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- * A "minimal" expression has a minimum number of terms, each with a minimum number of variables. (For some functions, it is possible to have more than one minimal expressions, i.e., more than one expressions with the same complexity.)

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- * A "minimal" expression has a minimum number of terms, each with a minimum number of variables. (For some functions, it is possible to have more than one minimal expressions, i.e., more than one expressions with the same complexity.)
- * A minimal expression can be implemented with fewer gates.

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А	В	С	Y
0	0	0	0
0	0	1	1
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0	1	1	0
1	0	0	Х
1	0	1	0
1	1	0	0
1	1	1	1

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А	В	С	Υ
0	0	0	0
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0	1	0	1
0	1	1	0
1	0	0	Х
1	0	1	0
1	1	0	0
1	1	1	1

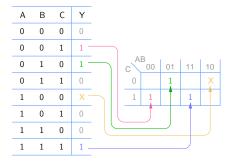
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0	0	0	0
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0	1	1	0
1	0	0	Х
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1	1	0	0
-			

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0 1	0
0 1	0
1 0	0
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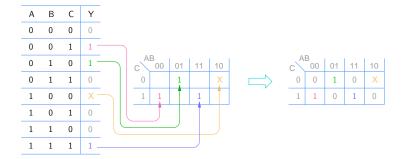
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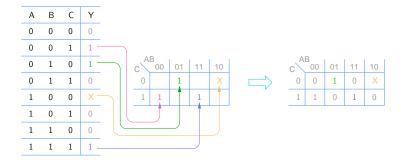
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0	0	1	1 -												
0	1	0	1 -		С	AB 00	01	11	10		A C	B 00	01	11	10
0	1	1	0			0	1		X	\Box	0	0	1	0	Х
1	0	0	Χ-	$\overline{}$		1 1		1			1	1	0	1	0
1	0	1	0			Ĵ		Î							
1	1	0	0	Ľ											
1	1	1	1.												

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* A K map is the same as the truth table of a function except for the way the entries are arranged.

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- * A K map is the same as the truth table of a function except for the way the entries are arranged.
- * In a K map, the adjacent rows or columns differ only in *one* variable. For example, in going from the column AB = 01 to AB = 11, there is only one change, viz., $A = 0 \rightarrow A = 1$.

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K maps: example with four variables

01 11

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1 0 X

0 1 1

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Consider the following functions (written in the standard sum-of-products form): $X_1(A) = A + \overline{A}$. $X_2(A, B) = AB + A\overline{B} + \overline{A}B + \overline{A}\overline{B}$. $X_3(A, B, C) = ABC + AB\overline{C} + \overline{A}BC + \overline{A}B\overline{C} + A\overline{B}C + \overline{A}\overline{B}C + \overline{A}\overline{B}C$.



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By using theorems and identities seen earlier, we can show that $X_1 = X_2 = X_3 = 1$. Another way to explain why X_1 , X_2 , X_3 are each equal to 1 is the following.

For example, consider $X_2 \equiv Y_1 + Y_2 + Y_3 + Y_4 + = AB + A\overline{B} + \overline{A}B + \overline{A}\overline{B}$.

A	В	Y ₁	Y ₂	Y ₃	Y ₄	X ₂
0	0	0	0 0 1 0	0	1	1
0	1	0	0	1	0	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

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A	В	Y ₁	Y ₂	Y ₃	Y ₄	X ₂
0	0	0	0 0 1 0	0	1	1
0	1	0	0	1	0	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

From the truth table, it is clear that X_2 is equal to 1 because it includes *all* possible minterms that we can make with two variables.

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A	В	Y_1	Y ₂	Y ₃	Y ₄	<i>X</i> ₂
0	0	0 0 0 1	0	0	1	1
0	1	0	0	1	0	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

From the truth table, it is clear that X_2 is equal to 1 because it includes *all* possible minterms that we can make with two variables.

For any combination of A and B, one of the minterms is $1 \Rightarrow X_2 = 1$.

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By using theorems and identities seen earlier, we can show that $X_1 = X_2 = X_3 = 1$. Another way to explain why X_1 , X_2 , X_3 are each equal to 1 is the following.

For example, consider $X_2 \equiv Y_1 + Y_2 + Y_3 + Y_4 + = AB + A\overline{B} + \overline{A}B + \overline{A}\overline{B}$.

A	В	Y_1	Y ₂	Y ₃	Y ₄	<i>X</i> ₂
0	0	0 0 0	0	0	1	1
0	1	0	0	1	0	1
1	0	0	1	0	0	1
1	1	1	0	0	0	1

From the truth table, it is clear that X_2 is equal to 1 because it includes *all* possible minterms that we can make with two variables.

For any combination of A and B, one of the minterms is $1 \Rightarrow X_2 = 1$.

Conclusion: "1" can be replaced by a suitable expansion in 1, 2, 3 (or more) variables. We will find this useful in understanding K maps.

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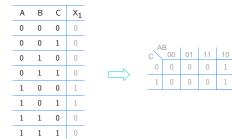
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А	В	С	X_1
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

A	В			
c	00	01	11	10
0	0	0	0	1
1	0	0	0	1



1

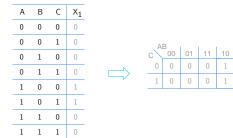


 $X_1 = A\overline{B} = (A\overline{B}) \cdot 1 = (A\overline{B}) \cdot (C + \overline{C}) = A\overline{B}C + A\overline{B}\overline{C}.$

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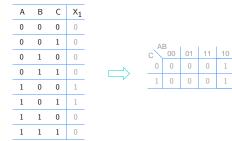
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 $X_1 = A\overline{B} = (A\overline{B}) \cdot 1 = (A\overline{B}) \cdot (C + \overline{C}) = A\overline{B}C + A\overline{B}\overline{C}.$ $\rightarrow X_1$ is composed of 2¹ minterms \rightarrow two 1s in the K map.

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$$\begin{split} X_1 &= A \,\overline{B} = (A \,\overline{B}) \cdot 1 = (A \,\overline{B}) \cdot (C + \overline{C}) = A \,\overline{B} \,C + A \,\overline{B} \,\overline{C}. \\ &\rightarrow X_1 \text{ is composed of } 2^1 \text{ minterms } \rightarrow \text{ two 1s in the K map.} \\ \text{Further, these } 2^1 \text{ minterms appear in adjacent boxes, making up a rectangle.} \end{split}$$

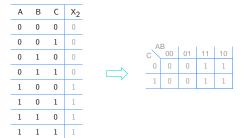
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А	В	С	х2
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

A	В			
c	00	01	11	10
0	0	0	1	1
1	0	0	1	1

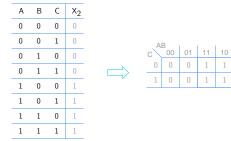




 $X_2 = A = A \cdot 1 = A \cdot (BC + B\overline{C} + \overline{B}C + \overline{B}\overline{C}) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\overline{C}.$

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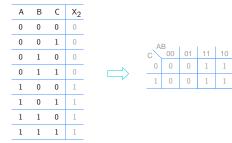
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 $X_2 = A = A \cdot 1 = A \cdot (B \ C + B \ \overline{C} + \overline{B} \ C + \overline{B} \ \overline{C}) = A B \ C + A B \ \overline{C} + A \overline{B} \ C + A \overline{B} \ \overline{C}.$ $\rightarrow X_2 \text{ is composed of } 2^2 \text{ minterms} \rightarrow \text{ four 1s in the K map.}$

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 $X_2 = A = A \cdot 1 = A \cdot (B C + B \overline{C} + \overline{B} C + \overline{B} \overline{C}) = A B C + A \overline{B} \overline{C} + A \overline{B} \overline{C} + A \overline{B} \overline{C}.$ $\rightarrow X_2 \text{ is composed of } 2^2 \text{ minterms} \rightarrow \text{four 1s in the K map.}$ Further, these 2^2 minterms appear in adjacent boxes, *making up a rectangle*.

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Let us now look at the *reverse* problem: Given a rectangle of 2^N 1s in the K map, what is the corresponding function (call it Y)? Consider the following example:

A	B 00	01	11	10	
0	0	1	1	0	
1	0	1	1	0	

We note that Y is independent of C (since the C = 0 and C = 1 boxes are identical). Y is also independent of A (since the A = 0 and A = 1 boxes are identical). $\rightarrow Y = B$ or $Y = \overline{B}$.

By inspection, Y = B (since Y is 1 in the B = 1 boxes).

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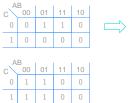
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C OO		01	11	10
0	0	1	1	0
1	0	0	0	0

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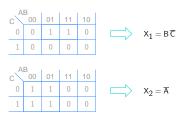


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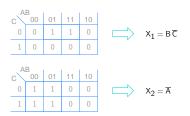


 \square $X_1 = B\overline{C}$



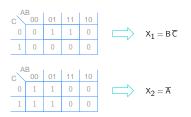






* It should be now clear why we must have no more than one variable changing between adjacent columns (or rows).

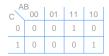
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- * It should be now clear why we must have no more than one variable changing between adjacent columns (or rows).
- * If this format is followed, terms that can be combined appear in rectangles of 2^1 , 2^2 , 2^3 , \cdots and can be easily combined by inspection.

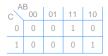
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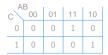
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Although the number of 1s is a power of 2 (2^1) , they cannot be combined because they are not adjacent (i.e., they do not form a rectangle).



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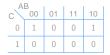


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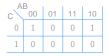
 \rightarrow the function $(A B \overline{C} + A \overline{B} C)$ cannot be minimized.



C A	B 00	01	11	10
0	1	0	0	1
1	0	0	0	0



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Let us redraw the K map by changing the order of the columns cyclically.



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Can the 1s shown in the K map be combined? Let us redraw the K map by changing the order of the columns cyclically. The two 1s are, in fact, adjacent and can be combined to give $\overline{B} \overline{C}$.



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The two 1s are, in fact, adjacent and can be combined to give $\overline{B} \overline{C}$.

 \rightarrow Columns AB = 00 and AB = 10 in the K map on the left are indeed "logically adjacent" (although they are not geometrically adjacent) since they differ only in one variable (A).

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We could have therefore combined the 1s without actually redrawing the K map.



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	A				
С	D	00	01	11	10
	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1



CD 00 01 11 10 00 1 0 0 1 01 0 0 0 0 11 0 0 0 0 11 0 0 0 0 10 1 0 0 1		A	В	1		
01 0 0 0 0 11 0 0 0 0	С	D	00	01	11	10
11 0 0 0 0		00	1	0	0	1
		01	0	0	0	0
10 1 0 0 1		11	0	0	0	0
		10	1	0	0	1

CD	A	00 B	01	11	10
	00	1	0	0	1
С)1	0	0	0	0
1	.1	0	0	0	0
1	.0	1	0	0	1

 \square $X_1 = \overline{B}\overline{D}$

	A	В			
С	D	00	01	11	10
	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

 \square $X_1 = \overline{B} \overline{D}$

	A	R			
С	D	00	01	11	10
	00	1	0	0	1
	01	1	0	0	1
	11	0	0	0	0
	10	0	0	0	0

	A	В			
С	D	00	01	11	10
	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

 \square $X_1 = \overline{B} \overline{D}$

	A	B			
C	:D	00	01	11	10
	00	1	0	0	1
	01	1	0	0	1
	11	0	0	0	0
	10	0	0	0	0

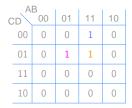
	AB					
С	D	00	01	11	10	
	00	1	0	0	1	
	01	0	0	0	0	
	11	0	0	0	0	
	10	1	0	0	1	

 \square $X_1 = \overline{B} \overline{D}$

С	A D	B 00	01	11	10
	00	1	0	0	1
	01	1	0	0	1
	11	0	0	0	0
	10	0	0	0	0

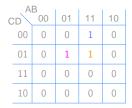
 \square $X_2 = \overline{BC}$





Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

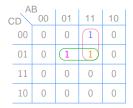
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Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

$$\begin{aligned} X_1 &= A B \overline{C} \overline{D} + A B \overline{C} D + A B \overline{C} D + \overline{A} B \overline{C} D & (using Y = Y + Y) \\ &= A B \overline{C} (\overline{D} + D) + B \overline{C} D (A + \overline{A}) \\ &= A B \overline{C} + B \overline{C} D \end{aligned}$$

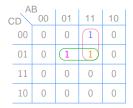
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Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

$$\begin{aligned} X_1 &= A B \overline{C} \overline{D} + A B \overline{C} D + A B \overline{C} D + \overline{A} B \overline{C} D & (using Y = Y + Y) \\ &= A B \overline{C} (\overline{D} + D) + B \overline{C} D (A + \overline{A}) \\ &= A B \overline{C} + B \overline{C} D \end{aligned}$$

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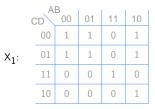
Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

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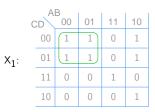
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$$\begin{split} X_1 &= A B \overline{C} \overline{D} + A B \overline{C} D + A B \overline{C} D + \overline{A} B \overline{C} D \qquad (using Y=Y+Y) \\ &= A B \overline{C} (\overline{D} + D) + B \overline{C} D (A + \overline{A}) \\ &= A B \overline{C} + B \overline{C} D \end{split}$$

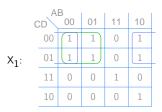
* A minterm can be combined with others more than once.



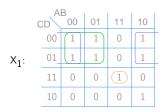
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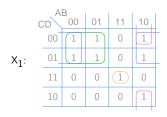
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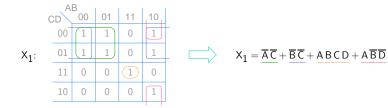
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