EE101: Bode plots

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  Interesting facts:

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  - Bell considered the telephone an intrusion and refused to put one in his office.
- Bel turned out to be too large in practice $\longrightarrow$ deciBel (i.e., one tenth of a Bel).

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- The voltage gain of an amplifier is

  A_V in dB $= 20\,\log\,(V_o/V_i)$,

  with V_i serving as the reference voltage.

Example

Given $V_i = 2.5\,\text{mV}$ and $A_V = 36.3\,\text{dB}$,
compute V_o in dBm and in mV.

(V_i and V_o are peak input and peak output voltages, respectively).

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V_i Amplifier V_o

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$$V_o = A_V \times V_i = 65 \times 2.5\,mV = 162.5\,mV.$$

dB in audio measurements

- When sound intensity is specified in dB, the reference pressure is $P_{\text{ref}} = 20\,\mu Pa$ (our hearing threshold).

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loud thunder	110 dB

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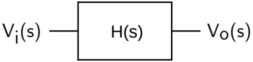
loudest sound human ear can tolerate	120 dB
windows break	163 dB

Bode plots



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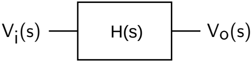

- The transfer function of a circuit such as an amplifier or a filter is given by, $H(s) = V_o(s)/V_i(s), \quad s = j\omega$.
  e.g., $H(s) = \dfrac{K}{1+s\tau} = \dfrac{K}{1+j\omega\tau}$

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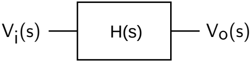

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  (a) a plot of $|H(j\omega)|$ versus ω.

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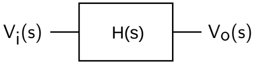

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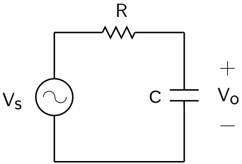
  (a) a plot of $|H(j\omega)|$ versus ω.
  (b) a plot of $\angle H(j\omega)$ versus ω.
- Bode gave simple rules which allow construction of the above "Bode plots" in an approximate (asymptotic) manner.

A simple transfer function



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1{+}s\,RC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

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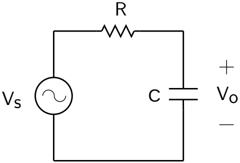

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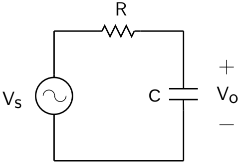

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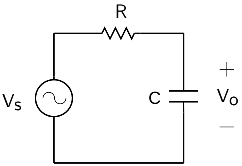

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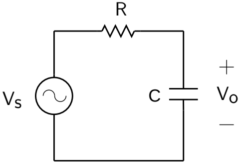

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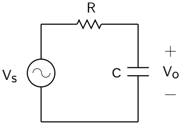
- We are generally interested in a large variation in ω (several orders), and its effect on $|H|$ and $\angle H$.
- The magnitude ($|H|$) varies by orders of magnitude as well.
  The phase ($\angle H$) varies from 0 (for $\omega \ll \omega_0$) to $-\pi/2$ (for $\omega \gg \omega_0$).

A simple transfer function: magnitude



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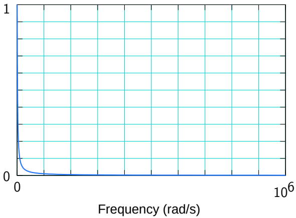

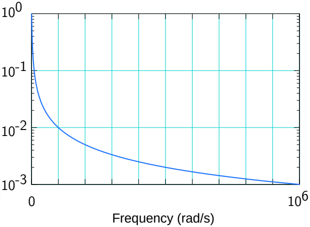

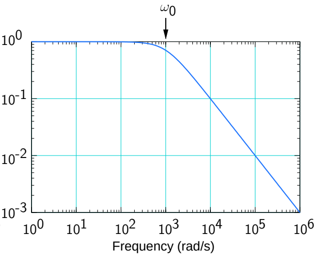

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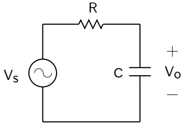

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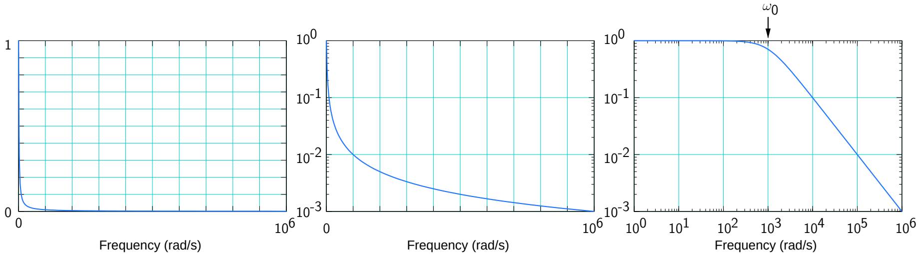

Since ω and $|H(j\,\omega)|$ vary by several orders of magnitude, a linear ω- or $|H|$-axis is not appropriate → $\log|H|$ is plotted against $\log\omega$.

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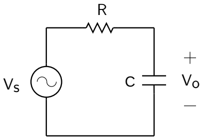

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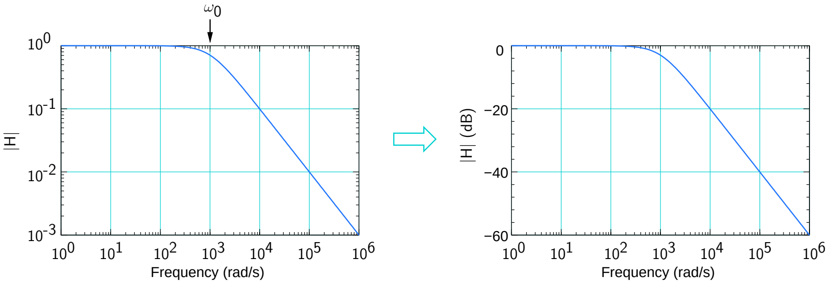

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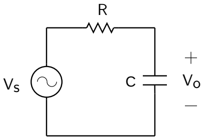

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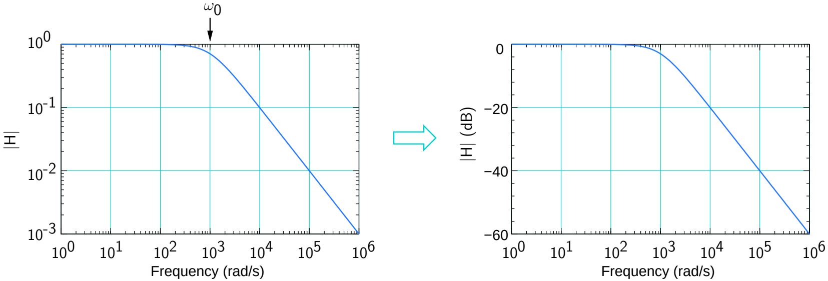
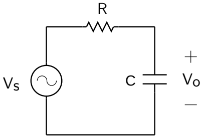


Note that the *shape* of the plot does not change.

$|H|$ (dB) $= 20 \log |H|$ is simply a scaled version of $\log |H|$.

A simple transfer function: magnitude



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

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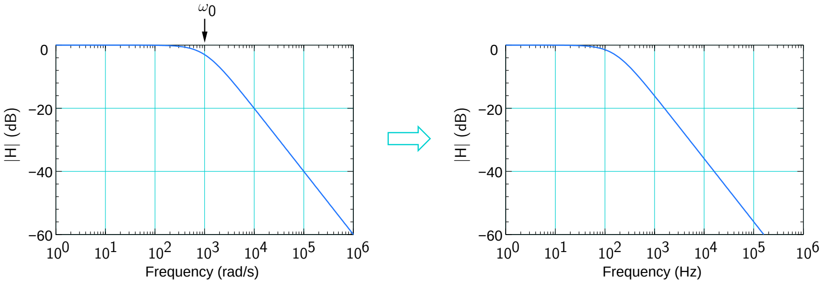

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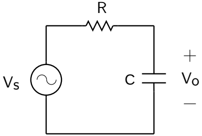

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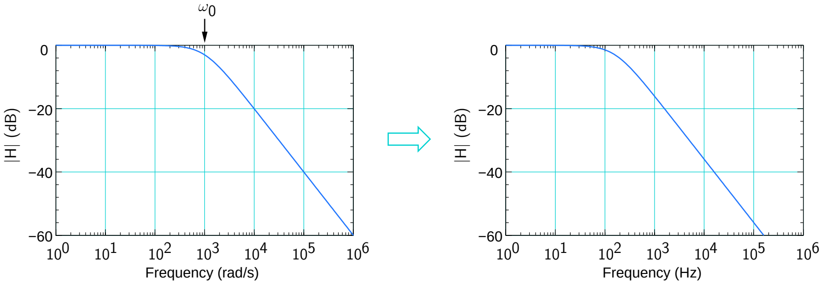
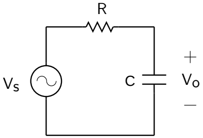


Since $\omega = 2\pi\, f$, the *shape* of the plot does not change.

A simple transfer function: phase



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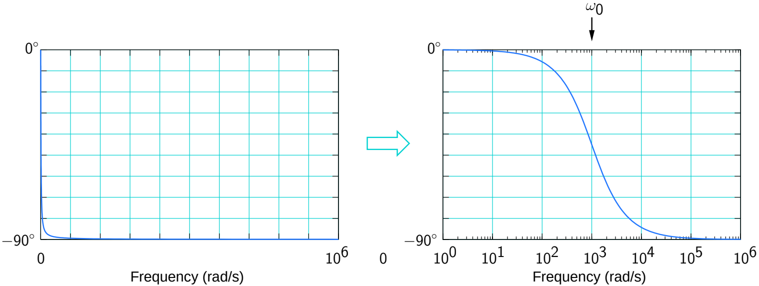

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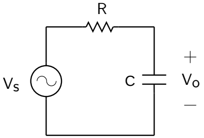

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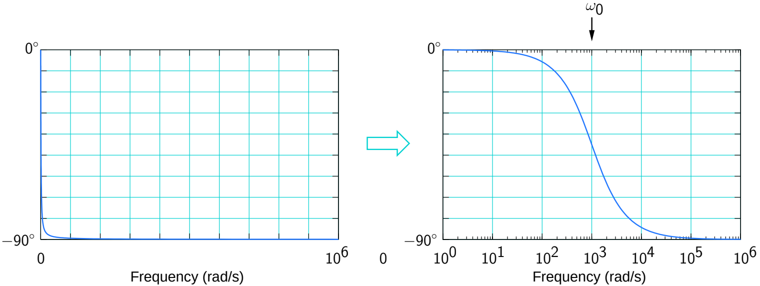

- ∗ Since $\angle H = -\tan^{-1}(\omega/\omega_0)$ varies in a limited range (0° to −90°), a linear axis is appropriate for $\angle H$.

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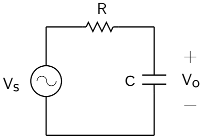

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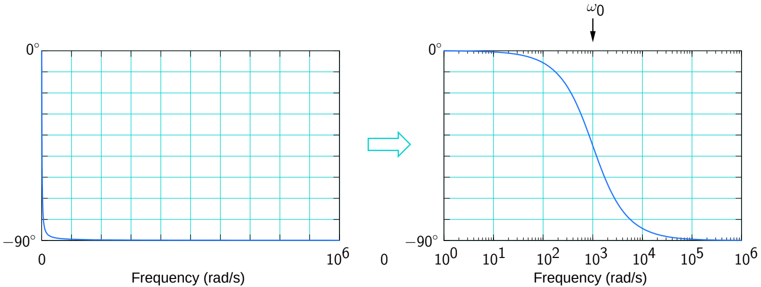

- * Since $\angle H = -\tan^{-1}(\omega/\omega_0)$ varies in a limited range (0° to -90°), a linear axis is appropriate for $\angle H$.
- * As in the magnitude plot, we use a log axis for ω, since we are interested in a wide range of ω.

Construction of Bode plots

Consider $H(s) = \dfrac{K\,(1+s/z_1)(1+s/z_2)\cdots(1+s/z_M)}{(1+s/p_1)(1+s/p_2)\cdots(1+s/p_N)}$.

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$-z_1$, $-z_2, \cdots$ are called the “zeros” of $H(s)$.

$-p_1$, $-p_2, \cdots$ are called the “poles” of $H(s)$.

(In addition, there could be terms like s, $s^2, \cdots$ in the numerator.)

We will assume, for simplicity, that the zeros (and poles) are real and distinct.

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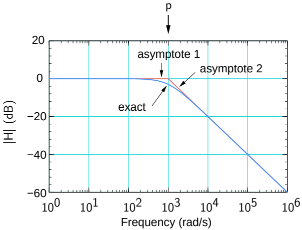
Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function ω.

(b) combining the various contributions to obtain $|H|$ and $\angle H$ versus ω.

Contribution of a pole: magnitude



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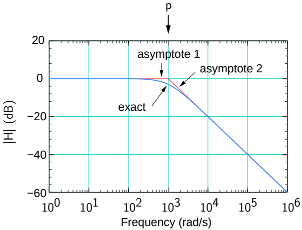

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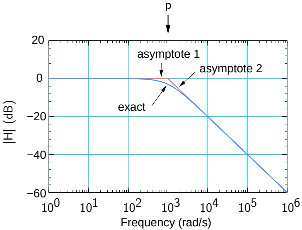

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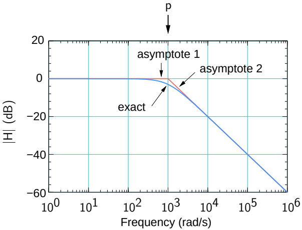

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Consider two values of ω: ω_1 and $10\,\omega_1$.

$|H|_1 = 20\ \log\ p - 20\ \log\omega_1\ \ (\mathrm{dB})$

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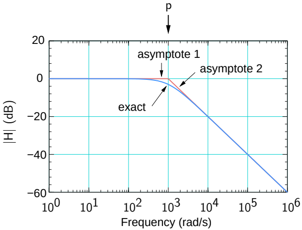

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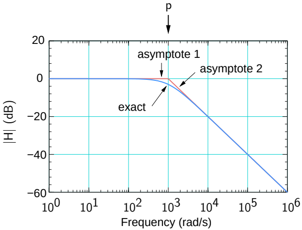

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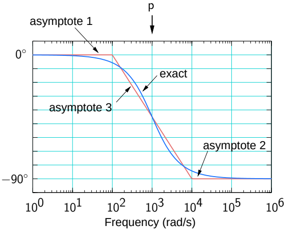
$\rightarrow$ $|H|$ versus ω has a slope of $-20\,\text{dB/decade}$.

Note that, at $\omega = p$, the actual value of $|H|$ is $1/\sqrt{2}$ (i.e., $-3\,\text{dB}$).

Contribution of a pole: phase



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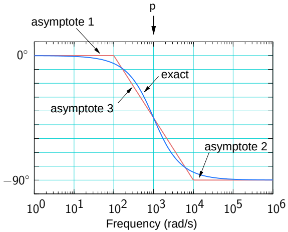

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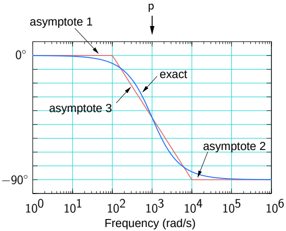

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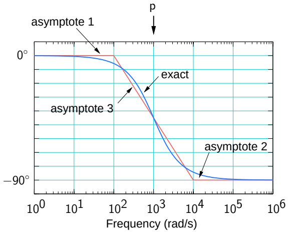

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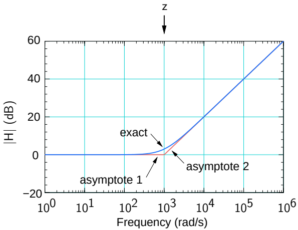
Contribution of a zero: magnitude



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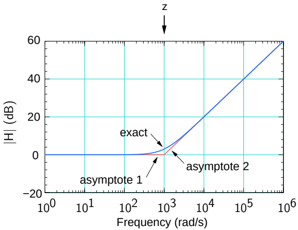

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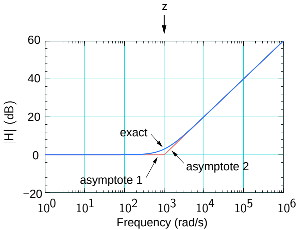

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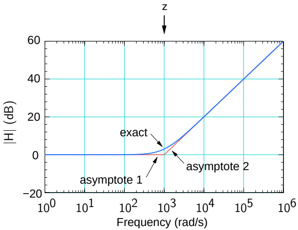

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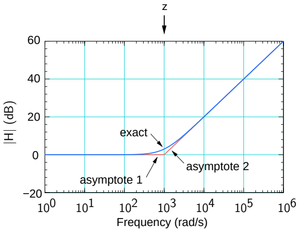

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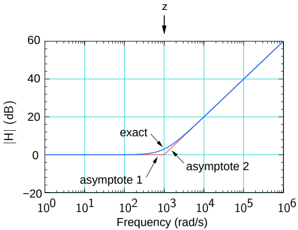

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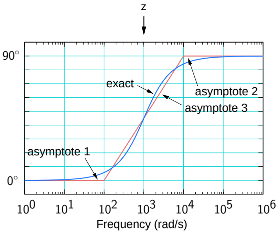

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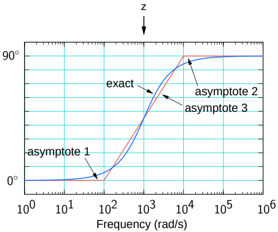

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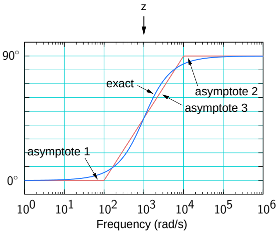

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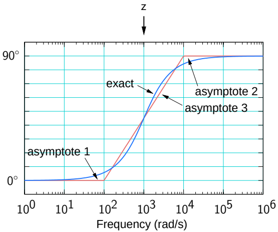

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For $H(s) = K$, $20\ \log|H| = 20\ \log\ K$ (a constant), and $\angle H = 0$.

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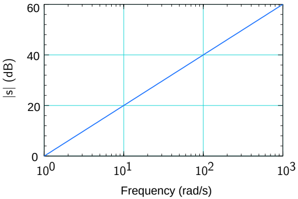

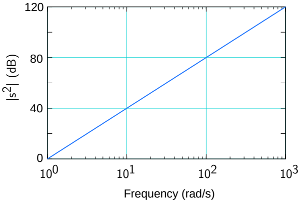

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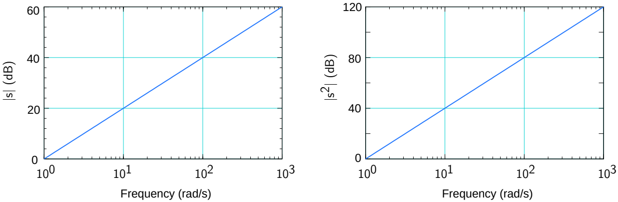

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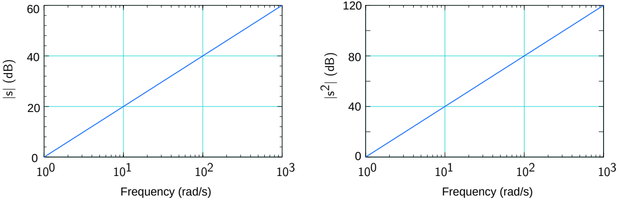

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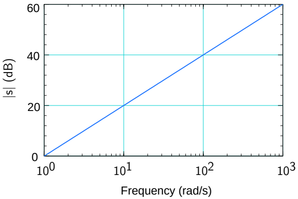

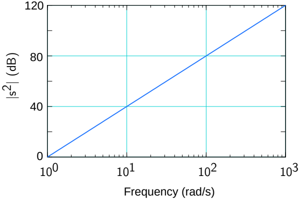

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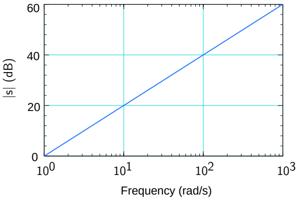

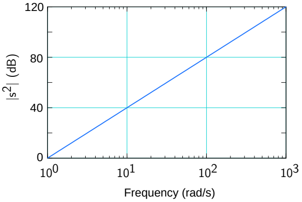

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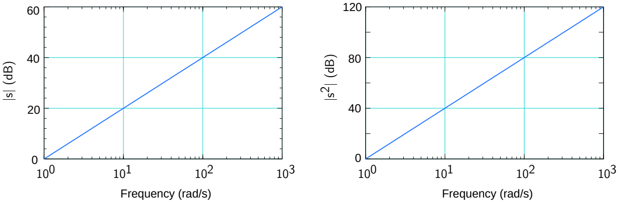

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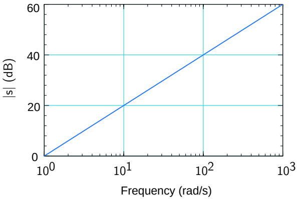

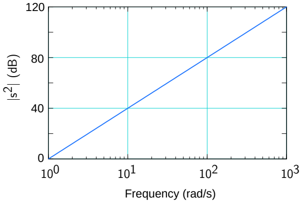

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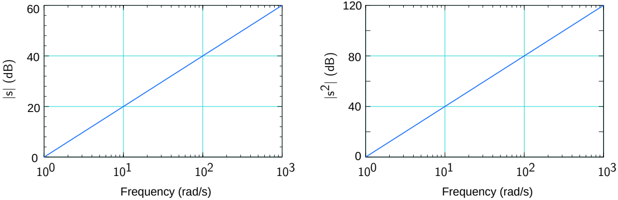

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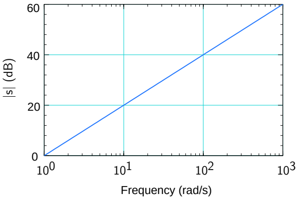

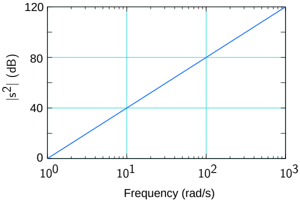

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Consider $H(s) = H_1(s) \times H_2(s)$.

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$H_1(j\omega)$ and $H_2(j\omega)$ are complex numbers.

At a given ω, let $H_1 = K_1 \angle \alpha = K_1\, e^{j\alpha}$, and $H_2 = K_2 \angle \beta = K_2\, e^{j\beta}$.

Then, $H_1 H_2 = K_1\, K_2\, e^{j(\alpha+\beta)} = K_1 K_2 \angle\,(\alpha + \beta)$.

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In the Bode phase plot, the contributions due to H_1 and H_2 also get added.

The same reasoning applies to more than two terms as well.

Combining different terms: example

Consider $H(s) = \dfrac{10\,s}{(1+s/10^2)\,(1+s/10^5)}$.

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Let $H(s) = H_1(s)\,H_2(s)\,H_3(s)\,H_4(s)$, where

$$H_1(s) = 10\,,$$
$$H_2(s) = s\,,$$
$$H_3(s) = \frac{1}{1+s/p_1}\,,\ p_1 = 10^2\ \text{rad/s},$$
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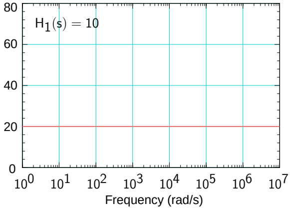
We can now plot the magnitude and phase of H_1, H_2, H_3, H_4 *individually* versus ω and then simply add them to obtain $|H|$ and $\angle H$.

Magnitude plot ($|H|$ in dB)



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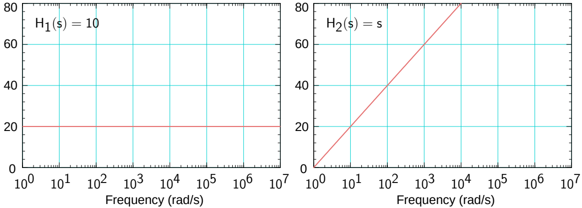

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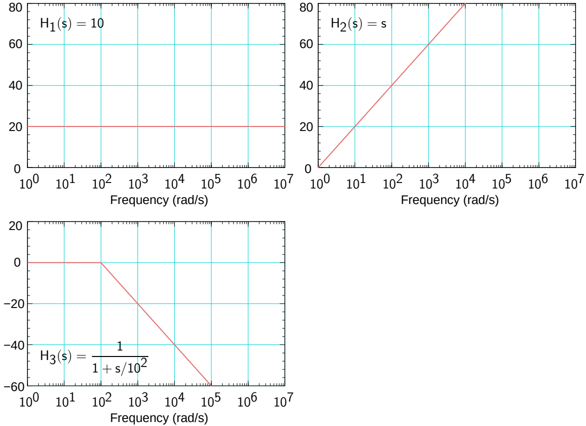

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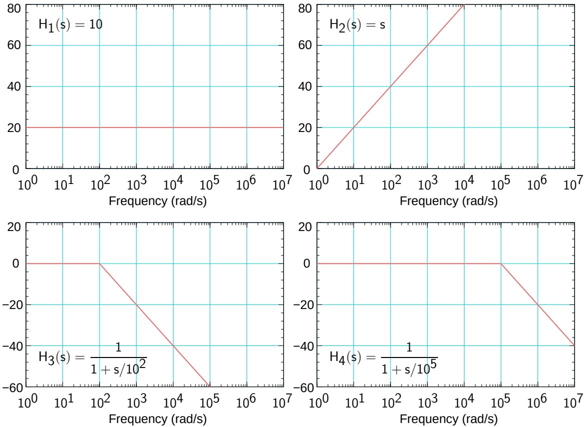

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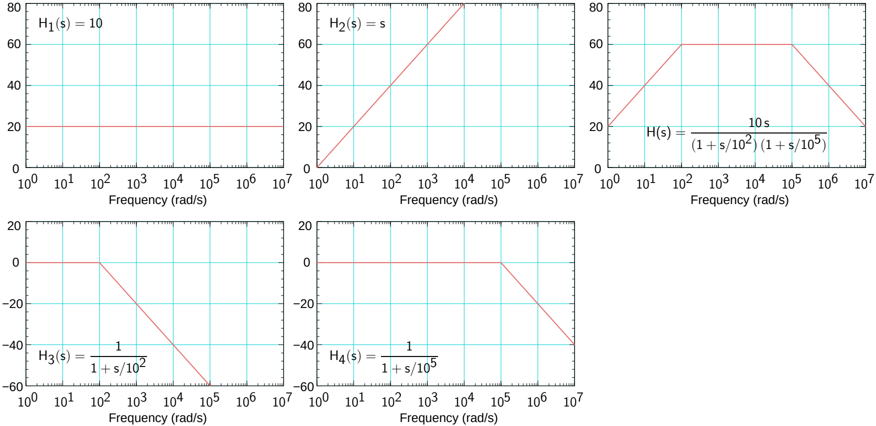

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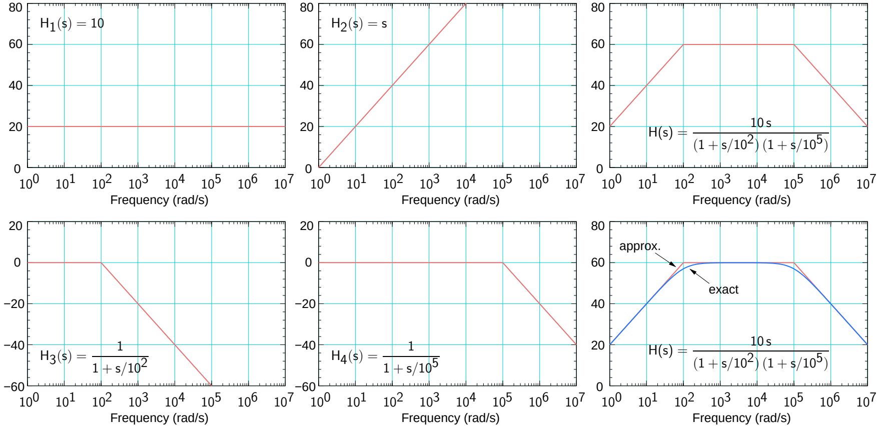
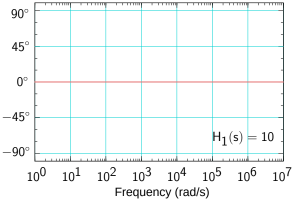
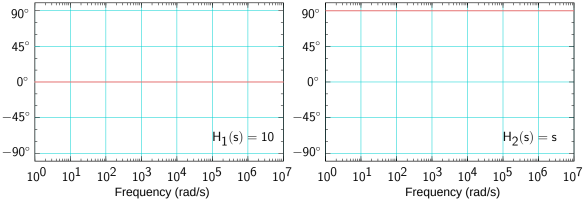


Phase plot



90°
45°
0°
−45°
−90°
$H_1(s) = 10$
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
Frequency (rad/s)

Phase plot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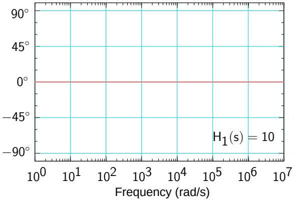
$H_1(s) = 10$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)
90°
45°
0°
−45°
−90°
$H_2(s) = s$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot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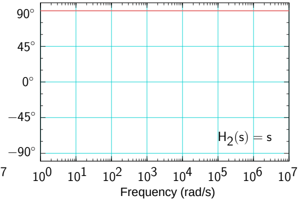
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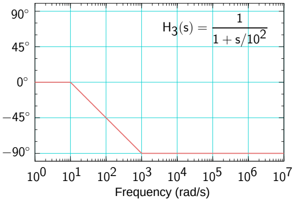
90°
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−45°
−90°
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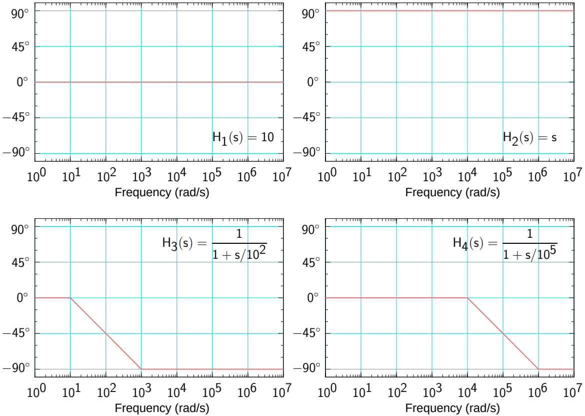
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot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−90°
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$H_2(s) = s$
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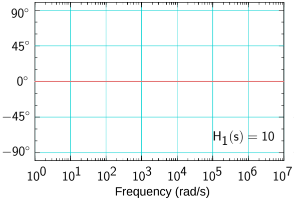
Frequency (rad/s)

Phase plot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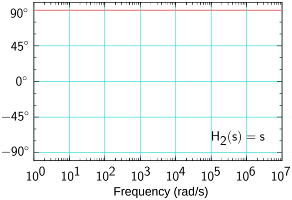
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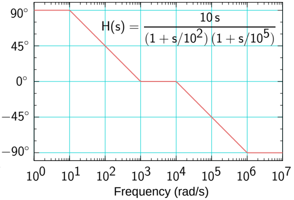
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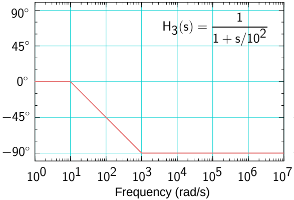
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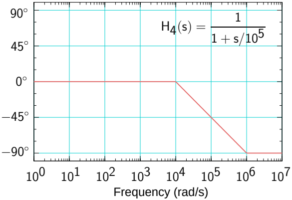
Frequency (rad/s)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−45°
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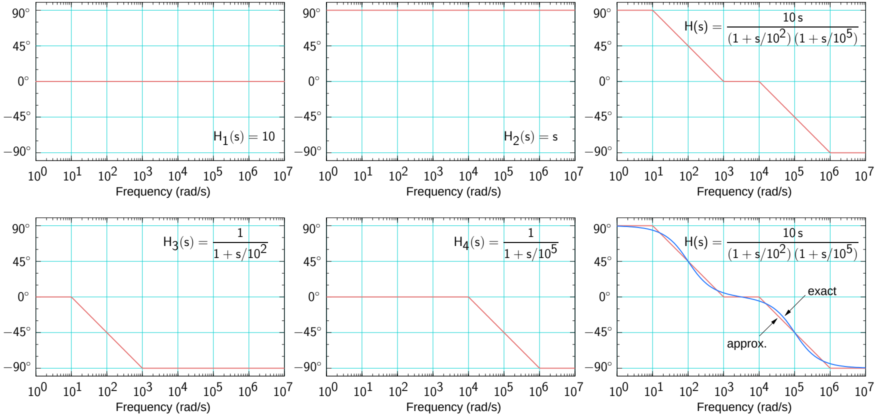
Frequency (rad/s)

Phase plot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$H(s) = \dfrac{10\,s}{(1+s/10^2)\,(1+s/10^5)}$
$H_3(s) = \dfrac{1}{1+s/10^2}$
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exact
approx.

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- If two poles p_1 and p_2 are close to each other (say, separated by less than a decade in ω), the error becomes larger (next slide).
- When the poles and zeros are not sufficiently separated, the Bode approximation should be used only for a rough estimate, follwed by a numerical calculation. However, even in such cases, it does give a good idea of the *asymptotic* magnitude and phase plots, which is valuable in amplifier design.

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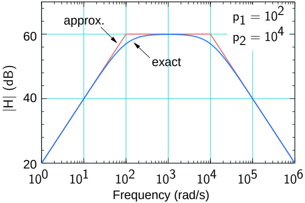

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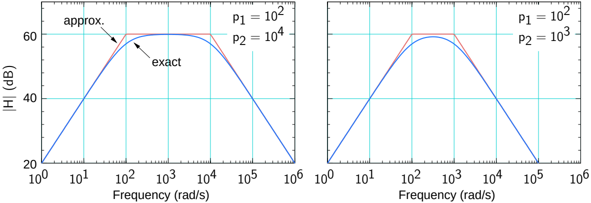

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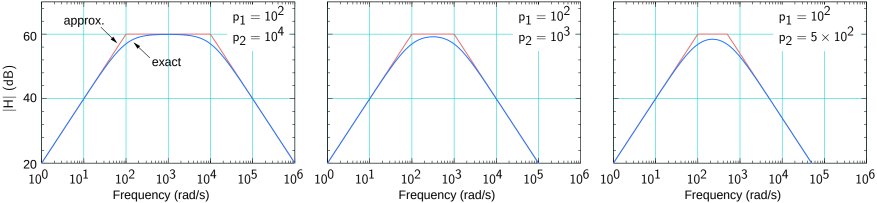

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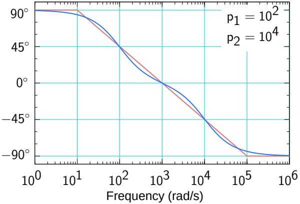

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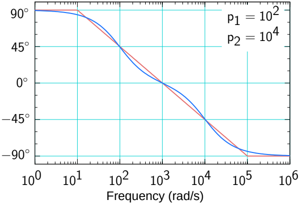

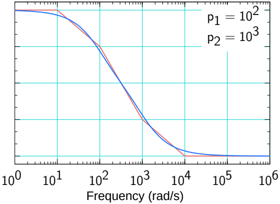

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