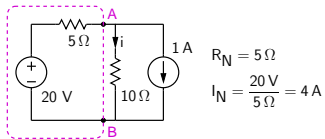
Example



Example

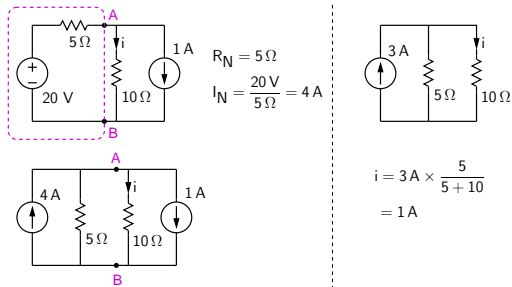


Example



$$i = 3A \times \frac{5}{5+10}$$
$$= 1A$$

Example

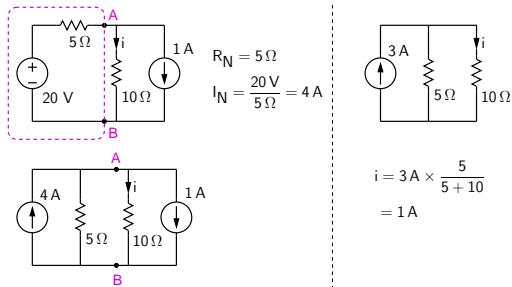


$$i = 3\text{ A} \times \frac{5}{5 + 10} = 1\text{ A}$$

Home work:

- * Find i by superposition and compare.

Example



Home work:

- * Find i by superposition and compare.
- * Compute the power absorbed by each element and verify that $\sum P_i = 0$.