

## EE101: Digital circuits (Part 5)

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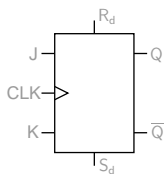


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Department of Electrical Engineering  
Indian Institute of Technology Bombay

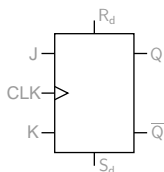
## JK flip-flop: asynchronous inputs



| $S_d$ | $R_d$ | CLK | J | K | $Q_{n+1}$        |
|-------|-------|-----|---|---|------------------|
| 0     | 1     | X   | X | X | 0                |
| 1     | 0     | X   | X | X | 1                |
| 1     | 1     | X   | X | X | invalid          |
| 0     | 0     | ↑   | 0 | 0 | $Q_n$            |
| 0     | 0     | ↑   | 0 | 1 | 0                |
| 0     | 0     | ↑   | 1 | 0 | 1                |
| 0     | 0     | ↑   | 1 | 1 | $\overline{Q_n}$ |

} normal operation

## JK flip-flop: asynchronous inputs

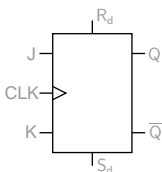


| $S_d$ | $R_d$ | CLK | J | K | $Q_{n+1}$        |
|-------|-------|-----|---|---|------------------|
| 0     | 1     | X   | X | X | 0                |
| 1     | 0     | X   | X | X | 1                |
| 1     | 1     | X   | X | X | invalid          |
| 0     | 0     | ↑   | 0 | 0 | $Q_n$            |
| 0     | 0     | ↑   | 0 | 1 | 0                |
| 0     | 0     | ↑   | 1 | 0 | 1                |
| 0     | 0     | ↑   | 1 | 1 | $\overline{Q_n}$ |

} normal operation

- \* Clocked flip-flops are also provided with *asynchronous* or *direct* Set and Reset inputs,  $S_d$  and  $R_d$ , (also called Preset and Clear, respectively) which override all other inputs (J, K, CLK).

## JK flip-flop: asynchronous inputs

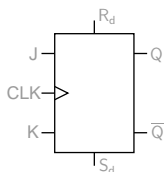


| $S_d$ | $R_d$ | CLK | J | K | $Q_{n+1}$        |
|-------|-------|-----|---|---|------------------|
| 0     | 1     | X   | X | X | 0                |
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| 0     | 0     | ↑   | 0 | 0 | $Q_n$            |
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- \* The  $S_d$  and  $R_d$  inputs may be active low; in that case, they are denoted by  $\overline{S_d}$  and  $\overline{R_d}$ .

## JK flip-flop: asynchronous inputs

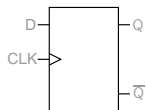


| $S_d$ | $R_d$ | CLK | J | K | $Q_{n+1}$        |
|-------|-------|-----|---|---|------------------|
| 0     | 1     | X   | X | X | 0                |
| 1     | 0     | X   | X | X | 1                |
| 1     | 1     | X   | X | X | invalid          |
| 0     | 0     | ↑   | 0 | 0 | $Q_n$            |
| 0     | 0     | ↑   | 0 | 1 | 0                |
| 0     | 0     | ↑   | 1 | 0 | 1                |
| 0     | 0     | ↑   | 1 | 1 | $\overline{Q_n}$ |

} normal operation

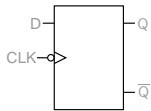
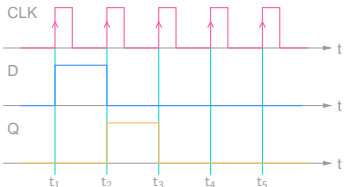
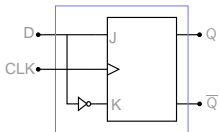
- \* Clocked flip-flops are also provided with *asynchronous* or *direct* Set and Reset inputs,  $S_d$  and  $R_d$ , (also called Preset and Clear, respectively) which override all other inputs (J, K, CLK).
- \* The  $S_d$  and  $R_d$  inputs may be active low; in that case, they are denoted by  $\overline{S_d}$  and  $\overline{R_d}$ .
- \* The asynchronous inputs are convenient for "starting up" a circuit in a known state.

# D flip-flop



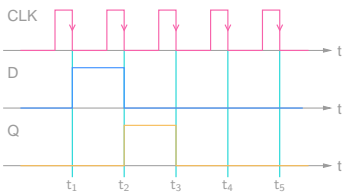
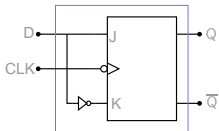
| CLK | D | $Q_{n+1}$ |
|-----|---|-----------|
| ↑   | 0 | 0         |
| ↑   | 1 | 1         |

positive edge-triggered D flip-flop

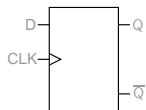


| CLK | D | $Q_{n+1}$ |
|-----|---|-----------|
| ↓   | 0 | 0         |
| ↓   | 1 | 1         |

negative edge-triggered D flip-flop

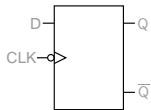
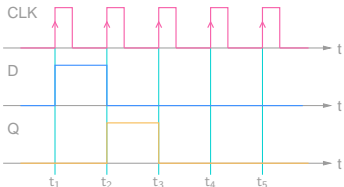
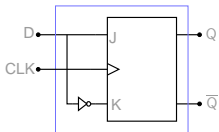


# D flip-flop



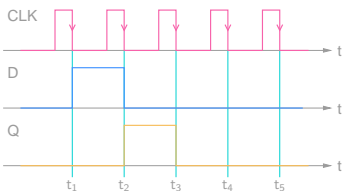
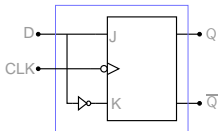
positive edge-triggered D flip-flop

| CLK | D | $Q_{n+1}$ |
|-----|---|-----------|
| ↑   | 0 | 0         |
| ↑   | 1 | 1         |



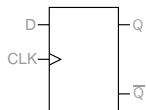
negative edge-triggered D flip-flop

| CLK | D | $Q_{n+1}$ |
|-----|---|-----------|
| ↓   | 0 | 0         |
| ↓   | 1 | 1         |



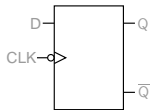
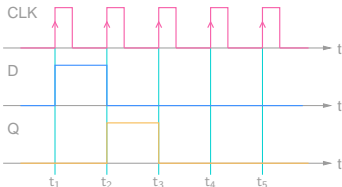
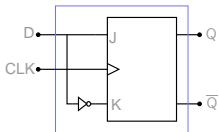
\* The D flip-flop can be used to *delay* the Data (D) signal by one clock period.

## D flip-flop



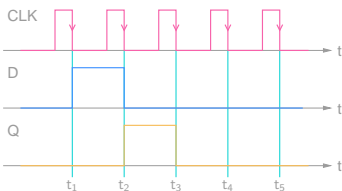
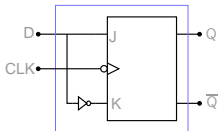
| CLK | D | $Q_{n+1}$ |
|-----|---|-----------|
| ↑   | 0 | 0         |
| ↑   | 1 | 1         |

positive edge-triggered D flip-flop



| CLK | D | $Q_{n+1}$ |
|-----|---|-----------|
| ↓   | 0 | 0         |
| ↓   | 1 | 1         |

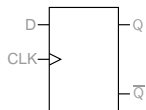
negative edge-triggered D flip-flop



- \* The D flip-flop can be used to *delay* the Data (D) signal by one clock period.
- \* With  $J = D$ ,  $K = \bar{D}$ , we have either  $J = 0$ ,  $K = 1$  or  $J = 1$ ,  $K = 0$ ; the next  $Q$  is 0 in the first case, 1 in the second case.

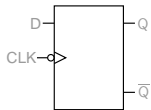
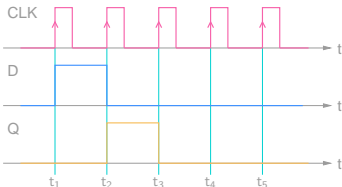
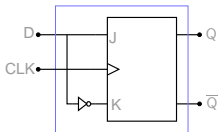


## D flip-flop



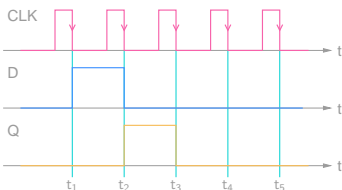
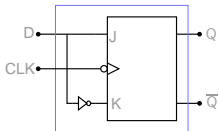
| CLK | D | $Q_{n+1}$ |
|-----|---|-----------|
| ↑   | 0 | 0         |
| ↑   | 1 | 1         |

positive edge-triggered D flip-flop



| CLK | D | $Q_{n+1}$ |
|-----|---|-----------|
| ↓   | 0 | 0         |
| ↓   | 1 | 1         |

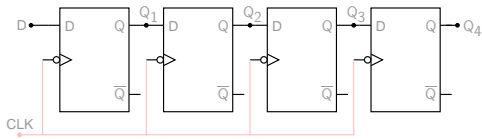
negative edge-triggered D flip-flop



- \* The D flip-flop can be used to *delay* the Data (D) signal by one clock period.
- \* With  $J = D$ ,  $K = \overline{D}$ , we have either  $J = 0$ ,  $K = 1$  or  $J = 1$ ,  $K = 0$ ; the next  $Q$  is 0 in the first case, 1 in the second case.
- \* Instead of a JK flip-flop, an RS flip-flop can also be used to make a D flip-flop, with  $S = D$ ,  $R = \overline{D}$ .

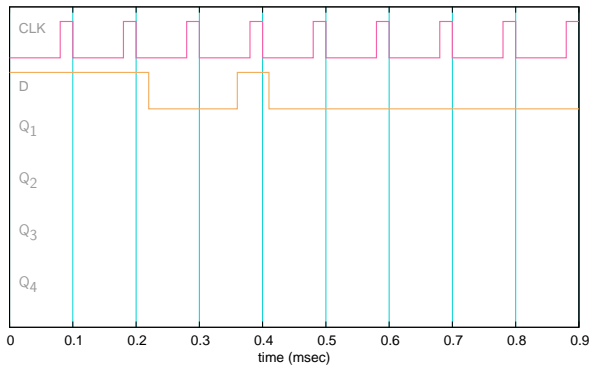
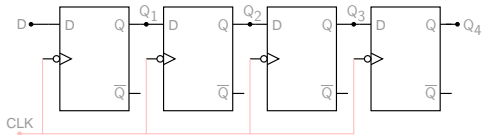
# Shift register

Let  $Q_1 = Q_2 = Q_3 = Q_4 = 0$  initially.



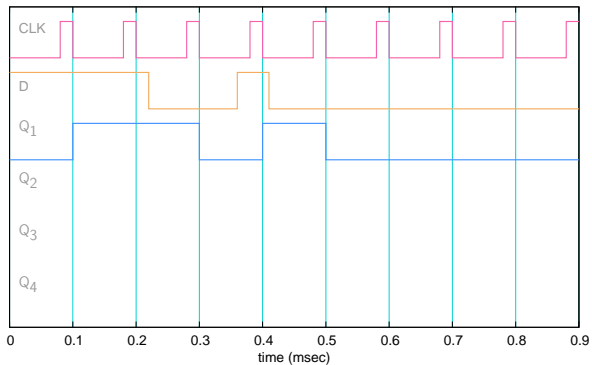
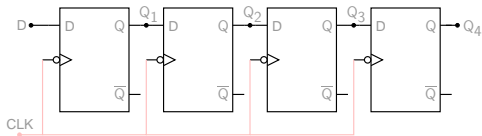
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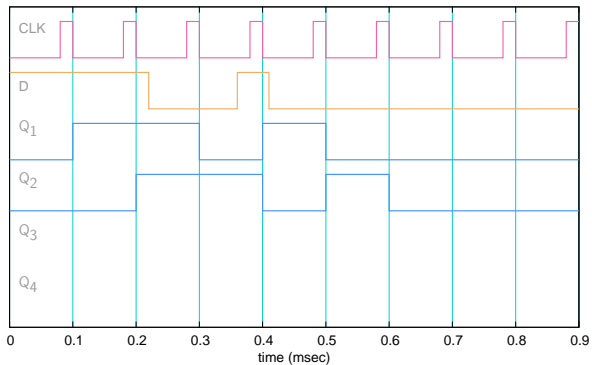
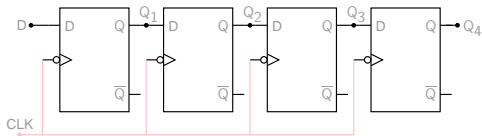
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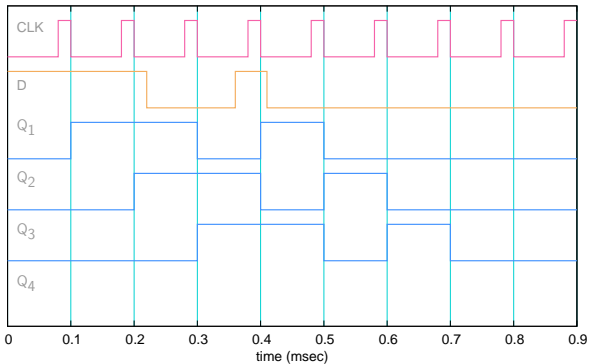
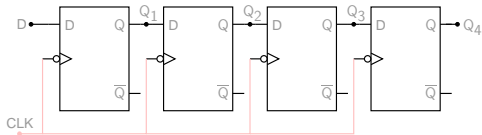
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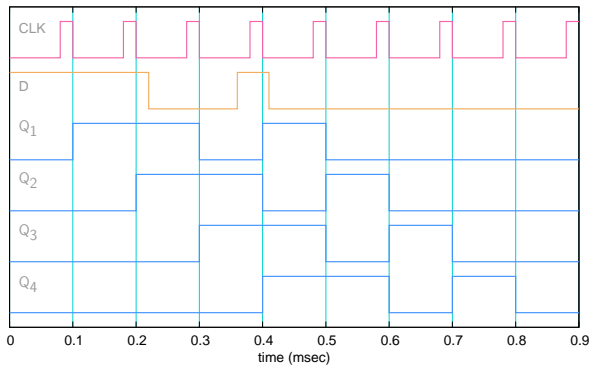
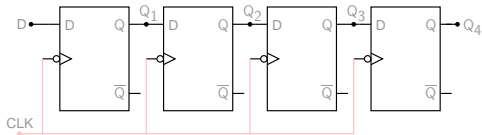
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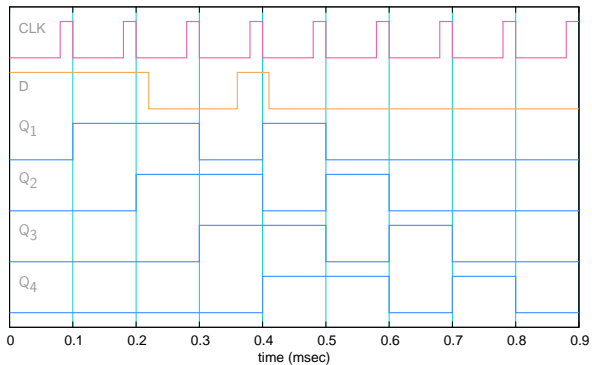
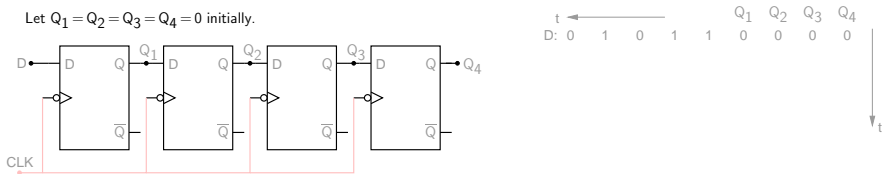
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# Shift register

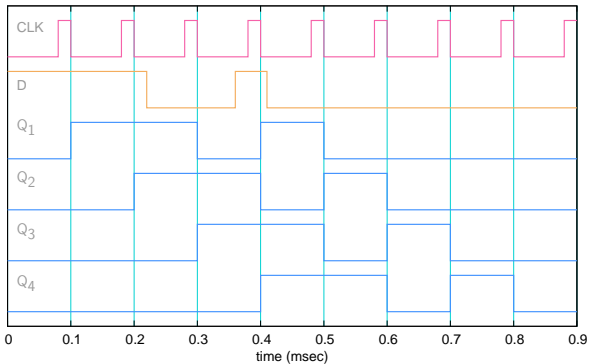
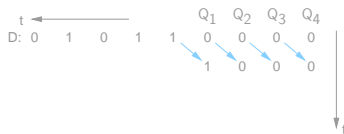
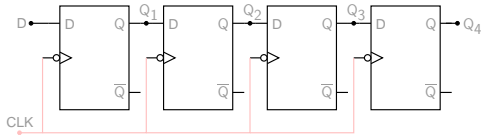
Let  $Q_1 = Q_2 = Q_3 = Q_4 = 0$  initially.





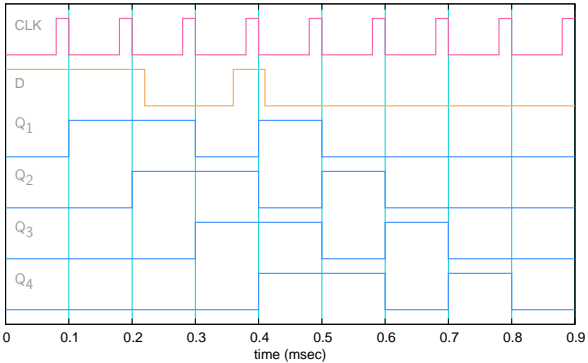
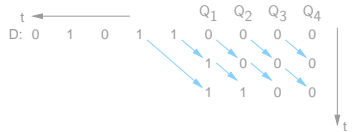
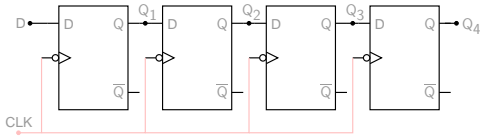
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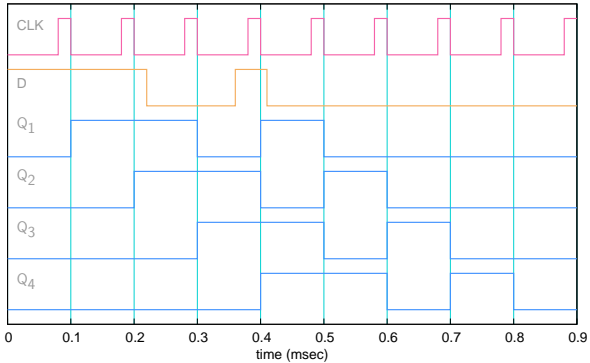
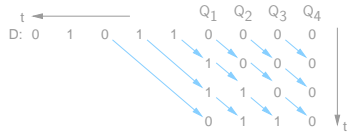
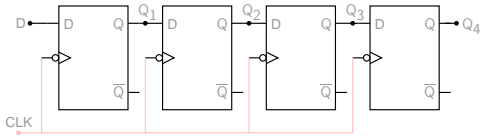
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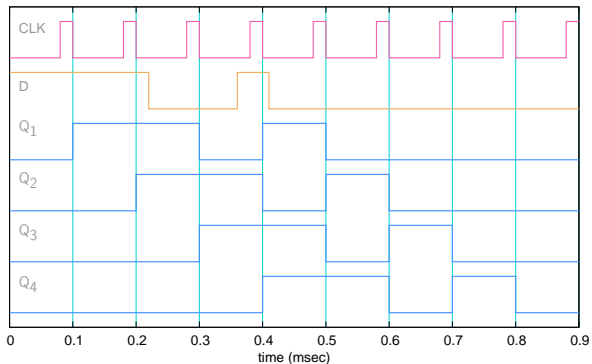
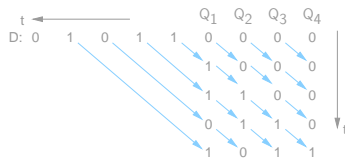
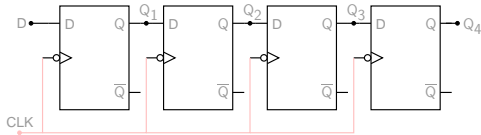
# Shift register

Let  $Q_1 = Q_2 = Q_3 = Q_4 = 0$  initially.



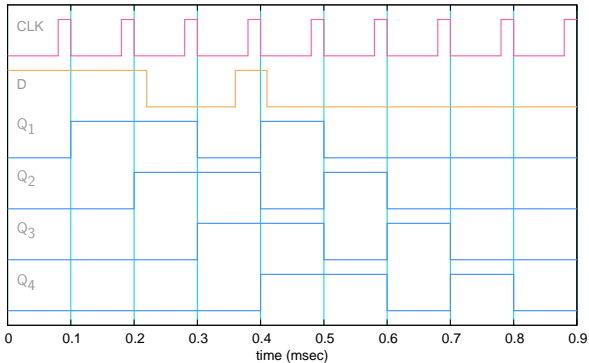
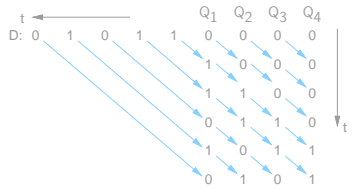
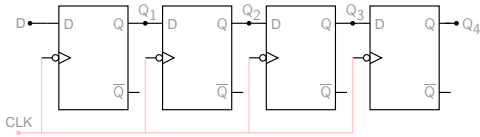
# Shift register

Let  $Q_1 = Q_2 = Q_3 = Q_4 = 0$  initially.



# Shift register

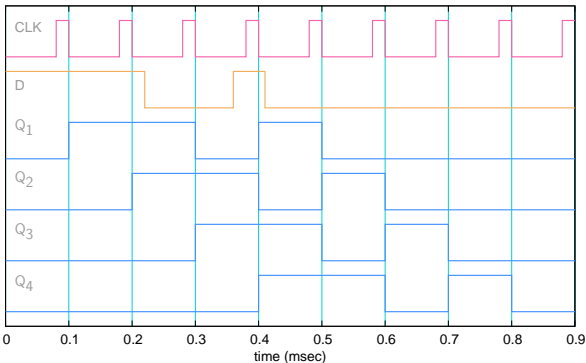
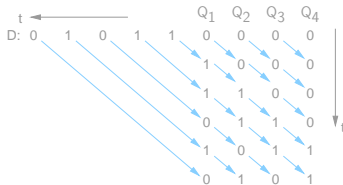
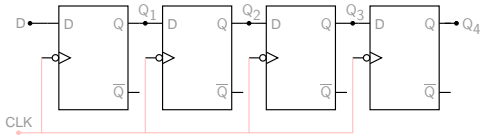
Let  $Q_1 = Q_2 = Q_3 = Q_4 = 0$  initially.



(SEQUENCE file: ee101\_shift\_reg\_1.sqproj)

# Shift register

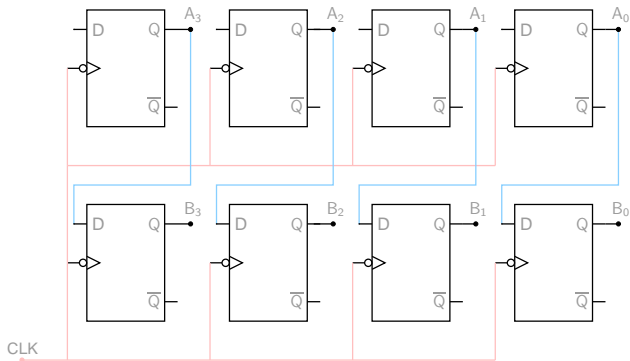
Let  $Q_1 = Q_2 = Q_3 = Q_4 = 0$  initially.



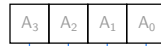
(SEQUENCE file: ee101\_shift\_reg\_1.sqproj)

\* The data (D) keeps shifting right after each active clock edge.

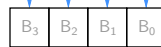
# Parallel transfer between shift registers



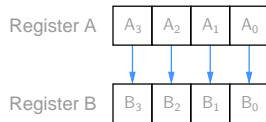
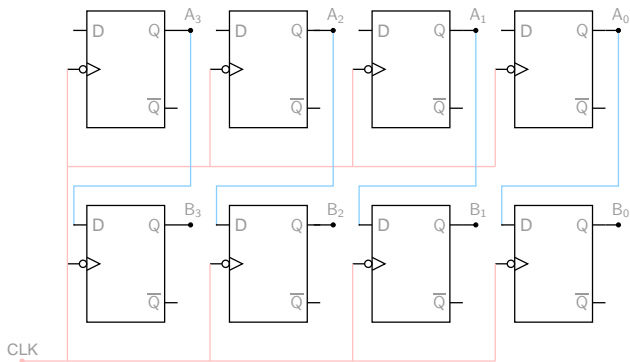
Register A



Register B



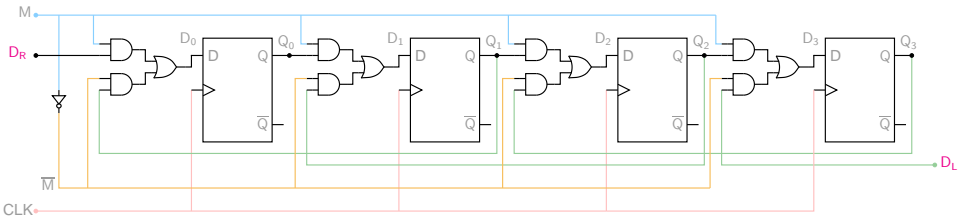
## Parallel transfer between shift registers



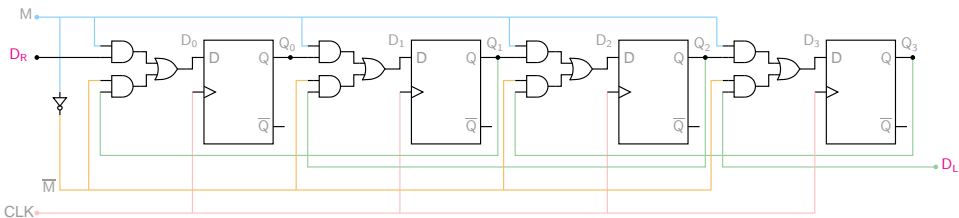
- \* After the active clock edge, the contents of the A register ( $A_3A_2A_1A_0$ ) are copied to the B register.



# Bidirectional shift register

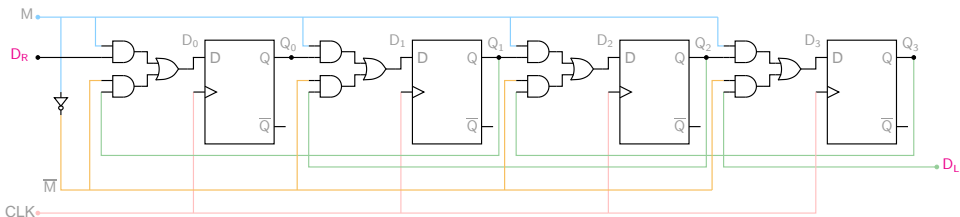


## Bidirectional shift register



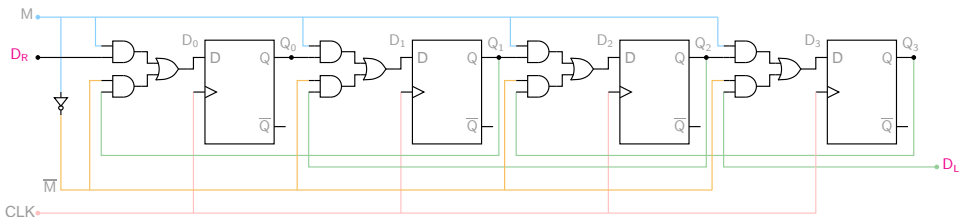
- \* When the mode input ( $M$ ) is 1, we have  
 $D_0 = D_R, D_1 = Q_0, D_2 = Q_1, D_3 = Q_2$ .

## Bidirectional shift register



- \* When the mode input ( $M$ ) is 1, we have  $D_0 = D_R$ ,  $D_1 = Q_0$ ,  $D_2 = Q_1$ ,  $D_3 = Q_2$ .
- \* When the mode input ( $M$ ) is 0, we have  $D_0 = Q_1$ ,  $D_1 = Q_2$ ,  $D_2 = Q_3$ ,  $D_3 = D_L$ .

## Bidirectional shift register



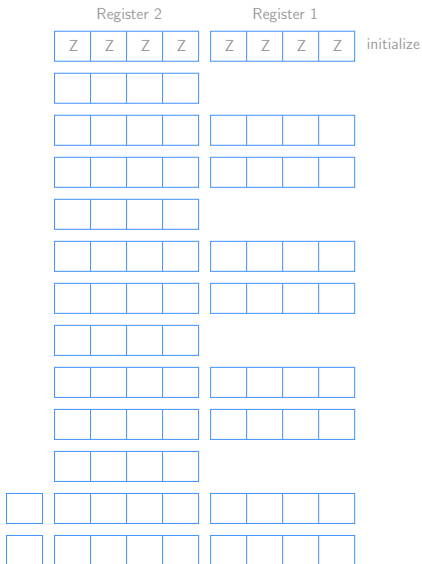
- \* When the mode input ( $M$ ) is 1, we have  $D_0 = D_R, D_1 = Q_0, D_2 = Q_1, D_3 = Q_2$ .
- \* When the mode input ( $M$ ) is 0, we have  $D_0 = Q_3, D_1 = Q_2, D_2 = Q_1, D_3 = Q_0$ .
- \*  $M = 1 \rightarrow$  shift right operation.  
 $M = 0 \rightarrow$  shift left operation.



# Multiplication using shift and add

$$\begin{array}{r}
 \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \\
 \phantom{+} \phantom{+} \phantom{+} \phantom{+} 1\ 0\ 1\ 1 \quad A_3A_2A_1A_0 \quad (\text{decimal } 11) \\
 \times \phantom{+} \phantom{+} \phantom{+} \phantom{+} 1\ 1\ 0\ 1 \quad B_3B_2B_1B_0 \quad (\text{decimal } 13) \\
 \hline
 + \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} 1\ 0\ 1\ 1 \quad \text{since } B_0 = 1 \\
 \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} 0\ 0\ 0\ 0\ Z \quad \text{since } B_1 = 0 \\
 \hline
 + \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} 0\ 1\ 0\ 1\ 1 \quad \text{addition} \\
 \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} 1\ 0\ 1\ 1\ Z\ Z \quad \text{since } B_2 = 1 \\
 \hline
 + \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} 1\ 1\ 0\ 1\ 1\ 1 \quad \text{addition} \\
 \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} \phantom{+} 1\ 0\ 1\ 1\ Z\ Z\ Z \quad \text{since } B_3 = 1 \\
 \hline
 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1 \quad \text{addition} \quad (\text{decimal } 143)
 \end{array}$$

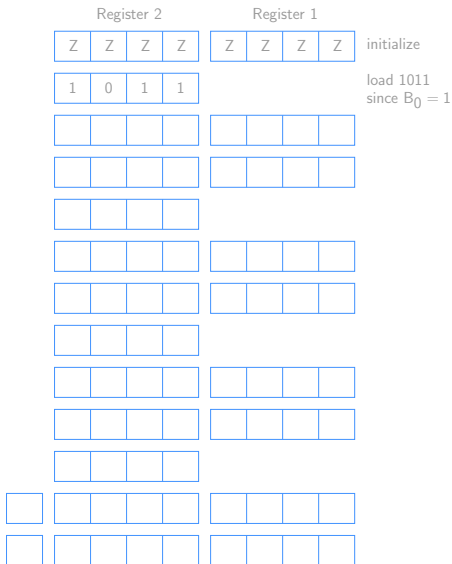
Note that  $Z = 0$ . We use  $Z$  to denote 0s which are independent of the numbers being multiplied.



# Multiplication using shift and add

|  |                 |                             |
|--|-----------------|-----------------------------|
|  | 1 0 1 1         | $A_3A_2A_1A_0$ (decimal 11) |
| ×  | 1 1 0 1         | $B_3B_2B_1B_0$ (decimal 13) |
| <hr style="border: 0.5px solid black;"/> |                 |                             |
| +  | 1 0 1 1         | since $B_0 = 1$             |
|  | 0 0 0 0 Z       | since $B_1 = 0$             |
| <hr style="border: 0.5px solid black;"/> |                 |                             |
| +  | 0 1 0 1 1       | addition                    |
|  | 1 0 1 1 Z Z     | since $B_2 = 1$             |
| <hr style="border: 0.5px solid black;"/> |                 |                             |
| +  | 1 1 0 1 1 1     | addition                    |
|  | 1 0 1 1 Z Z Z   | since $B_3 = 1$             |
| <hr style="border: 0.5px solid black;"/> |                 |                             |
|  | 1 0 0 0 1 1 1 1 | addition (decimal 143)      |

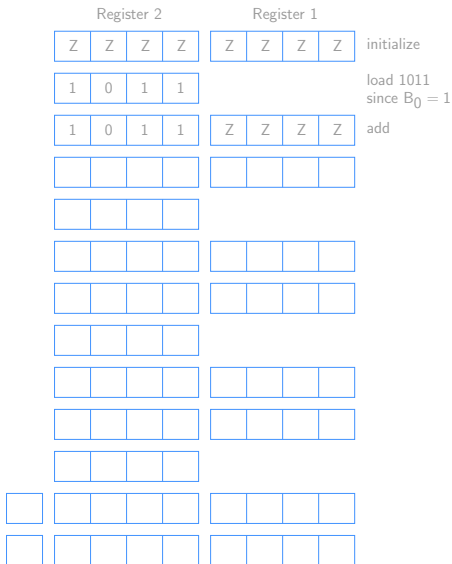
Note that  $Z = 0$ . We use  $Z$  to denote 0s which are independent of the numbers being multiplied.



# Multiplication using shift and add

|   |                 |                             |
|---|-----------------|-----------------------------|
|   | 1 0 1 1         | $A_3A_2A_1A_0$ (decimal 11) |
| × | 1 1 0 1         | $B_3B_2B_1B_0$ (decimal 13) |
|   |                 |                             |
| + | 1 0 1 1         | since $B_0 = 1$             |
|   | 0 0 0 0 Z       | since $B_1 = 0$             |
|   |                 |                             |
| + | 0 1 0 1 1       | addition                    |
|   | 1 0 1 1 Z Z     | since $B_2 = 1$             |
|   |                 |                             |
| + | 1 1 0 1 1 1     | addition                    |
|   | 1 0 1 1 Z Z Z   | since $B_3 = 1$             |
|   |                 |                             |
|   | 1 0 0 0 1 1 1 1 | addition (decimal 143)      |

Note that  $Z = 0$ . We use  $Z$  to denote 0s which are independent of the numbers being multiplied.



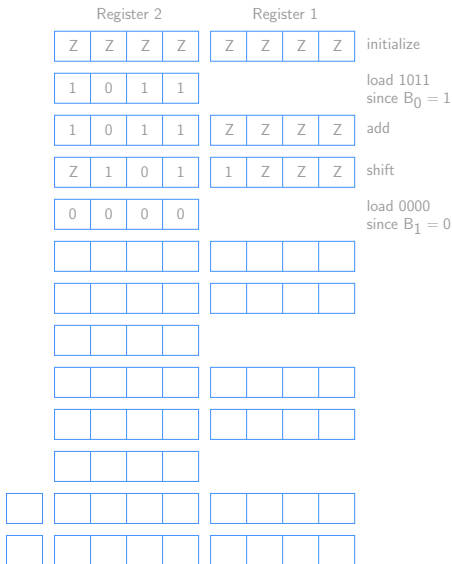




# Multiplication using shift and add

|   |                 |                             |
|---|-----------------|-----------------------------|
|   | 1 0 1 1         | $A_3A_2A_1A_0$ (decimal 11) |
| × | 1 1 0 1         | $B_3B_2B_1B_0$ (decimal 13) |
|   |                 |                             |
| + | 1 0 1 1         | since $B_0 = 1$             |
|   | 0 0 0 0 Z       | since $B_1 = 0$             |
|   |                 |                             |
|   | 0 1 0 1 1       | addition                    |
| + | 1 0 1 1 Z Z     | since $B_2 = 1$             |
|   |                 |                             |
|   | 1 1 0 1 1 1     | addition                    |
| + | 1 0 1 1 Z Z Z   | since $B_3 = 1$             |
|   |                 |                             |
|   | 1 0 0 0 1 1 1 1 | addition (decimal 143)      |

Note that  $Z = 0$ . We use  $Z$  to denote 0s which are independent of the numbers being multiplied.













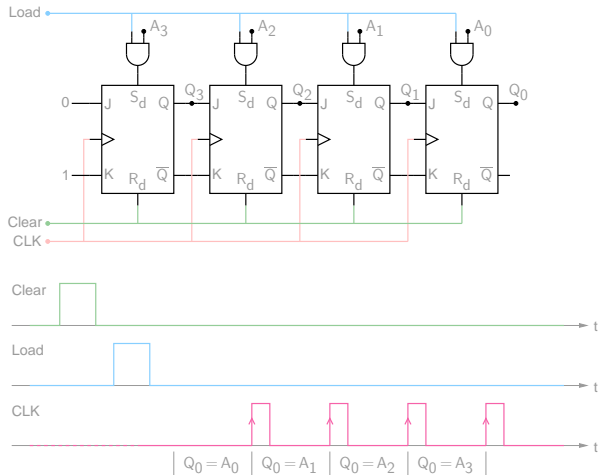




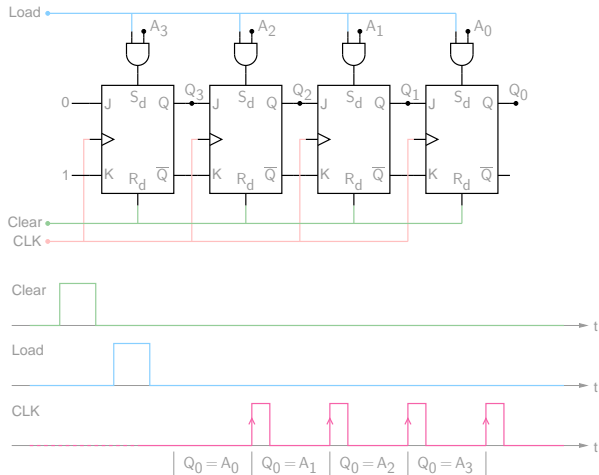




# Parallel in-serial out data movement

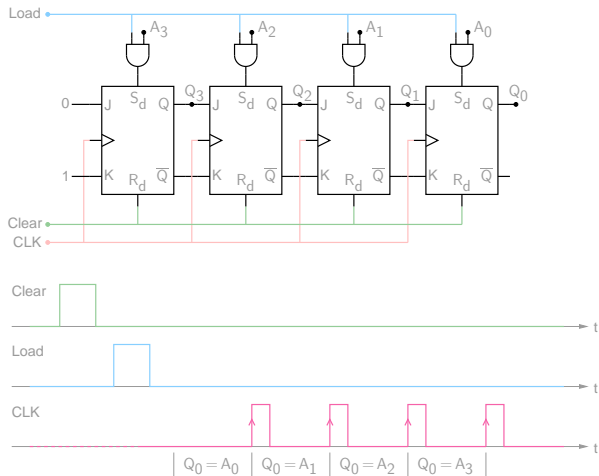


## Parallel in-serial out data movement



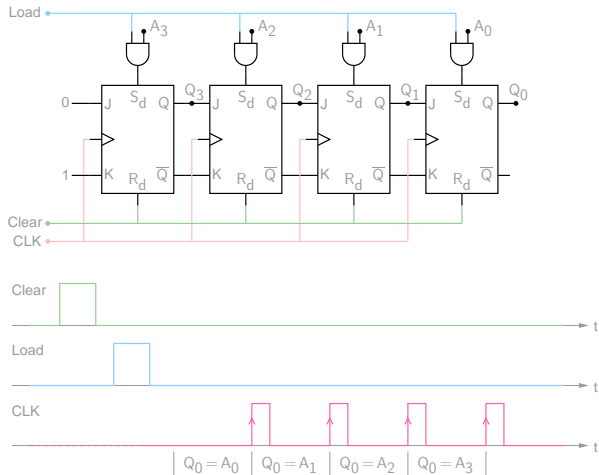
\* All flip-flops are cleared in the beginning (with  $R_d = \text{Clear} = 1$ ,  $S_d = 0$ ).

## Parallel in-serial out data movement



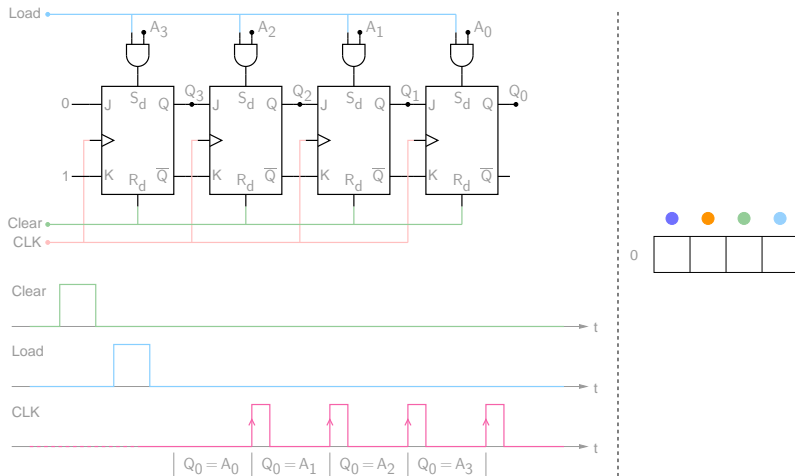
- \* All flip-flops are cleared in the beginning (with  $R_d = \text{Clear} = 1$ ,  $S_d = 0$ ).
- \* When  $\text{Load} = 1$ ,  $S_d = A_i$ ,  $R_d = 0 \rightarrow A_i$  gets loaded into the  $i^{\text{th}}$  flip-flop. (We will assume that CLK has been made 0 in this initial phase.)

## Parallel in-serial out data movement



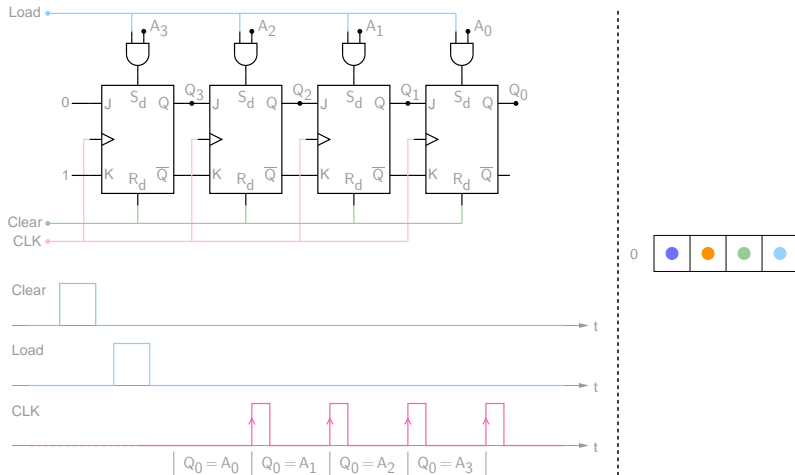
- \* All flip-flops are cleared in the beginning (with  $R_d = \text{Clear} = 1$ ,  $S_d = 0$ ).
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- \* Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output  $Q_0$ .

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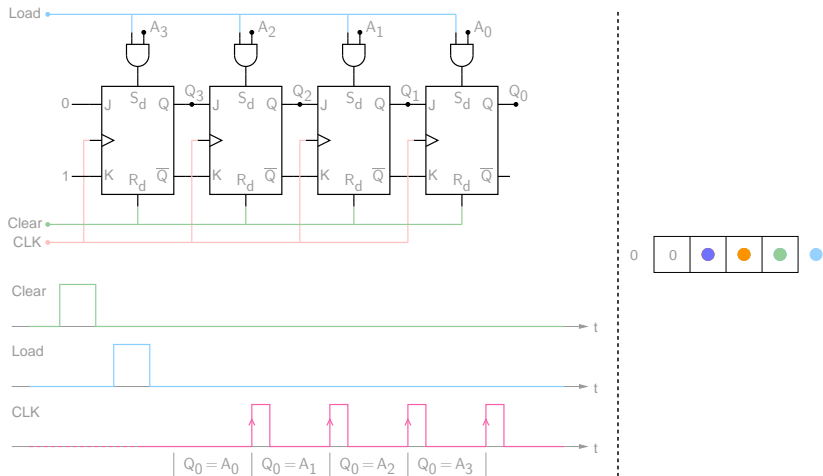
## Parallel in-serial out data movement



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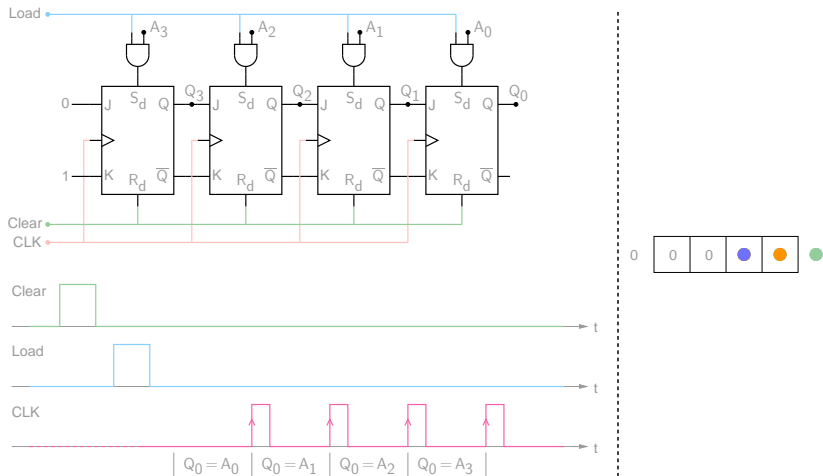


## Parallel in-serial out data movement



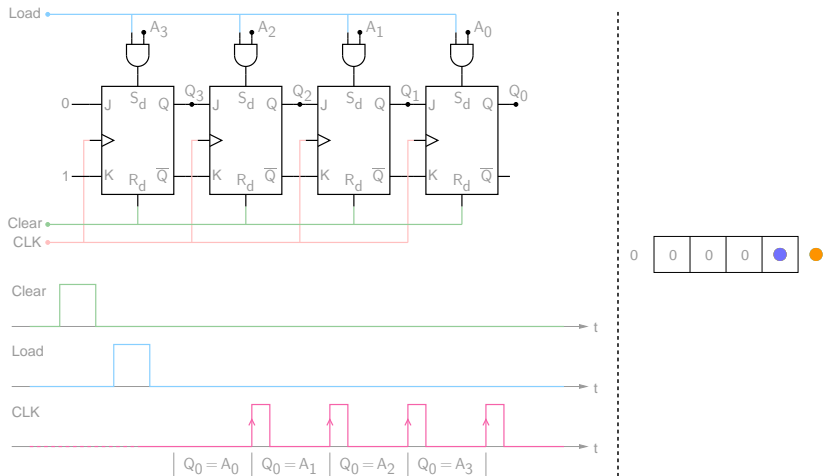
- \* All flip-flops are cleared in the beginning (with  $R_d = \text{Clear} = 1$ ,  $S_d = 0$ ).
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## Parallel in-serial out data movement



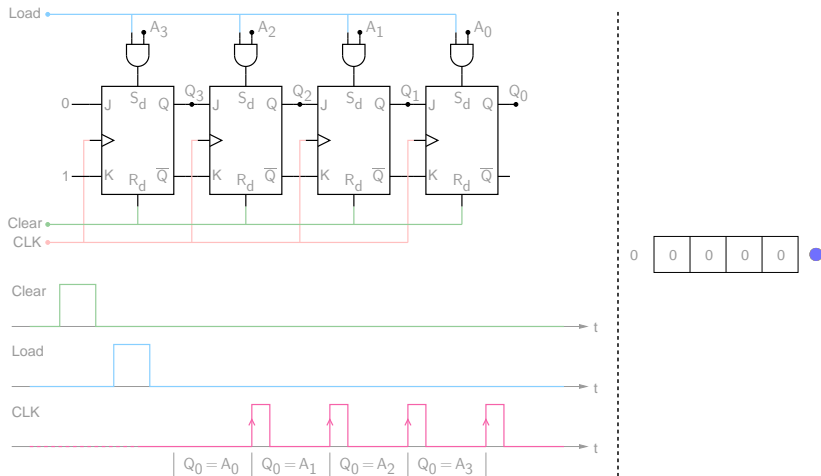
- \* All flip-flops are cleared in the beginning (with  $R_d = \text{Clear} = 1$ ,  $S_d = 0$ ).
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## Parallel in-serial out data movement



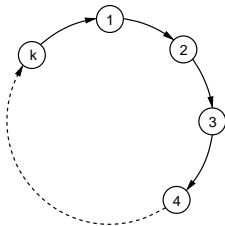
- \* All flip-flops are cleared in the beginning (with  $R_d = \text{Clear} = 1$ ,  $S_d = 0$ ).
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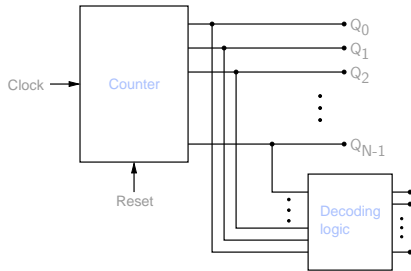


- \* All flip-flops are cleared in the beginning (with  $R_d = \text{Clear} = 1, S_d = 0$ ).
- \* When  $\text{Load} = 1, S_d = A_i, R_d = 0 \rightarrow A_i$  gets loaded into the  $i^{\text{th}}$  flip-flop. (We will assume that CLK has been made 0 in this initial phase.)
- \* Subsequently, with every clock pulse, the data shifts right and appears *serially* at the output  $Q_0$ .

# Counters

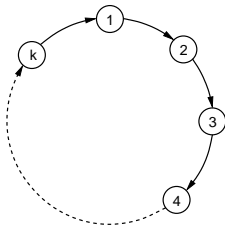


State transition diagram

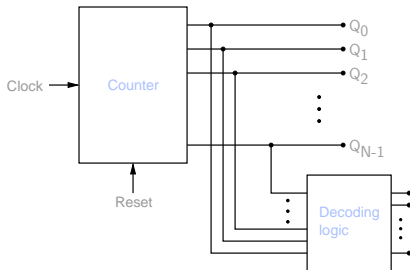


General configuration

# Counters

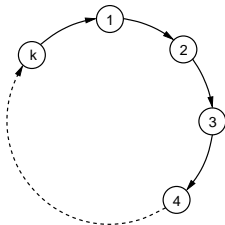


State transition diagram

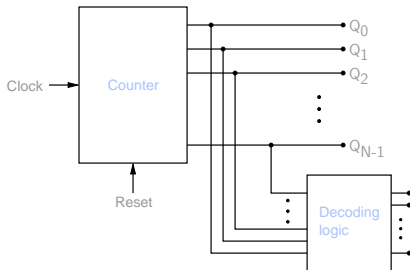


General configuration

- \* A counter with  $k$  states is called a modulo- $k$  (mod- $k$ ) counter.

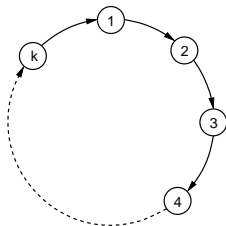


State transition diagram

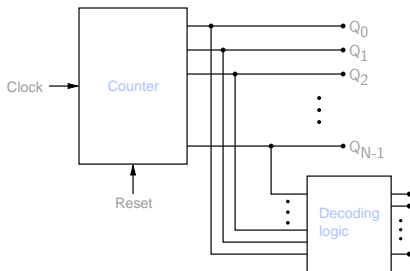


General configuration

- \* A counter with  $k$  states is called a modulo- $k$  (mod- $k$ ) counter.
- \* A counter can be made with flip-flops, each flip-flop serving as a memory element with two states (0 or 1).



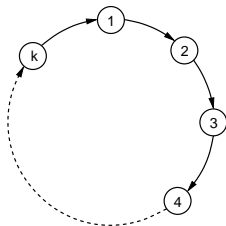
State transition diagram



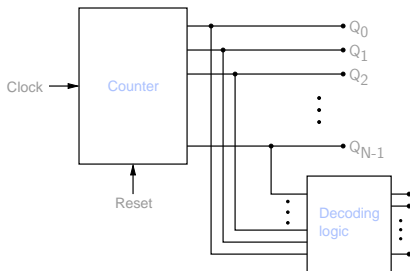
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- \* A counter can be made with flip-flops, each flip-flop serving as a memory element with two states (0 or 1).
- \* If there are  $N$  flip-flops in a counter, there are  $2^N$  possible states (since each flip-flop can have  $Q = 0$  or  $Q = 1$ ). It is possible to exclude some of these states.  
→  $N$  flip-flops can be used to make a mod- $k$  counter with  $k \leq 2^N$ .





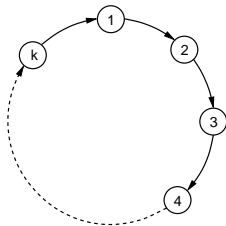
State transition diagram



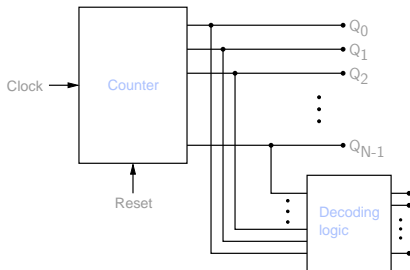
General configuration

- \* A counter with  $k$  states is called a modulo- $k$  (mod- $k$ ) counter.
- \* A counter can be made with flip-flops, each flip-flop serving as a memory element with two states (0 or 1).
- \* If there are  $N$  flip-flops in a counter, there are  $2^N$  possible states (since each flip-flop can have  $Q = 0$  or  $Q = 1$ ). It is possible to exclude some of these states.  
→  $N$  flip-flops can be used to make a mod- $k$  counter with  $k \leq 2^N$ .
- \* Typically, a reset facility is also provided, which can be used to force a certain state to initialize the counter.

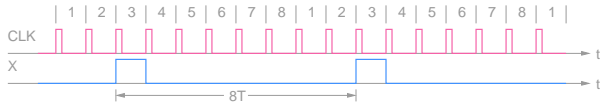
# Counters



State transition diagram

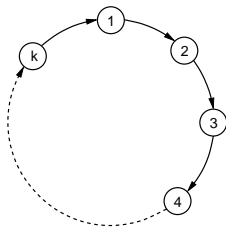


General configuration

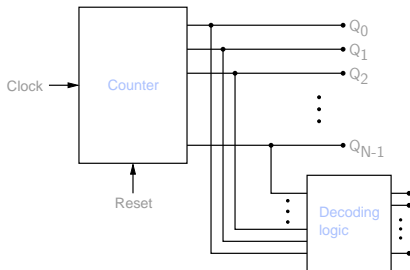


X is 1 for state 3; else, it is 0.

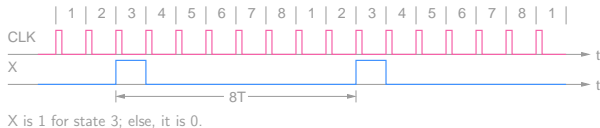
# Counters



State transition diagram

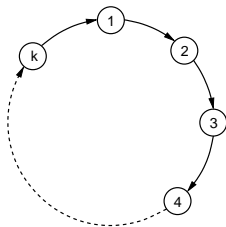


General configuration

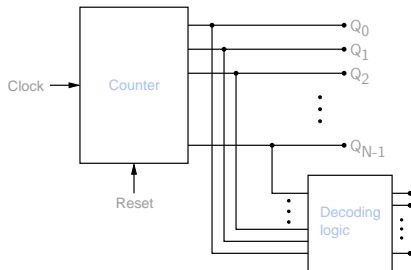


- \* The counter outputs (i.e., the flip-flop outputs,  $Q_0, Q_1, \dots, Q_{N-1}$ ) can be decoded using appropriate logic.

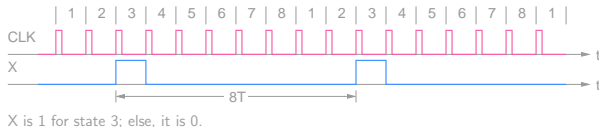
# Counters



State transition diagram

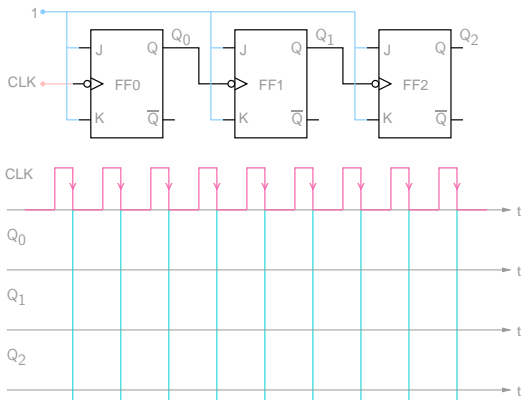


General configuration

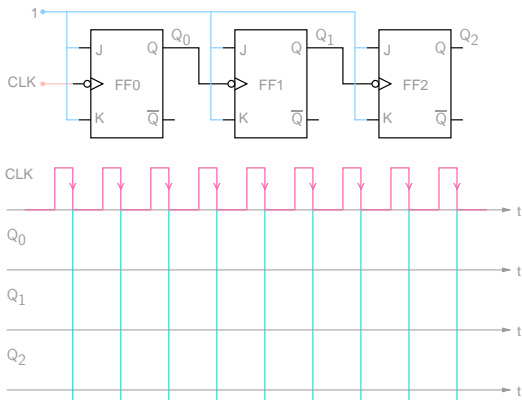


- \* The counter outputs (i.e., the flip-flop outputs,  $Q_0, Q_1, \dots, Q_{N-1}$ ) can be decoded using appropriate logic.
- \* In particular, it is possible to have a decoder output (say,  $X$ ) which is 1 only for state  $i$ , and 0 otherwise.  
→ For  $k$  clock pulses, we get a single pulse at  $X$ , i.e., the clock frequency has been divided by  $k$ . For this reason, a mod- $k$  counter is also called a divide-by- $k$  counter.

# A binary ripple counter

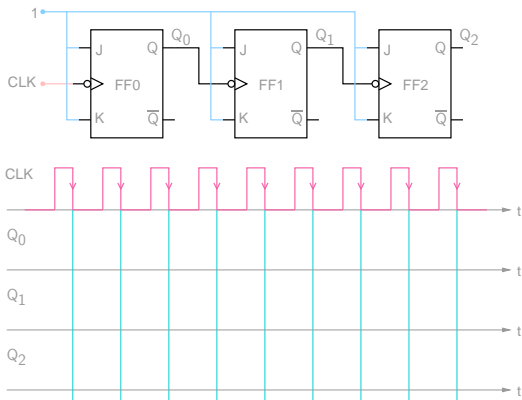


## A binary ripple counter



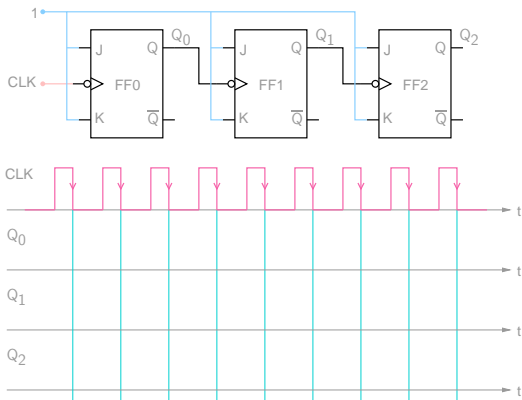
\*  $J = K = 1$  for all flip-flops. Let  $Q_0 = Q_1 = Q_2 = 0$  initially.

## A binary ripple counter



- \*  $J = K = 1$  for all flip-flops. Let  $Q_0 = Q_1 = Q_2 = 0$  initially.
- \* Since  $J = K = 1$ , each flip-flop will toggle when an active (in this case, negative) clock edge arrives.

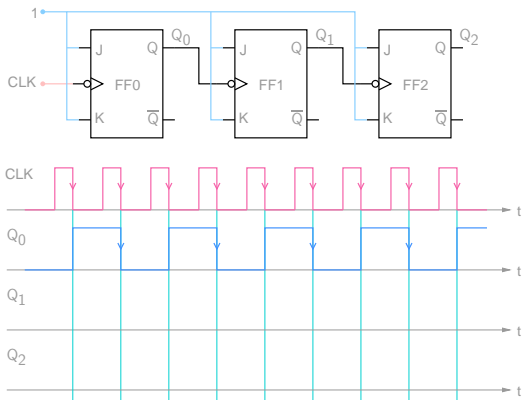
## A binary ripple counter



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- \* Since  $J = K = 1$ , each flip-flop will toggle when an active (in this case, negative) clock edge arrives.
- \* For FF1 and FF2,  $Q_0$  and  $Q_1$ , respectively, provide the clock.

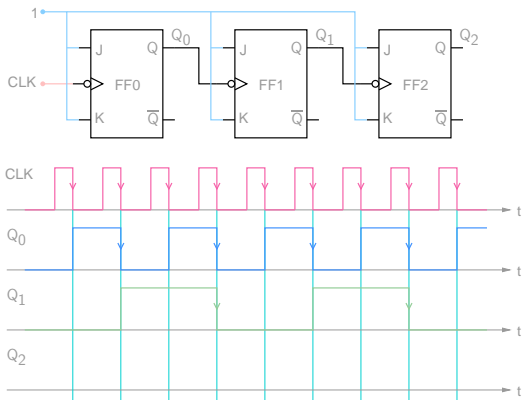


## A binary ripple counter



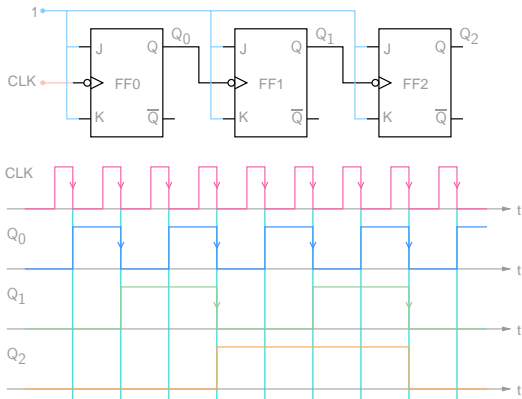
- \*  $J = K = 1$  for all flip-flops. Let  $Q_0 = Q_1 = Q_2 = 0$  initially.
- \* Since  $J = K = 1$ , each flip-flop will toggle when an active (in this case, negative) clock edge arrives.
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## A binary ripple counter



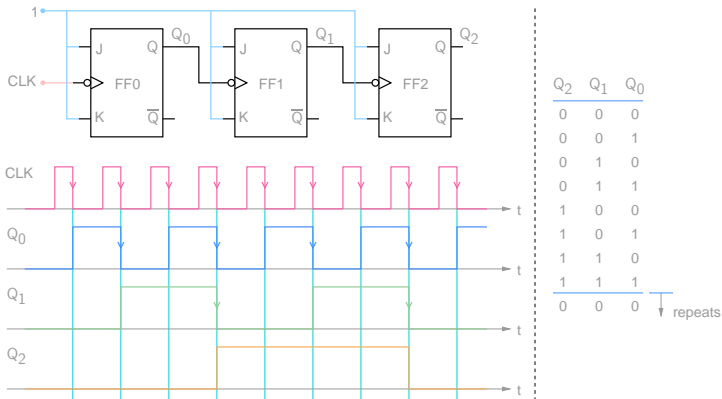
- \*  $J = K = 1$  for all flip-flops. Let  $Q_0 = Q_1 = Q_2 = 0$  initially.
- \* Since  $J = K = 1$ , each flip-flop will toggle when an active (in this case, negative) clock edge arrives.
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## A binary ripple counter



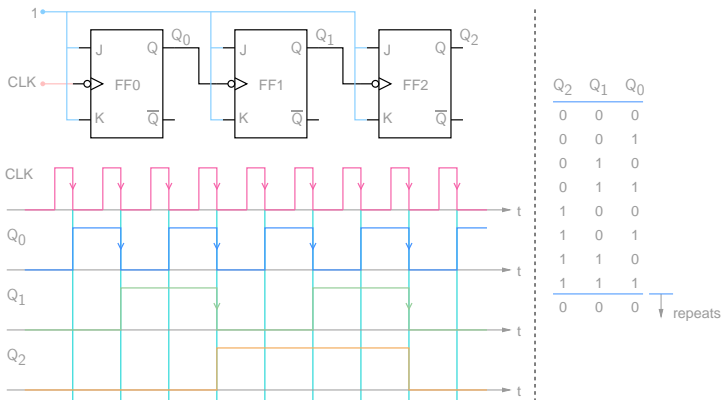
- \*  $J = K = 1$  for all flip-flops. Let  $Q_0 = Q_1 = Q_2 = 0$  initially.
- \* Since  $J = K = 1$ , each flip-flop will toggle when an active (in this case, negative) clock edge arrives.
- \* For FF1 and FF2,  $Q_0$  and  $Q_1$ , respectively, provide the clock.

## A binary ripple counter



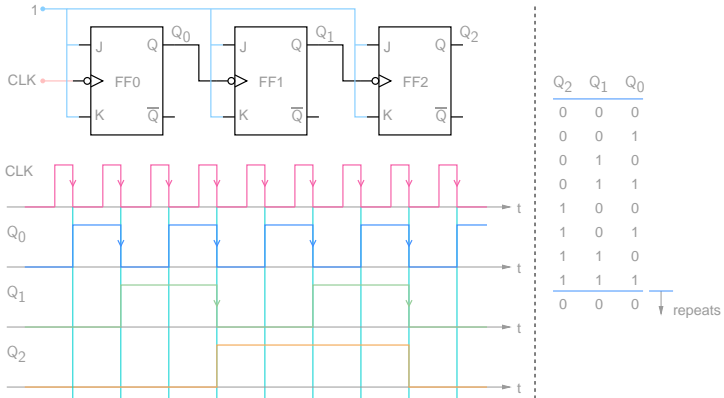
- \*  $J = K = 1$  for all flip-flops. Let  $Q_0 = Q_1 = Q_2 = 0$  initially.
- \* Since  $J = K = 1$ , each flip-flop will toggle when an active (in this case, negative) clock edge arrives.
- \* For FF1 and FF2,  $Q_0$  and  $Q_1$ , respectively, provide the clock.

## A binary ripple counter

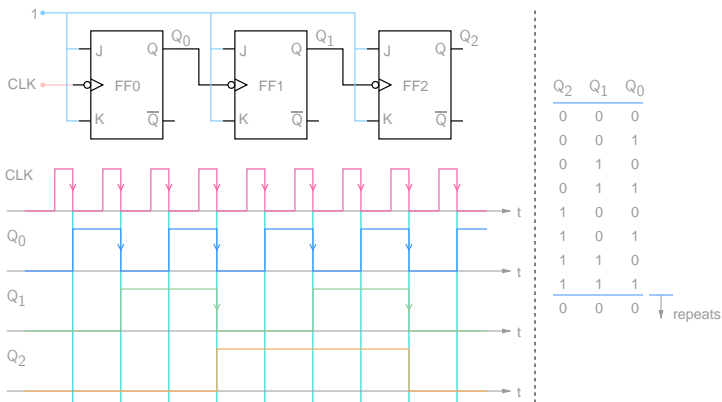


- \*  $J = K = 1$  for all flip-flops. Let  $Q_0 = Q_1 = Q_2 = 0$  initially.
- \* Since  $J = K = 1$ , each flip-flop will toggle when an active (in this case, negative) clock edge arrives.
- \* For FF1 and FF2,  $Q_0$  and  $Q_1$ , respectively, provide the clock.
- \* Note that the direct inputs  $S_d$  and  $R_d$  (not shown) are assumed to be  $S_d = R_d = 0$  for all flip-flops, allowing normal flip-flop operation.

# A binary ripple counter

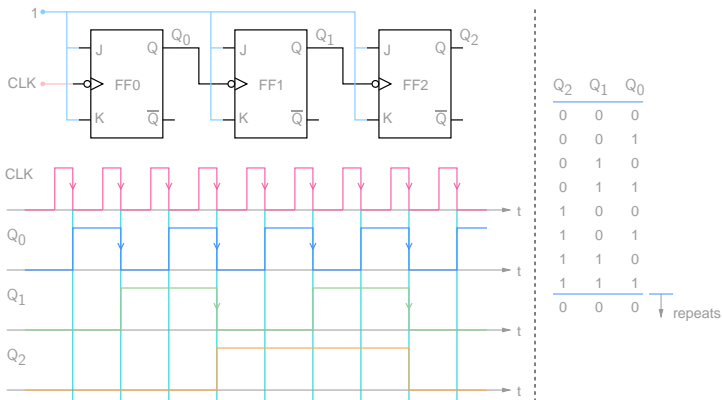


## A binary ripple counter



- \* The counter has 8 states,  $Q_2 Q_1 Q_0 = 000, 001, 010, 011, 100, 101, 110, 111$ .  
→ it is a mod-8 counter. In particular, it is a *binary, mod-8, up* counter (since it counts *up* from 000 to 111).

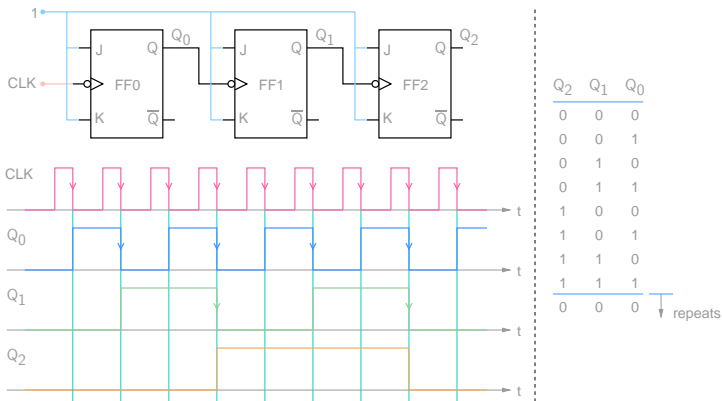
## A binary ripple counter



- \* The counter has 8 states,  $Q_2 Q_1 Q_0 = 000, 001, 010, 011, 100, 101, 110, 111$ .  
→ it is a *mod-8 counter*. In particular, it is a *binary, mod-8, up counter* (since it counts *up* from 000 to 111).
- \* If the clock frequency is  $f_c$ , the frequency at the  $Q_0, Q_1, Q_2$  outputs is  $f_c/2, f_c/4, f_c/8$ , respectively. For this counter, therefore, div-by-2, div-by-4, div-by-8 outputs are already available, without requiring decoding logic.

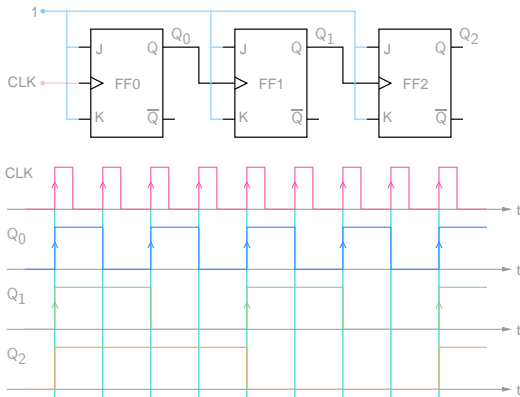


## A binary ripple counter



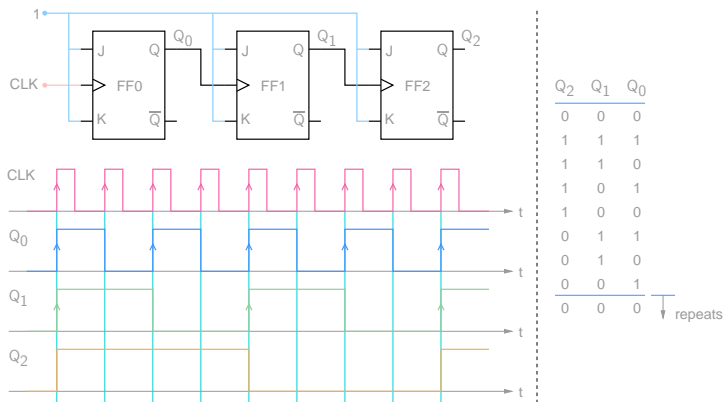
- \* The counter has 8 states,  $Q_2 Q_1 Q_0 = 000, 001, 010, 011, 100, 101, 110, 111$ .  
→ it is a *mod-8* counter. In particular, it is a *binary, mod-8, up* counter (since it counts *up* from 000 to 111).
- \* If the clock frequency is  $f_c$ , the frequency at the  $Q_0, Q_1, Q_2$  outputs is  $f_c/2, f_c/4, f_c/8$ , respectively. For this counter, therefore, div-by-2, div-by-4, div-by-8 outputs are already available, without requiring decoding logic.
- \* This type of counter is called a "ripple" counter since the clock transitions *ripple* through the flip-flops.

# A binary ripple counter



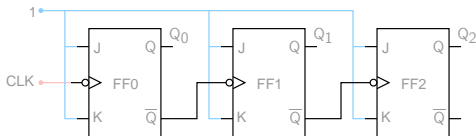
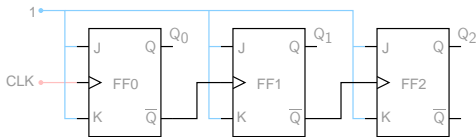
| Q <sub>2</sub> | Q <sub>1</sub> | Q <sub>0</sub> |
|----------------|----------------|----------------|
| 0              | 0              | 0              |
| 1              | 1              | 1              |
| 1              | 1              | 0              |
| 1              | 0              | 1              |
| 1              | 0              | 0              |
| 0              | 1              | 1              |
| 0              | 1              | 0              |
| 0              | 0              | 1              |
| 0              | 0              | 0              |
| ↓ repeats      |                |                |

## A binary ripple counter



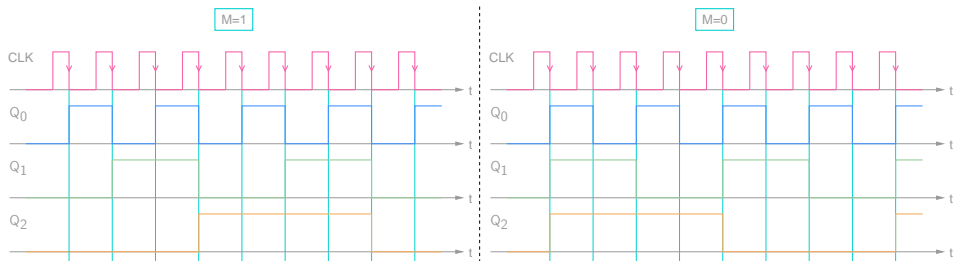
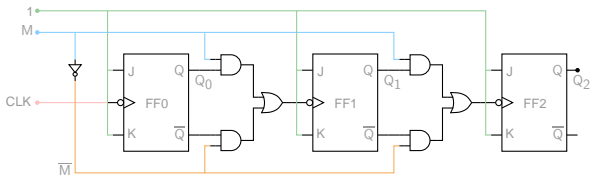
- \* If positive edge-triggered flip-flops are used, we get a binary *down* counter (counting down from 1111 to 0000).

## Binary ripple counters

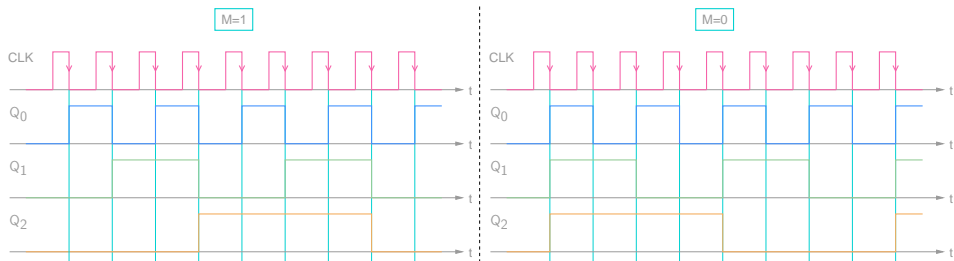
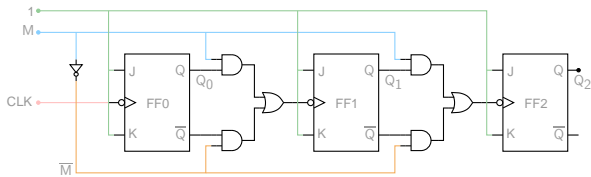


- \* Home work: Sketch the waveforms (CLK,  $Q_0$ ,  $Q_1$ ,  $Q_2$ ), and tabulate the counter states in each case.

# Up-down binary ripple counters

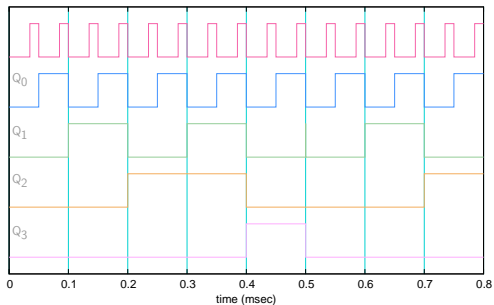
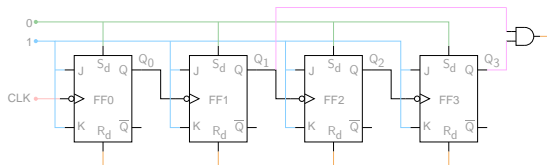


## Up-down binary ripple counters



- \* When Mode ( $M$ ) = 1, the counter counts up; else, it counts down.  
(SEQUEL file: ee101\_counter\_3.sqproj)

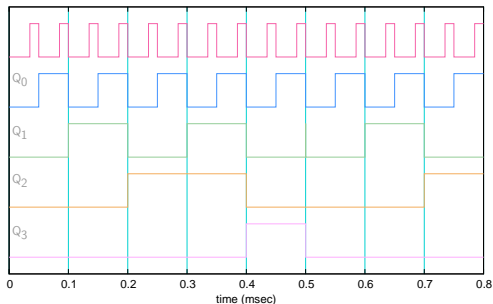
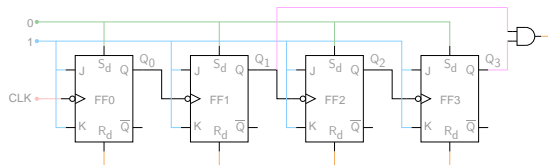
# Decade counter using direct inputs



| Q <sub>3</sub> | Q <sub>2</sub> | Q <sub>1</sub> | Q <sub>0</sub> |
|----------------|----------------|----------------|----------------|
| 0              | 0              | 0              | 0              |
| 0              | 0              | 0              | 1              |
| 0              | 0              | 1              | 0              |
| 0              | 0              | 1              | 1              |
| 0              | 1              | 0              | 0              |
| 0              | 1              | 0              | 1              |
| 0              | 1              | 1              | 0              |
| 0              | 1              | 1              | 1              |
| 1              | 0              | 0              | 0              |
| 1              | 0              | 0              | 1              |
| 0              | 0              | 0              | 0              |

↓ repeats

## Decade counter using direct inputs



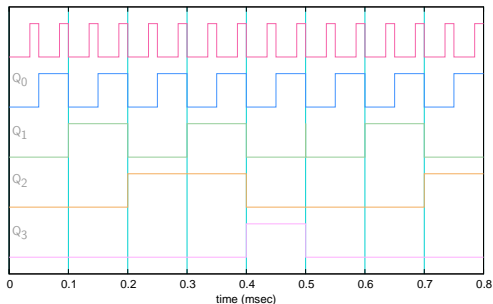
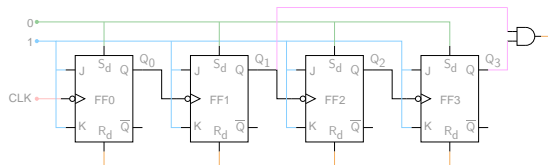
| $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ |
|-------|-------|-------|-------|
| 0     | 0     | 0     | 0     |
| 0     | 0     | 0     | 1     |
| 0     | 0     | 1     | 0     |
| 0     | 0     | 1     | 1     |
| 0     | 1     | 0     | 0     |
| 0     | 1     | 0     | 1     |
| 0     | 1     | 1     | 0     |
| 0     | 1     | 1     | 1     |
| 1     | 0     | 0     | 0     |
| 1     | 0     | 0     | 1     |
| 0     | 0     | 0     | 0     |

↓ repeats

- \* When the counter reaches  $Q_3 Q_2 Q_1 Q_0 = 1010$  (i.e., decimal 10),  $Q_3 Q_1 = 1$ , and the flip-flops are cleared to  $Q_3 Q_2 Q_1 Q_0 = 0000$ .



## Decade counter using direct inputs

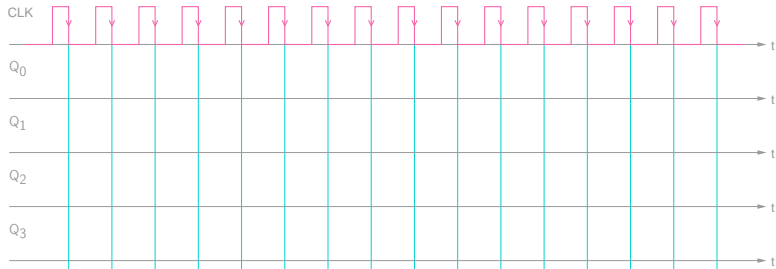
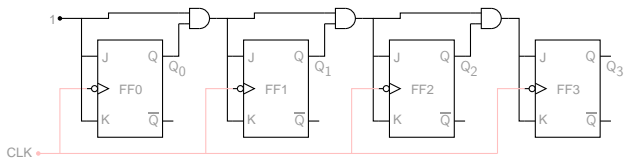


| $Q_3$ | $Q_2$ | $Q_1$ | $Q_0$ |
|-------|-------|-------|-------|
| 0     | 0     | 0     | 0     |
| 0     | 0     | 0     | 1     |
| 0     | 0     | 1     | 0     |
| 0     | 0     | 1     | 1     |
| 0     | 1     | 0     | 0     |
| 0     | 1     | 0     | 1     |
| 0     | 1     | 1     | 0     |
| 0     | 1     | 1     | 1     |
| 1     | 0     | 0     | 0     |
| 1     | 0     | 0     | 1     |
| 0     | 0     | 0     | 0     |

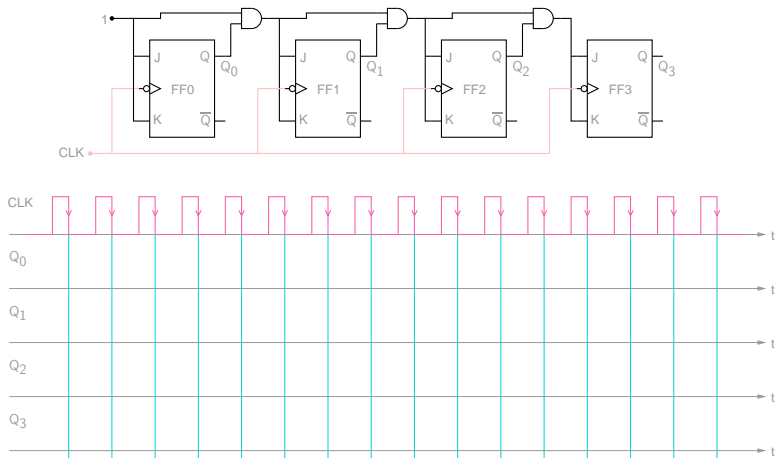
↓ repeats

- \* When the counter reaches  $Q_3Q_2Q_1Q_0 = 1010$  (i.e., decimal 10),  $Q_3Q_1 = 1$ , and the flip-flops are cleared to  $Q_3Q_2Q_1Q_0 = 0000$ .
- \* The counter counts from 0000 (decimal 0) to 1001 (decimal 9) → “decade counter.” (SEQUEL file: ee101\_counter\_5.sqproj)

# A synchronous counter

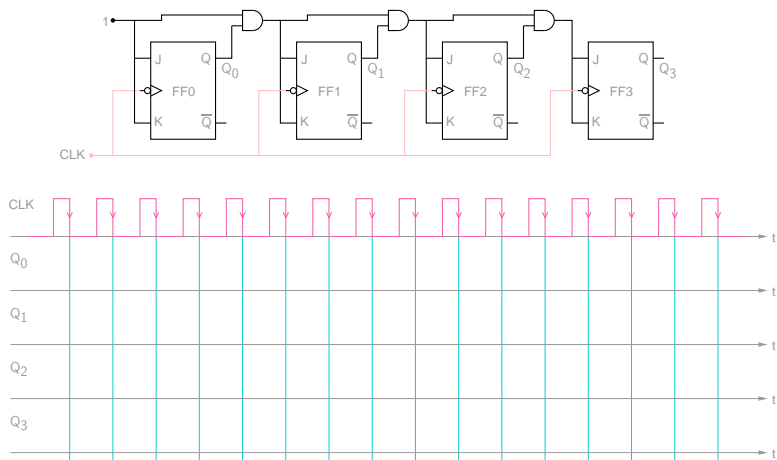


## A synchronous counter



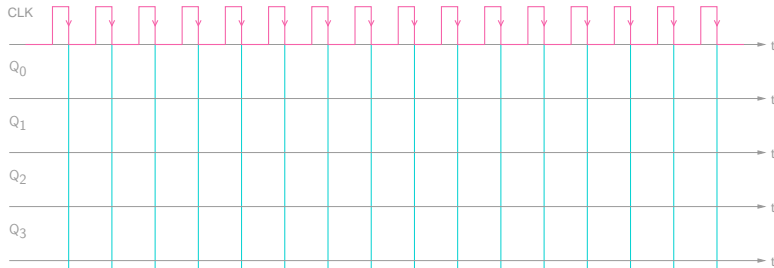
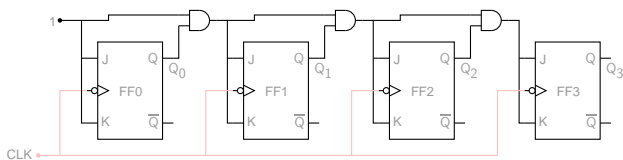
- \* Since all flip-flops are driven by the same clock, the counter is called a “synchronous” counter.

## A synchronous counter



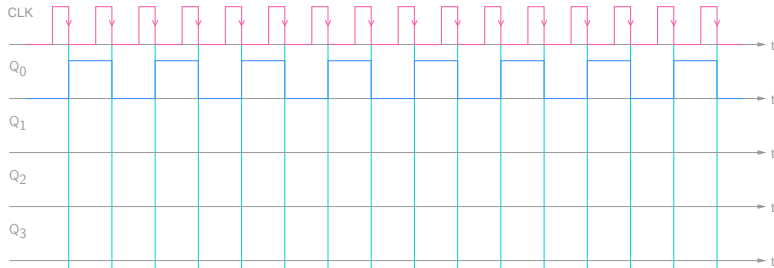
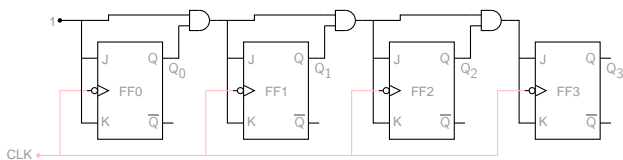
- \* Since all flip-flops are driven by the same clock, the counter is called a "synchronous" counter.
- \*  $J_0 = K_0 = 1$ ,  $J_1 = K_1 = Q_0$ ,  $J_2 = K_2 = Q_1 Q_0$ ,  $J_3 = K_3 = Q_2 Q_1 Q_0$ .

## A synchronous counter



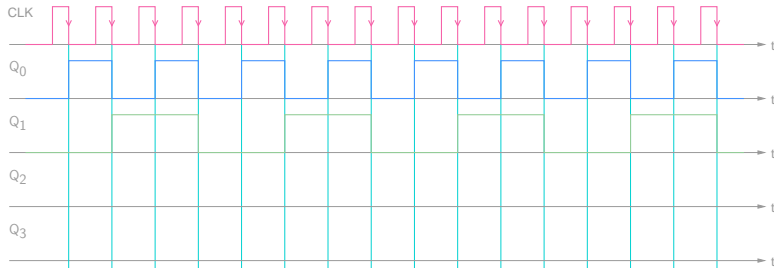
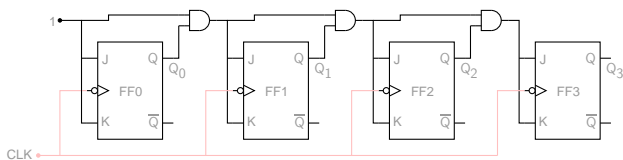
- \* Since all flip-flops are driven by the same clock, the counter is called a “synchronous” counter.
- \*  $J_0 = K_0 = 1$ ,  $J_1 = K_1 = Q_0$ ,  $J_2 = K_2 = Q_1 Q_0$ ,  $J_3 = K_3 = Q_2 Q_1 Q_0$ .
- \* FF0 toggles at every active edge.  
FF1 toggles if  $Q_0 = 1$  (just before the active clock edge); else, it retains its previous state.  
Similar comments apply to FF2 and FF3.

## A synchronous counter



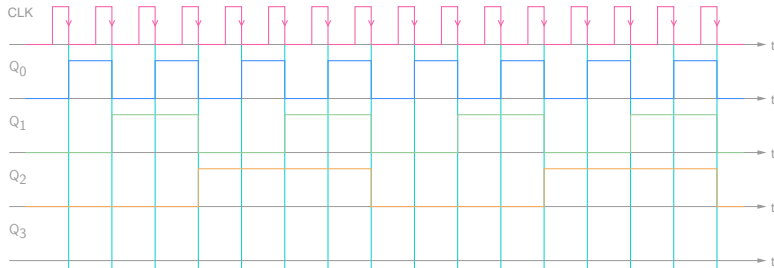
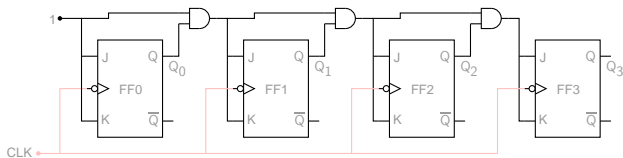
- \* Since all flip-flops are driven by the same clock, the counter is called a “synchronous” counter.
- \*  $J_0 = K_0 = 1$ ,  $J_1 = K_1 = Q_0$ ,  $J_2 = K_2 = Q_1 Q_0$ ,  $J_3 = K_3 = Q_2 Q_1 Q_0$ .
- \* FF0 toggles at every active edge.  
FF1 toggles if  $Q_0 = 1$  (just before the active clock edge); else, it retains its previous state.  
Similar comments apply to FF2 and FF3.

## A synchronous counter



- \* Since all flip-flops are driven by the same clock, the counter is called a “synchronous” counter.
- \*  $J_0 = K_0 = 1$ ,  $J_1 = K_1 = Q_0$ ,  $J_2 = K_2 = Q_1 Q_0$ ,  $J_3 = K_3 = Q_2 Q_1 Q_0$ .
- \* FF0 toggles at every active edge.  
FF1 toggles if  $Q_0 = 1$  (just before the active clock edge); else, it retains its previous state.  
Similar comments apply to FF2 and FF3.

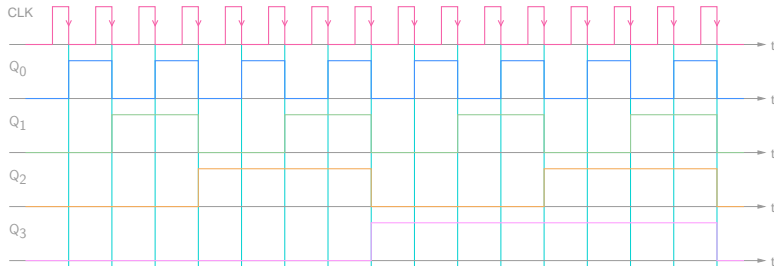
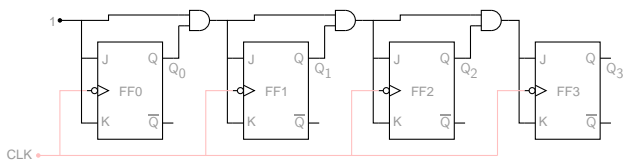
## A synchronous counter



- \* Since all flip-flops are driven by the same clock, the counter is called a “synchronous” counter.
- \*  $J_0 = K_0 = 1$ ,  $J_1 = K_1 = Q_0$ ,  $J_2 = K_2 = Q_1 Q_0$ ,  $J_3 = K_3 = Q_2 Q_1 Q_0$ .
- \* FF0 toggles at every active edge.  
FF1 toggles if  $Q_0 = 1$  (just before the active clock edge); else, it retains its previous state.  
Similar comments apply to FF2 and FF3.

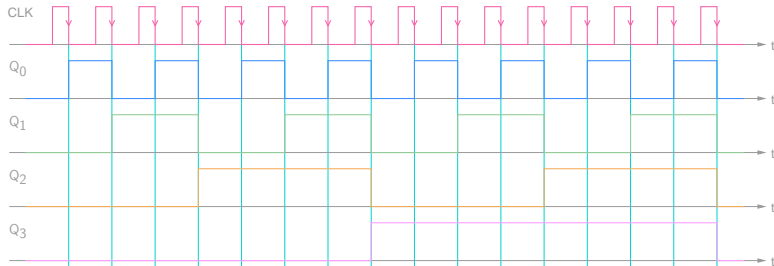
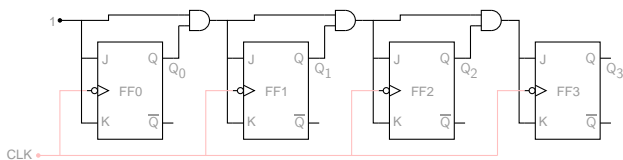


## A synchronous counter



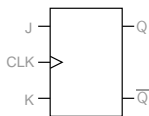
- \* Since all flip-flops are driven by the same clock, the counter is called a “synchronous” counter.
- \*  $J_0 = K_0 = 1$ ,  $J_1 = K_1 = Q_0$ ,  $J_2 = K_2 = Q_1 Q_0$ ,  $J_3 = K_3 = Q_2 Q_1 Q_0$ .
- \* FF0 toggles at every active edge.  
FF1 toggles if  $Q_0 = 1$  (just before the active clock edge); else, it retains its previous state.  
Similar comments apply to FF2 and FF3.

## A synchronous counter



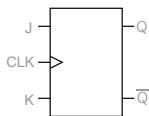
- \* Since all flip-flops are driven by the same clock, the counter is called a “synchronous” counter.
- \*  $J_0 = K_0 = 1$ ,  $J_1 = K_1 = Q_0$ ,  $J_2 = K_2 = Q_1 Q_0$ ,  $J_3 = K_3 = Q_2 Q_1 Q_0$ .
- \* FF0 toggles at every active edge.  
FF1 toggles if  $Q_0 = 1$  (just before the active clock edge); else, it retains its previous state. Similar comments apply to FF2 and FF3.
- \* From the waveforms, we see that it is a binary up counter. <img alt="Navigation icons: back, forward, search, etc."/>

## Design of synchronous counters



| CLK | J | K | $Q_{n+1}$        |
|-----|---|---|------------------|
| ↑   | 0 | 0 | $Q_n$            |
| ↑   | 0 | 1 | 0                |
| ↑   | 1 | 0 | 1                |
| ↑   | 1 | 1 | $\overline{Q_n}$ |

## Design of synchronous counters

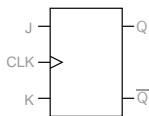


| CLK | J | K | $Q_{n+1}$        |
|-----|---|---|------------------|
| ↑   | 0 | 0 | $Q_n$            |
| ↑   | 0 | 1 | 0                |
| ↑   | 1 | 0 | 1                |
| ↑   | 1 | 1 | $\overline{Q_n}$ |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |

- \* Consider the *reverse* problem: We are given  $Q_n$  and the next desired state ( $Q_{n+1}$ ). What should  $J$  and  $K$  be in order to make that happen?

## Design of synchronous counters

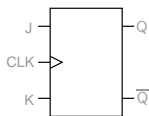


| CLK | J | K | $Q_{n+1}$        |
|-----|---|---|------------------|
| ↑   | 0 | 0 | $Q_n$            |
| ↑   | 0 | 1 | 0                |
| ↑   | 1 | 0 | 1                |
| ↑   | 1 | 1 | $\overline{Q_n}$ |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |

- \* Consider the *reverse* problem: We are given  $Q_n$  and the next desired state ( $Q_{n+1}$ ). What should  $J$  and  $K$  be in order to make that happen?
- \*  $Q_n = 0, Q_{n+1} = 0$ : We can either force  $Q_{n+1} = 0$  with  $J = 0, K = 1$ , or let  $Q_{n+1} = Q_n = 0$  by making  $J = 0, K = 0$ .

## Design of synchronous counters

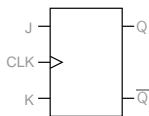


| CLK | J | K | $Q_{n+1}$        |
|-----|---|---|------------------|
| ↑   | 0 | 0 | $Q_n$            |
| ↑   | 0 | 1 | 0                |
| ↑   | 1 | 0 | 1                |
| ↑   | 1 | 1 | $\overline{Q_n}$ |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |

- \* Consider the *reverse* problem: We are given  $Q_n$  and the next desired state ( $Q_{n+1}$ ). What should  $J$  and  $K$  be in order to make that happen?
- \*  $Q_n = 0, Q_{n+1} = 0$ : We can either force  $Q_{n+1} = 0$  with  $J = 0, K = 1$ , or let  $Q_{n+1} = Q_n = 0$  by making  $J = 0, K = 0$ .  
→  $J = 0, K = X$  (i.e.,  $K$  can be 0 or 1).

## Design of synchronous counters

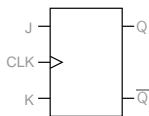


| CLK | J | K | $Q_{n+1}$        |
|-----|---|---|------------------|
| ↑   | 0 | 0 | $Q_n$            |
| ↑   | 0 | 1 | 0                |
| ↑   | 1 | 0 | 1                |
| ↑   | 1 | 1 | $\overline{Q_n}$ |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |

- \* Consider the *reverse* problem: We are given  $Q_n$  and the next desired state ( $Q_{n+1}$ ). What should  $J$  and  $K$  be in order to make that happen?
- \*  $Q_n = 0, Q_{n+1} = 0$ : We can either force  $Q_{n+1} = 0$  with  $J = 0, K = 1$ , or let  $Q_{n+1} = Q_n = 0$  by making  $J = 0, K = 0$ .  
→  $J = 0, K = X$  (i.e.,  $K$  can be 0 or 1).

## Design of synchronous counters



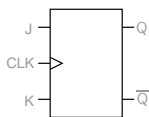
| CLK | J | K | $Q_{n+1}$        |
|-----|---|---|------------------|
| ↑   | 0 | 0 | $Q_n$            |
| ↑   | 0 | 1 | 0                |
| ↑   | 1 | 0 | 1                |
| ↑   | 1 | 1 | $\overline{Q_n}$ |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |
|     |       |           |   |   |

- \* Consider the *reverse* problem: We are given  $Q_n$  and the next desired state ( $Q_{n+1}$ ). What should  $J$  and  $K$  be in order to make that happen?
- \*  $Q_n = 0, Q_{n+1} = 0$ : We can either force  $Q_{n+1} = 0$  with  $J = 0, K = 1$ , or let  $Q_{n+1} = Q_n = 0$  by making  $J = 0, K = 0$ .  
→  $J = 0, K = X$  (i.e.,  $K$  can be 0 or 1).
- \* Similarly, work out the other entries in the table.



## Design of synchronous counters

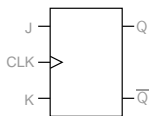


| CLK | J | K | $Q_{n+1}$        |
|-----|---|---|------------------|
| ↑   | 0 | 0 | $Q_n$            |
| ↑   | 0 | 1 | 0                |
| ↑   | 1 | 0 | 1                |
| ↑   | 1 | 1 | $\overline{Q_n}$ |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

- \* Consider the *reverse* problem: We are given  $Q_n$  and the next desired state ( $Q_{n+1}$ ). What should  $J$  and  $K$  be in order to make that happen?
- \*  $Q_n = 0, Q_{n+1} = 0$ : We can either force  $Q_{n+1} = 0$  with  $J = 0, K = 1$ , or let  $Q_{n+1} = Q_n = 0$  by making  $J = 0, K = 0$ .  
→  $J = 0, K = X$  (i.e.,  $K$  can be 0 or 1).
- \* Similarly, work out the other entries in the table.

## Design of synchronous counters



| CLK | J | K | $Q_{n+1}$        |
|-----|---|---|------------------|
| ↑   | 0 | 0 | $Q_n$            |
| ↑   | 0 | 1 | 0                |
| ↑   | 1 | 0 | 1                |
| ↑   | 1 | 1 | $\overline{Q_n}$ |

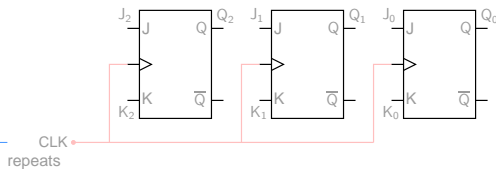
| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

- \* Consider the *reverse* problem: We are given  $Q_n$  and the next desired state ( $Q_{n+1}$ ). What should  $J$  and  $K$  be in order to make that happen?
- \*  $Q_n = 0, Q_{n+1} = 0$ : We can either force  $Q_{n+1} = 0$  with  $J = 0, K = 1$ , or let  $Q_{n+1} = Q_n = 0$  by making  $J = 0, K = 0$ .  
→  $J = 0, K = X$  (i.e.,  $K$  can be 0 or 1).
- \* Similarly, work out the other entries in the table.
- \* The table for a negative edge-triggered flip-flop would be identical except for the active edge.

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ |
|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     |
| 2     | 0     | 0     | 1     |
| 3     | 0     | 1     | 0     |
| 4     | 0     | 1     | 1     |
| 5     | 1     | 0     | 0     |
| 1     | 0     | 0     | 0     |

↓ repeats



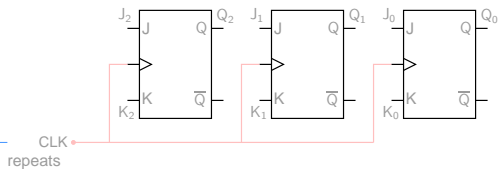
| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

Design a synchronous mod-5 counter with the given state transition table.

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ |
|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     |
| 2     | 0     | 0     | 1     |
| 3     | 0     | 1     | 0     |
| 4     | 0     | 1     | 1     |
| 5     | 1     | 0     | 0     |
| 1     | 0     | 0     | 0     |

↓ repeats



| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

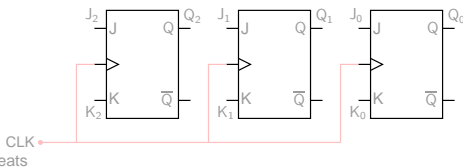
Design a synchronous mod-5 counter with the given state transition table.

Outline of method:

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ |
|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     |
| 2     | 0     | 0     | 1     |
| 3     | 0     | 1     | 0     |
| 4     | 0     | 1     | 1     |
| 5     | 1     | 0     | 0     |
| 1     | 0     | 0     | 0     |

↓ repeats



| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

Design a synchronous mod-5 counter with the given state transition table.

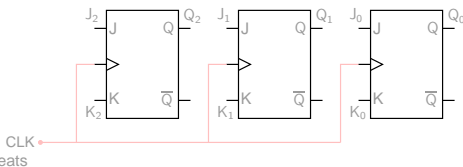
Outline of method:

- \* State 1  $\rightarrow$  State 2 means
  - $Q_2: 0 \rightarrow 0,$
  - $Q_1: 0 \rightarrow 0,$
  - $Q_0: 0 \rightarrow 1.$

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ |
|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     |
| 2     | 0     | 0     | 1     |
| 3     | 0     | 1     | 0     |
| 4     | 0     | 1     | 1     |
| 5     | 1     | 0     | 0     |
| 1     | 0     | 0     | 0     |

↓ repeats



| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

Design a synchronous mod-5 counter with the given state transition table.

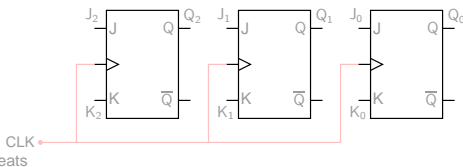
Outline of method:

- \* State 1  $\rightarrow$  State 2 means
  - $Q_2: 0 \rightarrow 0,$
  - $Q_1: 0 \rightarrow 0,$
  - $Q_0: 0 \rightarrow 1.$
- \* Refer to the right table. For  $Q_2: 0 \rightarrow 0,$  we must have  $J_2 = 0, K_2 = X,$  and so on.

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ |
|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     |
| 2     | 0     | 0     | 1     |
| 3     | 0     | 1     | 0     |
| 4     | 0     | 1     | 1     |
| 5     | 1     | 0     | 0     |
| 1     | 0     | 0     | 0     |

↓ repeats



| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

Design a synchronous mod-5 counter with the given state transition table.

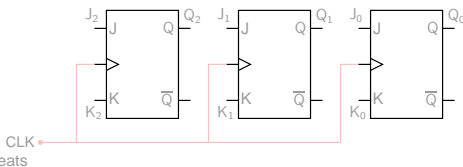
Outline of method:

- \* State 1  $\rightarrow$  State 2 means  
 $Q_2: 0 \rightarrow 0,$   
 $Q_1: 0 \rightarrow 0,$   
 $Q_0: 0 \rightarrow 1.$
- \* Refer to the right table. For  $Q_2: 0 \rightarrow 0$ , we must have  $J_2 = 0, K_2 = X$ , and so on.
- \* When we cover all transitions in the left table, we have the truth tables for  $J_0, K_0, J_1, K_1, J_2, K_2$  in terms of  $Q_1, Q_2, Q_3$ .

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ |
|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     |
| 2     | 0     | 0     | 1     |
| 3     | 0     | 1     | 0     |
| 4     | 0     | 1     | 1     |
| 5     | 1     | 0     | 0     |
| 1     | 0     | 0     | 0     |

↓ repeats



| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

Design a synchronous mod-5 counter with the given state transition table.

Outline of method:

- \* State 1  $\rightarrow$  State 2 means  
 $Q_2: 0 \rightarrow 0$ ,  
 $Q_1: 0 \rightarrow 0$ ,  
 $Q_0: 0 \rightarrow 1$ .
- \* Refer to the right table. For  $Q_2: 0 \rightarrow 0$ , we must have  $J_2 = 0$ ,  $K_2 = X$ , and so on.
- \* When we cover all transitions in the left table, we have the truth tables for  $J_0$ ,  $K_0$ ,  $J_1$ ,  $K_1$ ,  $J_2$ ,  $K_2$  in terms of  $Q_1$ ,  $Q_2$ ,  $Q_3$ .
- \* The last step is to come up with suitable functions for  $J_0$ ,  $K_0$ ,  $J_1$ ,  $K_1$ ,  $J_2$ ,  $K_2$  in terms of  $Q_1$ ,  $Q_2$ ,  $Q_3$ . This can be done with K-maps. (If the number of flip-flops is more than 4, other techniques can be employed.)



## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |
| 2     | 0     | 0     | 1     |       |       |       |       |       |       |
| 3     | 0     | 1     | 0     |       |       |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |
| 2     | 0     | 0     | 1     |       |       |       |       |       |       |
| 3     | 0     | 1     | 0     |       |       |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     |       |       |       |       |
| 2     | 0     | 0     | 1     |       |       |       |       |       |       |
| 3     | 0     | 1     | 0     |       |       |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | Q <sub>2</sub> | Q <sub>1</sub> | Q <sub>0</sub> | J <sub>2</sub> | K <sub>2</sub> | J <sub>1</sub> | K <sub>1</sub> | J <sub>0</sub> | K <sub>0</sub> |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1     | 0              | 0              | 0              | 0              | X              | 0              | X              |                |                |
| 2     | 0              | 0              | 1              |                |                |                |                |                |                |
| 3     | 0              | 1              | 0              |                |                |                |                |                |                |
| 4     | 0              | 1              | 1              |                |                |                |                |                |                |
| 5     | 1              | 0              | 0              |                |                |                |                |                |                |
| 1     | 0              | 0              | 0              |                |                |                |                |                |                |

| CLK | Q <sub>n</sub> | Q <sub>n+1</sub> | J | K |
|-----|----------------|------------------|---|---|
| ↑   | 0              | 0                | 0 | X |
| ↑   | 0              | 1                | 1 | X |
| ↑   | 1              | 0                | X | 1 |
| ↑   | 1              | 1                | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     |       |       |       |       |       |       |
| 3     | 0     | 1     | 0     |       |       |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | Q <sub>2</sub> | Q <sub>1</sub> | Q <sub>0</sub> | J <sub>2</sub> | K <sub>2</sub> | J <sub>1</sub> | K <sub>1</sub> | J <sub>0</sub> | K <sub>0</sub> |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1     | 0              | 0              | 0              | 0              | X              | 0              | X              | 1              | X              |
| 2     | 0              | 0              | 1              |                |                |                |                |                |                |
| 3     | 0              | 1              | 0              |                |                |                |                |                |                |
| 4     | 0              | 1              | 1              |                |                |                |                |                |                |
| 5     | 1              | 0              | 0              |                |                |                |                |                |                |
| 1     | 0              | 0              | 0              |                |                |                |                |                |                |

| CLK | Q <sub>n</sub> | Q <sub>n+1</sub> | J | K |
|-----|----------------|------------------|---|---|
| ↑   | 0              | 0                | 0 | X |
| ↑   | 0              | 1                | 1 | X |
| ↑   | 1              | 0                | X | 1 |
| ↑   | 1              | 1                | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     |       |       |       |       |
| 3     | 0     | 1     | 0     |       |       |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     |       |       |
| 3     | 0     | 1     | 0     |       |       |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |



## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     |       |       |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     |       |       |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     |       |       |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     |       |       |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     |       |       |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | Q <sub>2</sub> | Q <sub>1</sub> | Q <sub>0</sub> | J <sub>2</sub> | K <sub>2</sub> | J <sub>1</sub> | K <sub>1</sub> | J <sub>0</sub> | K <sub>0</sub> |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1     | 0              | 0              | 0              | 0              | X              | 0              | X              | 1              | X              |
| 2     | 0              | 0              | 1              | 0              | X              | 1              | X              | X              | 1              |
| 3     | 0              | 1              | 0              | 0              | X              | X              | 0              | 1              | X              |
| 4     | 0              | 1              | 1              |                |                |                |                |                |                |
| 5     | 1              | 0              | 0              |                |                |                |                |                |                |
| 1     | 0              | 0              | 0              |                |                |                |                |                |                |

| CLK | Q <sub>n</sub> | Q <sub>n+1</sub> | J | K |
|-----|----------------|------------------|---|---|
| ↑   | 0              | 0                | 0 | X |
| ↑   | 0              | 1                | 1 | X |
| ↑   | 1              | 0                | X | 1 |
| ↑   | 1              | 1                | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     |       |       |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     |       |       |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |



## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     |       |       |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     |       |       |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     | 0     | X     |       |       |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     | 0     | X     | 0     | X     |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     | 0     | X     | 0     | X     |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

- \* We now have the truth tables for  $J_0$ ,  $K_0$ ,  $J_1$ ,  $K_1$ ,  $J_2$ ,  $K_2$  in terms of  $Q_0$ ,  $Q_1$ ,  $Q_2$ . The next step is to find logical functions for each of them.

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     | 0     | X     | 0     | X     |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

| CLK | $Q_n$ | $Q_{n+1}$ | J | K |
|-----|-------|-----------|---|---|
| ↑   | 0     | 0         | 0 | X |
| ↑   | 0     | 1         | 1 | X |
| ↑   | 1     | 0         | X | 1 |
| ↑   | 1     | 1         | X | 0 |

- \* We now have the truth tables for  $J_0$ ,  $K_0$ ,  $J_1$ ,  $K_1$ ,  $J_2$ ,  $K_2$  in terms of  $Q_0$ ,  $Q_1$ ,  $Q_2$ . The next step is to find logical functions for each of them.
- \* Note that we have not tabulated the  $J$  and  $K$  values for those combinations of  $Q_0$ ,  $Q_1$ ,  $Q_2$  which do not occur in the state transition table (such as  $Q_2Q_1Q_0 = 110$ ). We treat these as don't care conditions (next slide).

# Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     | 0     | X     | 0     | X     |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

$J_2$

| $Q_0$ | $Q_2Q_1$ |   | 00 | 01 | 11 | 10 |
|-------|----------|---|----|----|----|----|
| 0     | 0        | 0 | 0  | 0  | X  | X  |
| 1     | 0        | 1 | 1  | X  | X  |    |

$J_1$

| $Q_0$ | $Q_2Q_1$ |   | 00 | 01 | 11 | 10 |
|-------|----------|---|----|----|----|----|
| 0     | 0        | X | X  | X  | 0  |    |
| 1     | 0        | 1 | 1  | X  | X  |    |

$J_0$

| $Q_0$ | $Q_2Q_1$ |   | 00 | 01 | 11 | 10 |
|-------|----------|---|----|----|----|----|
| 0     | 1        | 1 | 1  | 1  | X  | 0  |
| 1     | 1        | 1 | 1  | X  | X  | X  |

$K_2$

| $Q_0$ | $Q_2Q_1$ |   | 00 | 01 | 11 | 10 |
|-------|----------|---|----|----|----|----|
| 0     | X        | X | X  | X  | X  | 1  |
| 1     | X        | X | X  | X  | X  | X  |

$K_1$

| $Q_0$ | $Q_2Q_1$ |   | 00 | 01 | 11 | 10 |
|-------|----------|---|----|----|----|----|
| 0     | X        | 0 | X  | X  | X  | X  |
| 1     | X        | 1 | X  | X  | X  | X  |

$K_0$

| $Q_0$ | $Q_2Q_1$ |   | 00 | 01 | 11 | 10 |
|-------|----------|---|----|----|----|----|
| 0     | X        | X | X  | X  | X  | X  |
| 1     | 1        | 1 | 1  | X  | X  | X  |



## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     | 0     | X     | 0     | X     |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

$J_2$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 0  | 0  | X  | X  |
| 1        | 0  | 1  | X  | X  |

$J_1$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 0  | X  | X  | 0  |
| 1        | 1  | X  | X  | X  |

$J_0$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 1  | 1  | X  | 0  |
| 1        | X  | X  | X  | X  |

$K_2$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | X  | X  | 1  |
| 1        | X  | X  | X  | X  |

$K_1$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | 0  | X  | X  |
| 1        | X  | 1  | X  | X  |

$K_0$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | X  | X  | X  |
| 1        | 1  | 1  | X  | X  |

- \* We treat the unused states ( $Q_2Q_1Q_0 = 101, 110, 111$ ) as (additional) don't care conditions. Since these are different from the don't care conditions arising from the state transition table, we mark them with a different colour.

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     | 0     | X     | 0     | X     |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

$J_2$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 0  | 0  | X  | X  |
| 1        | 0  | 1  | X  | X  |

$J_1$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 0  | X  | X  | 0  |
| 1        | 1  | X  | X  | X  |

$J_0$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 1  | 1  | X  | 0  |
| 1        | X  | X  | X  | X  |

$K_2$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | X  | X  | 1  |
| 1        | X  | X  | X  | X  |

$K_1$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | 0  | X  | X  |
| 1        | X  | 1  | X  | X  |

$K_0$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | X  | X  | X  |
| 1        | 1  | 1  | X  | X  |

- \* We treat the unused states ( $Q_2Q_1Q_0 = 101, 110, 111$ ) as (additional) don't care conditions. Since these are different from the don't care conditions arising from the state transition table, we mark them with a different colour.
- \* We will assume that a suitable initialization facility is provided to ensure that the counter starts up in one of the five allowed states (say,  $Q_2Q_1Q_0 = 000$ ).

## Design of synchronous counters

| state | $Q_2$ | $Q_1$ | $Q_0$ | $J_2$ | $K_2$ | $J_1$ | $K_1$ | $J_0$ | $K_0$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 0     | 0     | 0     | 0     | X     | 0     | X     | 1     | X     |
| 2     | 0     | 0     | 1     | 0     | X     | 1     | X     | X     | 1     |
| 3     | 0     | 1     | 0     | 0     | X     | X     | 0     | 1     | X     |
| 4     | 0     | 1     | 1     | 1     | X     | X     | 1     | X     | 1     |
| 5     | 1     | 0     | 0     | X     | 1     | 0     | X     | 0     | X     |
| 1     | 0     | 0     | 0     |       |       |       |       |       |       |

$J_2$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 0  | 0  | X  | X  |
| 1        | 0  | 1  | X  | X  |

$J_1$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 0  | X  | X  | 0  |
| 1        | 1  | X  | X  | X  |

$J_0$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | 1  | 1  | X  | 0  |
| 1        | X  | X  | X  | X  |

$K_2$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | X  | X  | 1  |
| 1        | X  | X  | X  | X  |

$K_1$

| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | 0  | X  | X  |
| 1        | X  | 1  | X  | X  |

$K_0$

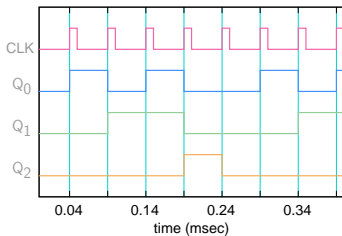
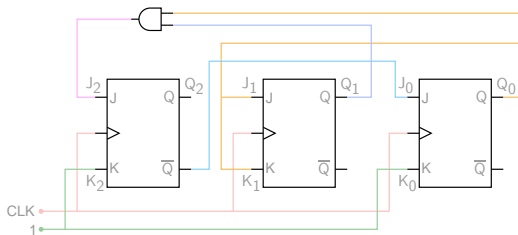
| $Q_2Q_1$ | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 0        | X  | X  | X  | X  |
| 1        | 1  | 1  | X  | X  |

- We treat the unused states ( $Q_2Q_1Q_0 = 101, 110, 111$ ) as (additional) don't care conditions. Since these are different from the don't care conditions arising from the state transition table, we mark them with a different colour.
- We will assume that a suitable initialization facility is provided to ensure that the counter starts up in one of the five allowed states (say,  $Q_2Q_1Q_0 = 000$ ).
- From the K-maps,  $J_2 = Q_1Q_0$ ,  $K_2 = 1$ ,  $J_1 = Q_0$ ,  $K_1 = Q_0$ ,  $J_0 = \overline{Q_2}$ ,  $K_0 = 1$ .





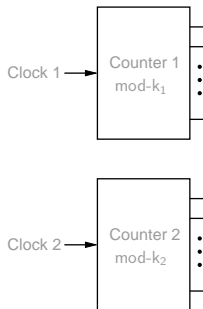
## Design of synchronous counters: verification



(SEQUEL file: ee101\_counter\_6.sqproj)

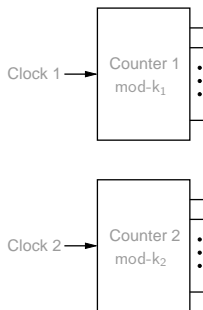
- \*  $J_2 = Q_1 Q_0$ ,  $K_2 = 1$ ,  $J_1 = Q_0$ ,  $K_1 = Q_0$ ,  $J_0 = \overline{Q_2}$ ,  $K_0 = 1$ .
- \* Note that the design is independent of whether positive or negative edge-triggered flip-flops are used.

## Combination of counters



- \* Consider two counters, Counter 1 (mod- $k_1$ ) and Counter 2 (mod- $k_2$ ). (Each of them can be ripple or synchronous type.)

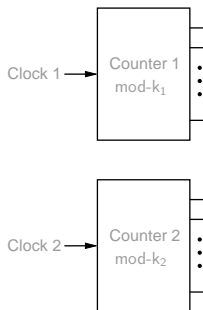
## Combination of counters



- \* Consider two counters, Counter 1 (mod- $k_1$ ) and Counter 2 (mod- $k_2$ ). (Each of them can be ripple or synchronous type.)
- \* Since Counter 1 has  $k_1$  states and Counter 2 has  $k_2$  states, we can get a new counter with  $k_1 k_2$  states if appropriate *synchronisation* is provided between the two clocks.

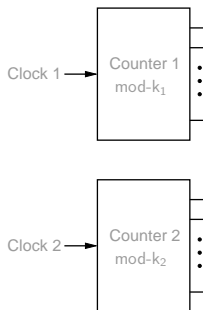


## Combination of counters



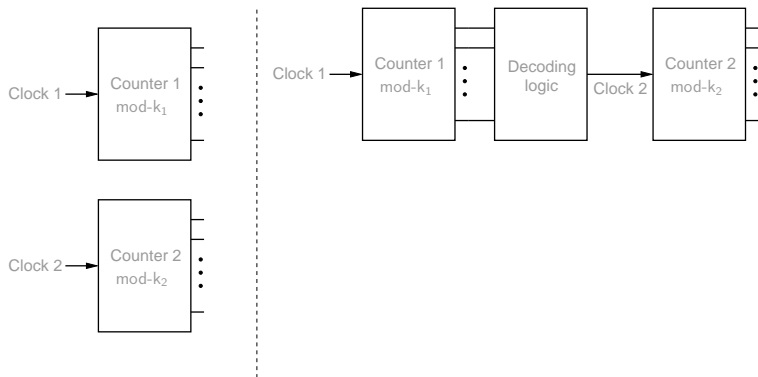
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- \* There are two ways of providing synchronisation:

## Combination of counters



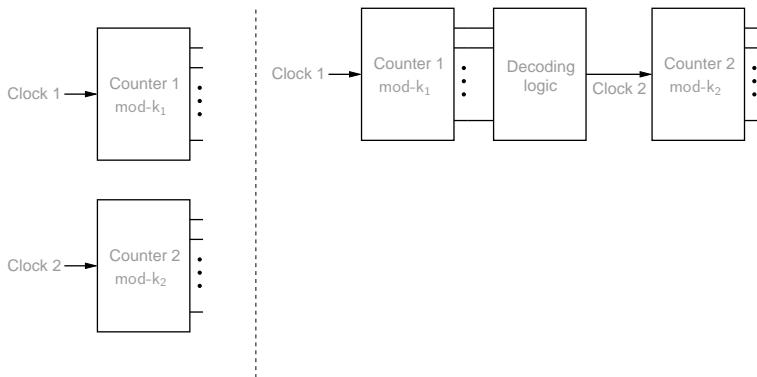
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  - derive Clock 2 from Clock 1 (using some decoding logic, if necessary)

## Combination of counters



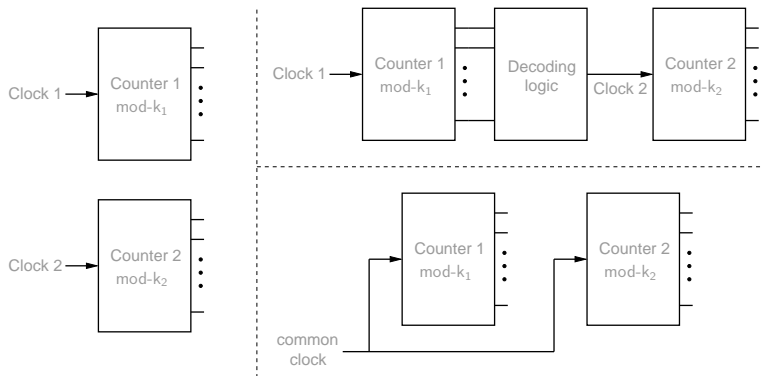
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## Combination of counters



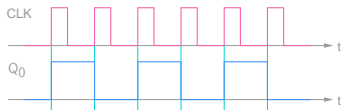
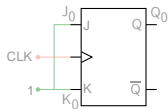
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  - drive the two counters with the same clock

## Combination of counters

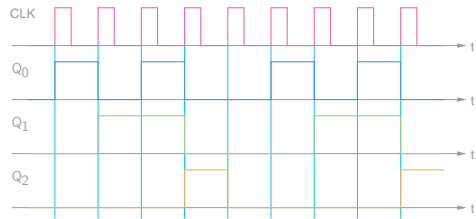
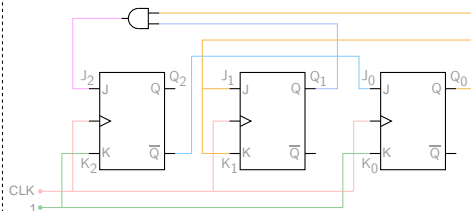


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# Combination of counters

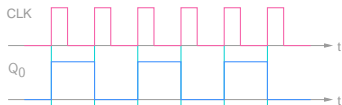
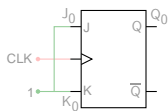


mod-2 counter

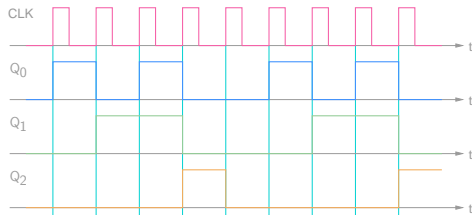
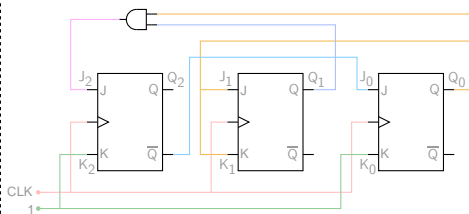


mod-5 counter

## Combination of counters



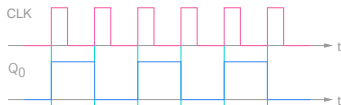
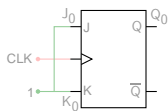
mod-2 counter



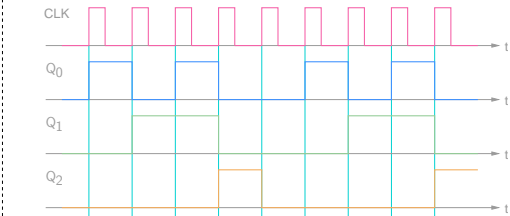
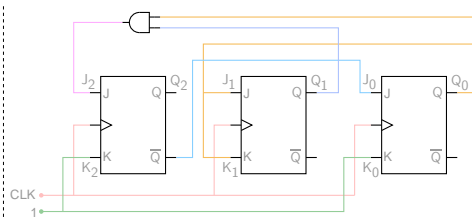
mod-5 counter

- \* Let us combine the mod-2 and mod-5 counters to make a mod-10 counter.

## Combination of counters



mod-2 counter



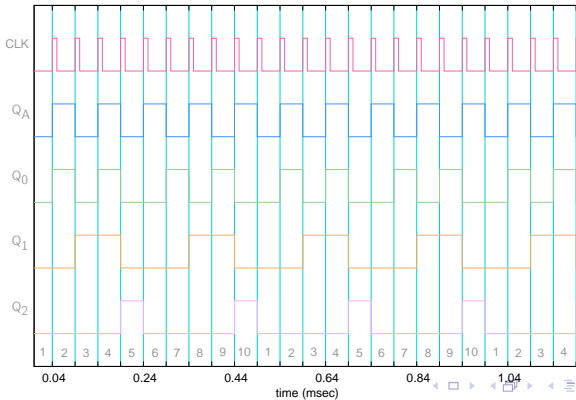
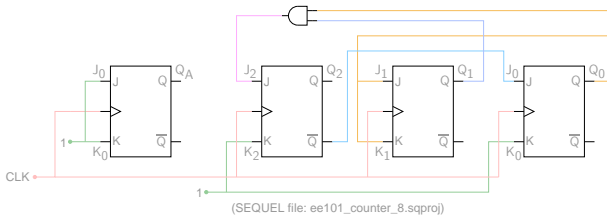
mod-5 counter

- \* Let us combine the mod-2 and mod-5 counters to make a mod-10 counter.
- \* We will follow two approaches (as described earlier):
  - A: The clock for the second (mod-5) counter is derived from the first (mod-2) counter.
  - B: A common clock is used to drive the mod-2 and mod-5 counters.





# Approach B



## Combination of counters

- \* Show that, by connecting the  $\overline{Q}$  output of the mod-2 counter (instead of the  $Q$  output) to the clock input of the mod-5 counter in the ripple connection (“Approach A”) circuit, we get a decade counter, counting up from 0000 to 1001.

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- \* Derive appropriate decoding logic for each of the ten counters states (i.e., the output should be 1 for only that particular state and 0 otherwise).
- \* Derive appropriate decoding logic which will give a symmetrical square wave (i.e., a duty cycle of 50 %) with a frequency of  $f_c/10$ , where  $f_c$  is the clock frequency.

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- \* Derive appropriate decoding logic which will give a symmetrical square wave (i.e., a duty cycle of 50 %) with a frequency of  $f_c/10$ , where  $f_c$  is the clock frequency.
- \* Verify your design by simulation.