

## EE101: Op Amp circuits (Part 2)

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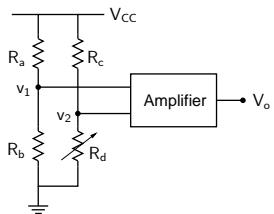
**M. B. Patil**

[mbpatil@ee.iitb.ac.in](mailto:mbpatil@ee.iitb.ac.in)

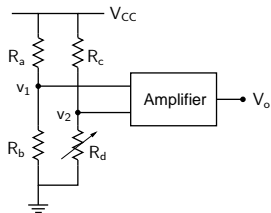
[www.ee.iitb.ac.in/~sequel](http://www.ee.iitb.ac.in/~sequel)

Department of Electrical Engineering  
Indian Institute of Technology Bombay

## Common-mode and differential-mode voltages



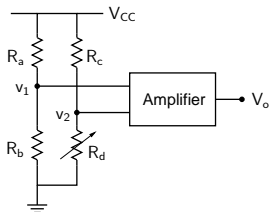
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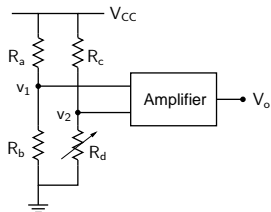


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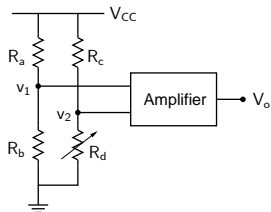
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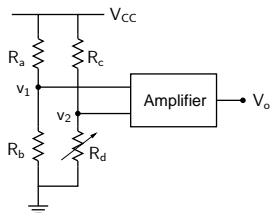
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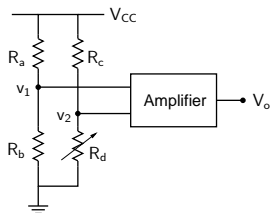
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where  $x = \Delta R/R$ .

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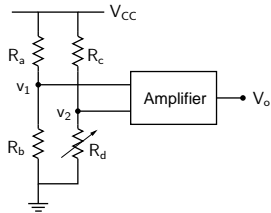
For example, with  $V_{CC} = 15 V$ ,  $R = 1 k$ ,  $\Delta R = 0.01 k$ ,

$$v_1 = 7.5 V ,$$

$$v_2 = 7.5 + 0.0375 V .$$

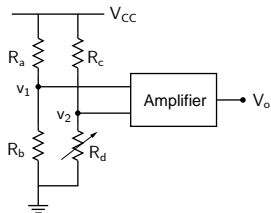


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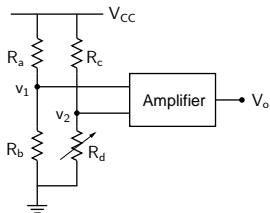
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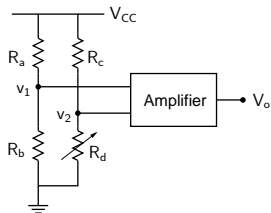
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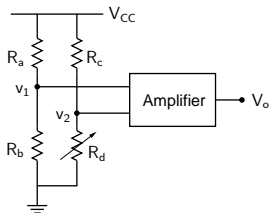
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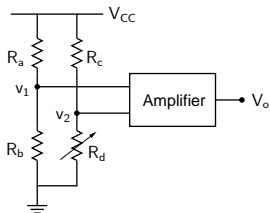
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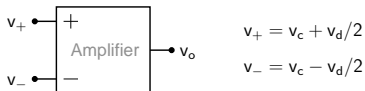
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Note that the common-mode voltage is quite large compared to the differential-mode voltage.

This is a common situation in transducer circuits.

## Common-Mode Rejection Ratio

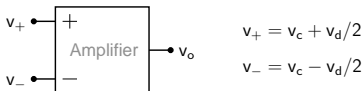


An ideal amplifier would only amplify the difference ( $v_+ - v_-$ ), giving

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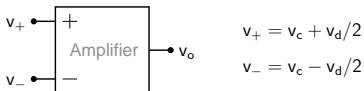
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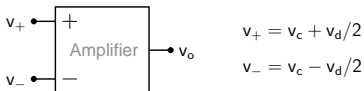
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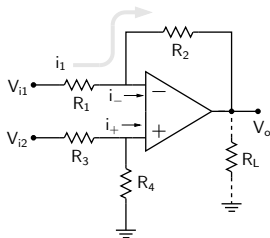
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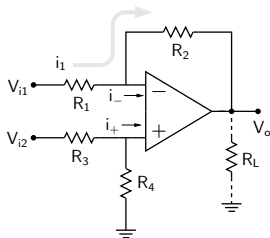
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For the 741 Op Amp, the CMRR is 90 dB ( $\simeq 30,000$ ), which may be considered to be infinite in many applications. In such cases, mismatch between circuit components will determine the overall common-mode rejection performance of the circuit.

## Op Amp circuits (linear region)



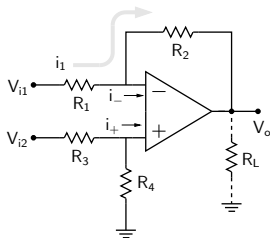
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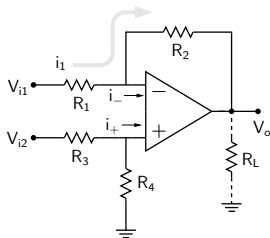


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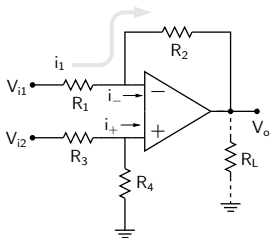
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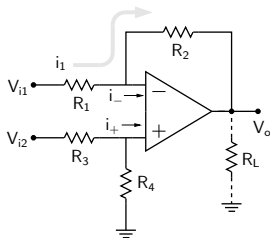
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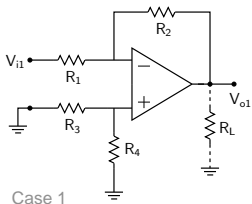
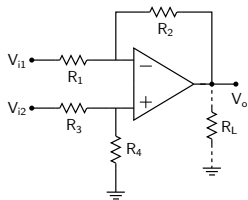
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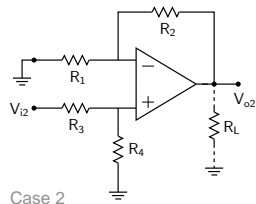
The circuit is a “difference amplifier.”



# Difference amplifier



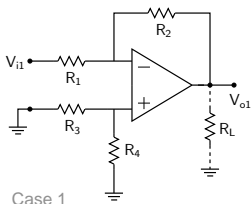
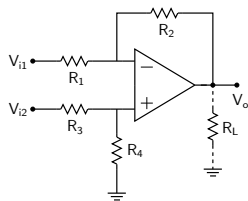
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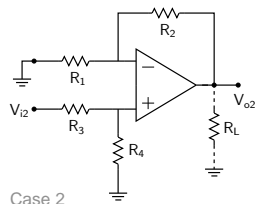
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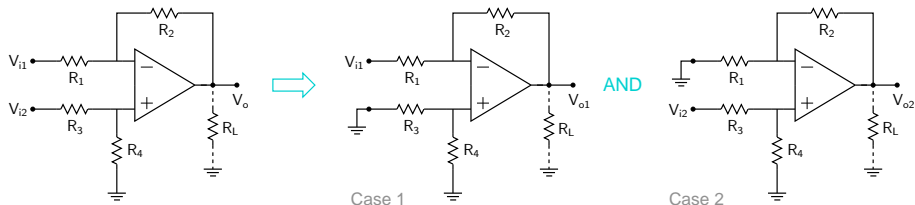
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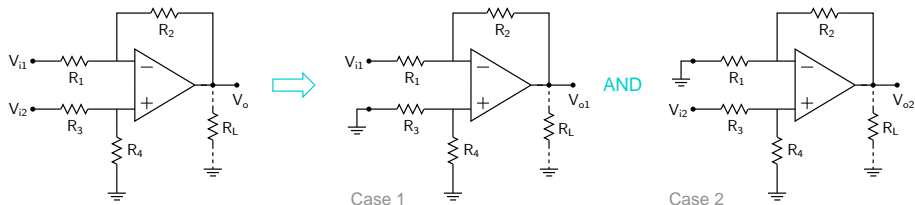
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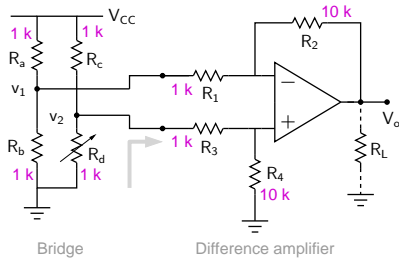
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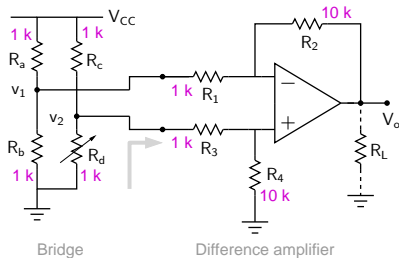
The net result is,

$$V_o = V_{o1} + V_{o2} = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1} V_{i1} = \frac{R_2}{R_1} (V_{i2} - V_{i1}), \text{ if } R_3/R_4 = R_1/R_2.$$

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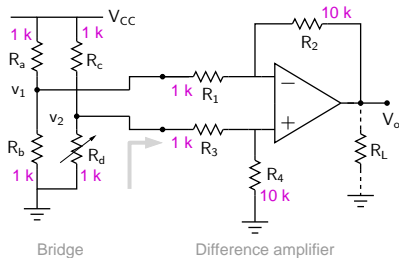


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The resistance seen from  $v_2$  is  $(R_3 + R_4)$  which is small enough to cause  $v_2$  to change. This is not desirable.

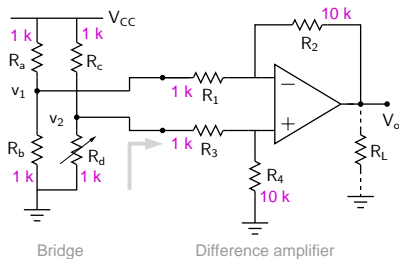
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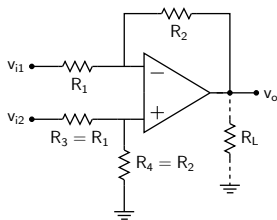
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We will discuss an improved difference amplifier later. Before we do that, let us discuss another problem with the above difference amplifier which can be important for some applications (next slide).



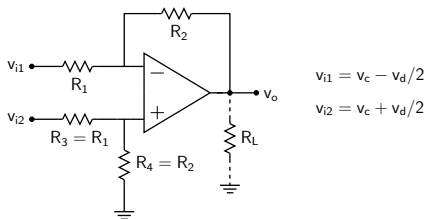
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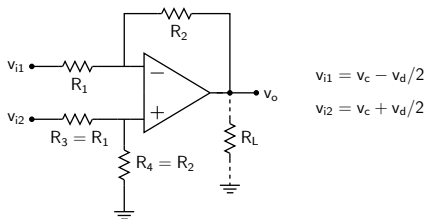
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The output voltage depends only on the differential-mode signal  $(v_{i2} - v_{i1})$ ,  
i.e.,  $A_c$  (common-mode gain) = 0.

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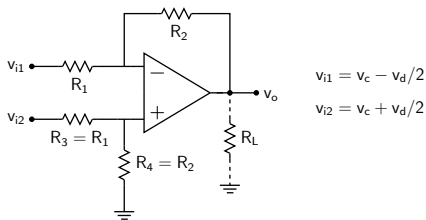


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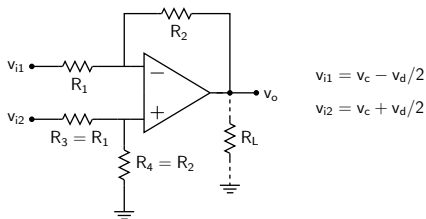
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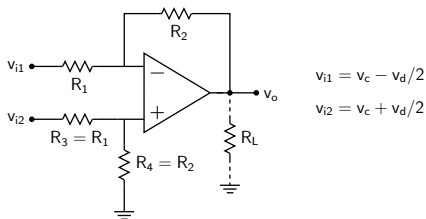
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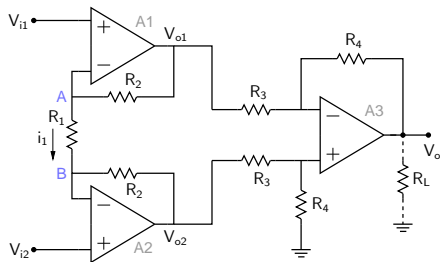
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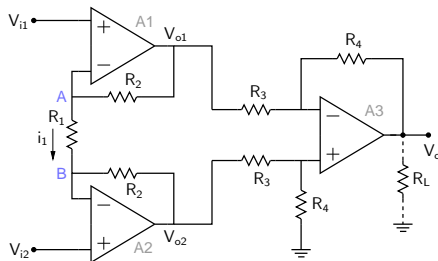
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However, since  $v_c$  can be large compared to  $v_d$ , the effect of  $A_c$  cannot be ignored.

## Improved difference amplifier



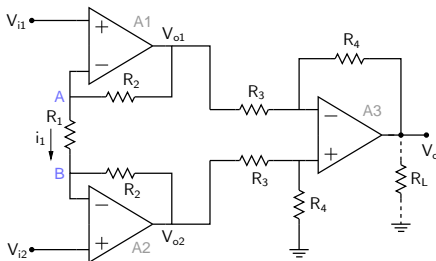
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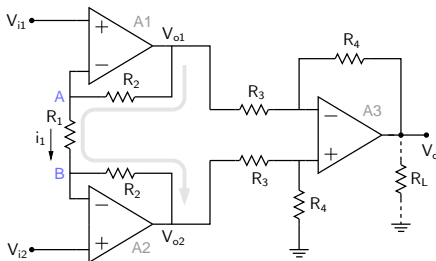
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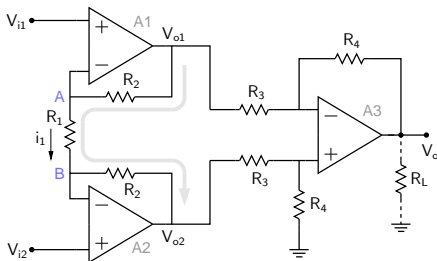
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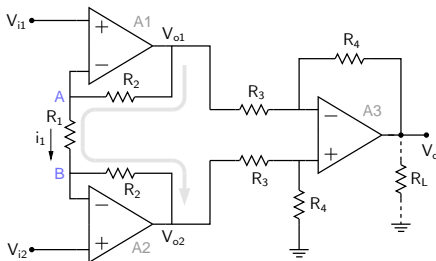


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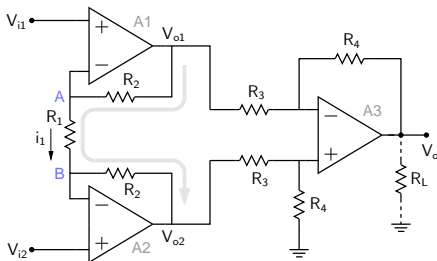
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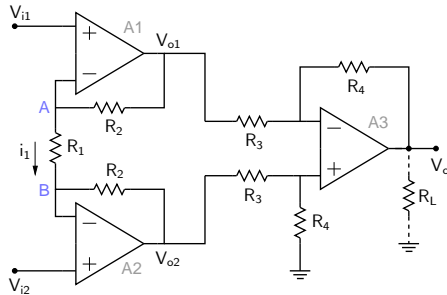
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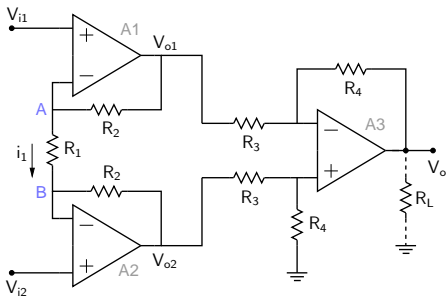
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This circuit is known as the "instrumentation amplifier."

# Instrumentation amplifier

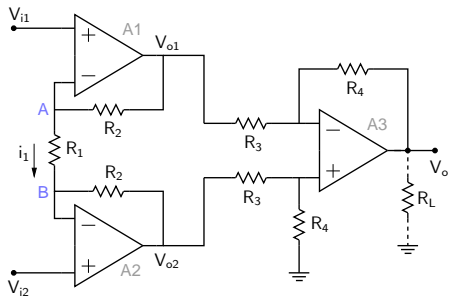


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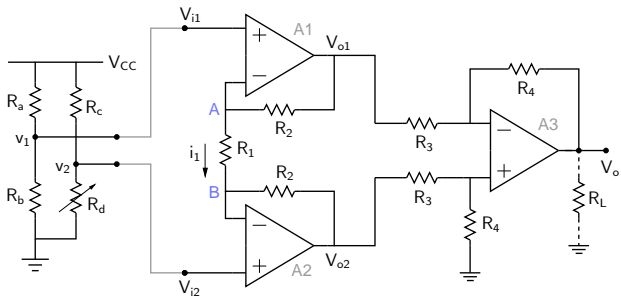


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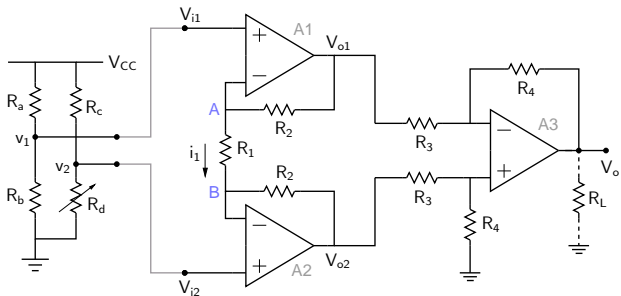
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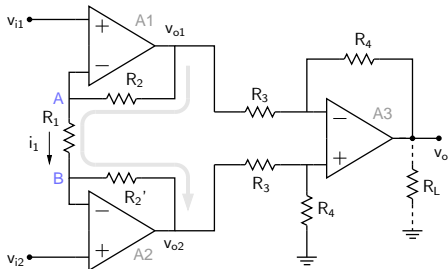


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As a result, the voltages  $v_1$  and  $v_2$  in the bridge circuit will remain essentially the same when the bridge circuit is connected to the instrumentation amplifier.

# Instrumentation amplifier

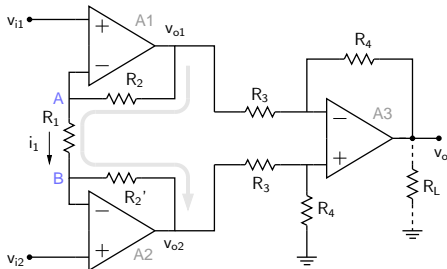


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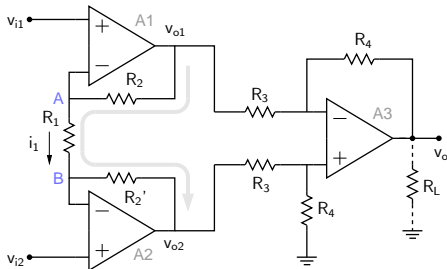
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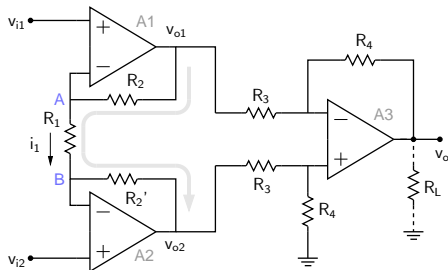
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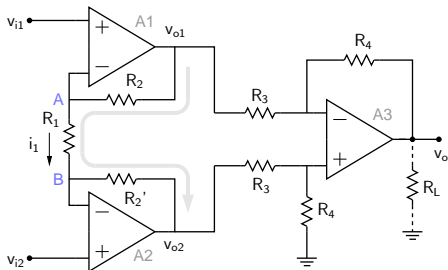
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→ The instrumentation amplifier is very effective in minimising the effect of the common-mode signal. (Note that component mismatch in the second stage will cause a finite CMRR, but the first stage has effectively amplified only  $v_d$  while leaving  $v_c$  unchanged; so the overall CMRR has improved.)

## Current-to-voltage conversion

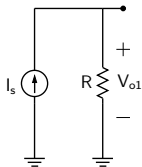
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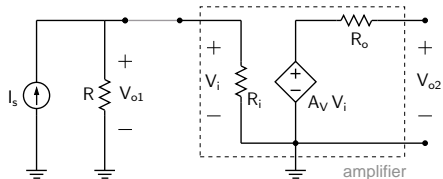
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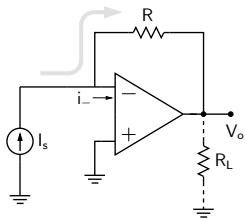
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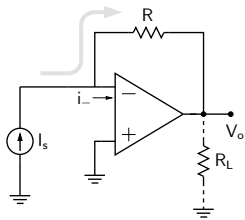


However, this simple approach will not work if the next stage in the circuit (such as an amplifier) has a finite  $R_i$ , since it will modify  $V_{o1}$  to  $V_{o1} = I_s (R_i \parallel R)$ , which is not desirable.

## Current-to-voltage conversion

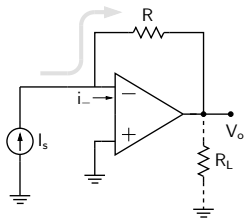


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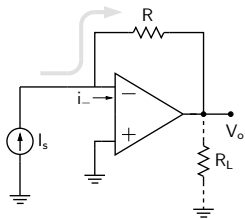
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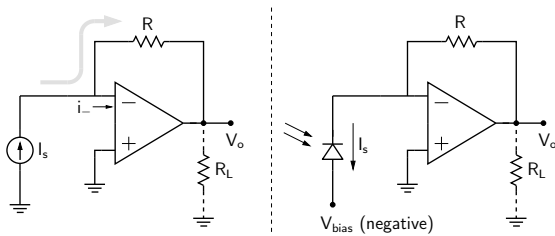


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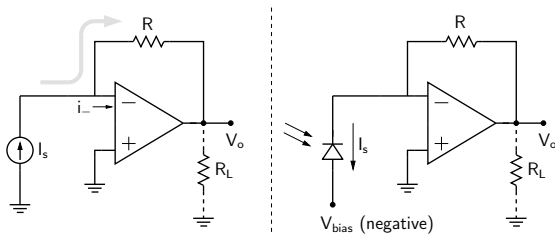


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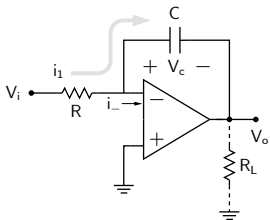
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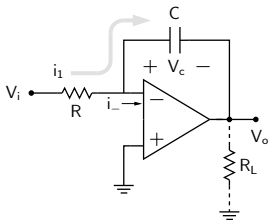
$V_o = I_s R$ . The diode is under a reverse bias, with  $V_n = 0 \text{ V}$  and  $V_p = V_{\text{bias}}$ .



## Op Amp circuits (linear region)

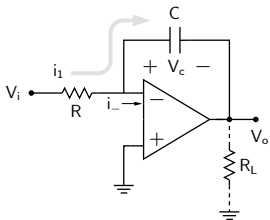


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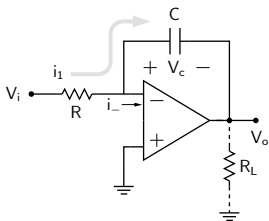


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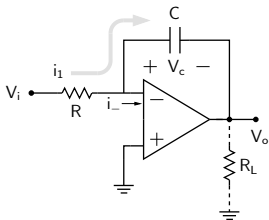
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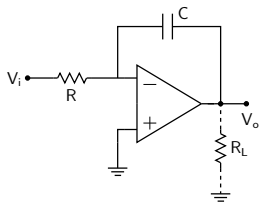
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The circuit works as an integrator.

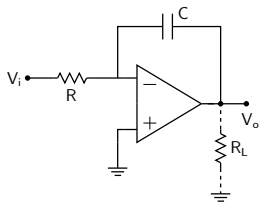
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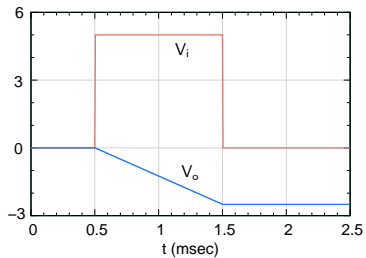
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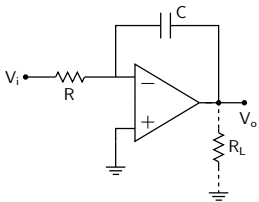


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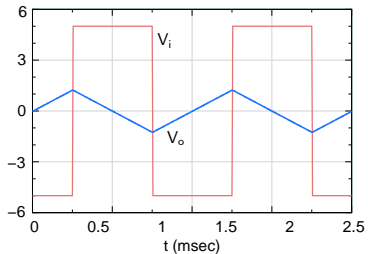
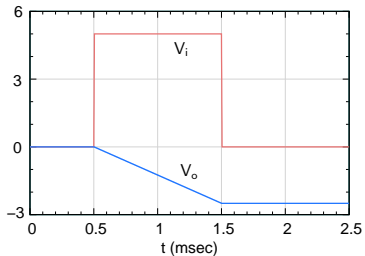


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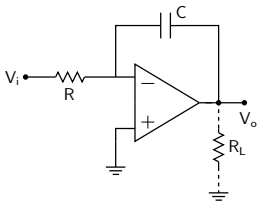
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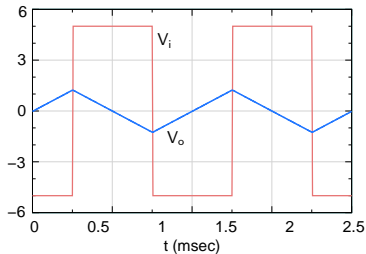
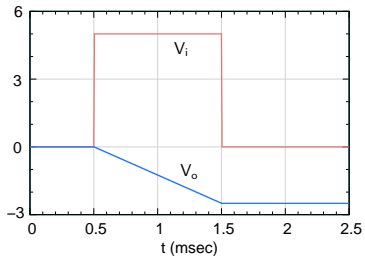


# Integrator



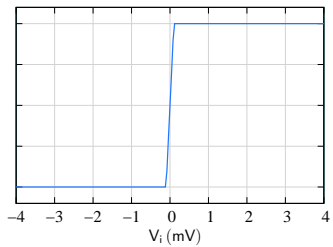
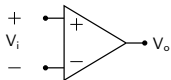
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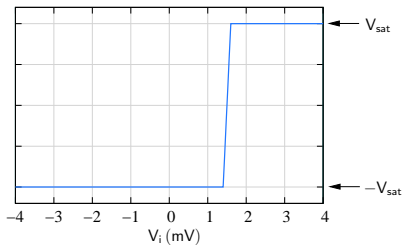
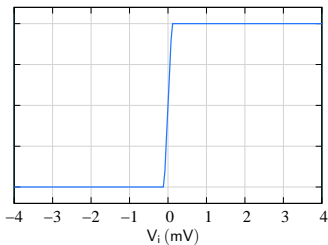
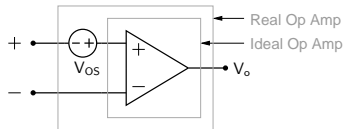
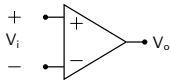


SEQUEL files: ee101\_integrator\_1.sqproj, ee101\_integrator\_2.sqproj

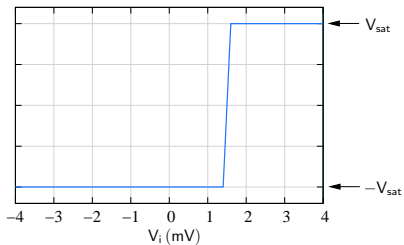
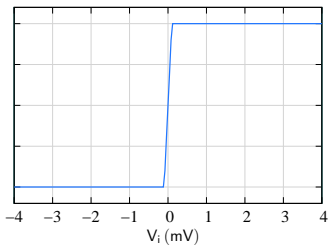
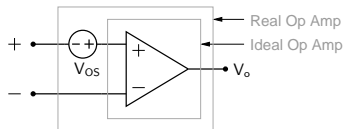
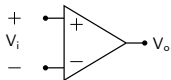
# Offset voltage



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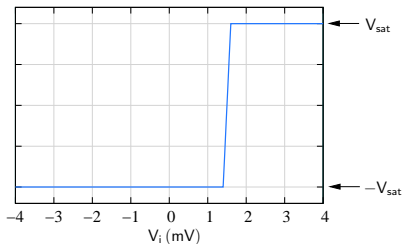
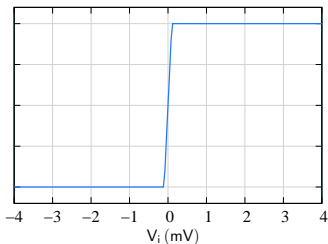
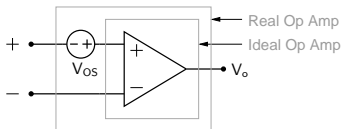
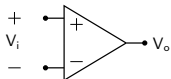


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For the real Op Amp,  $V_o = A_V((V_+ + V_{OS}) - V_-)$ .

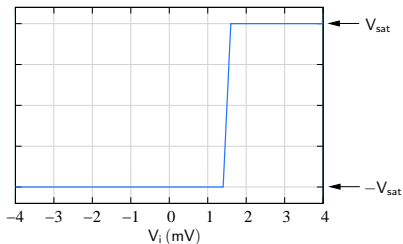
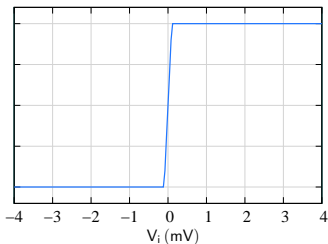
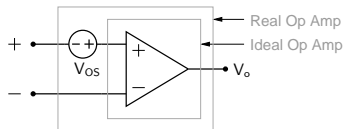
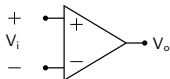
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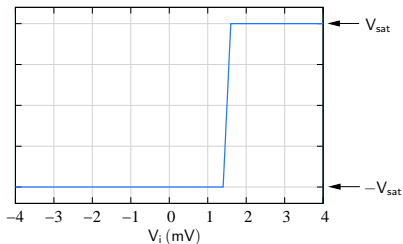
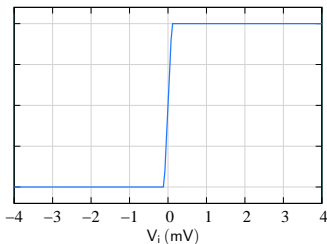
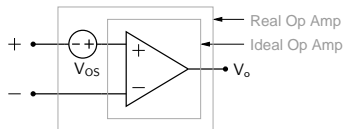
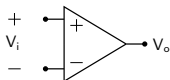


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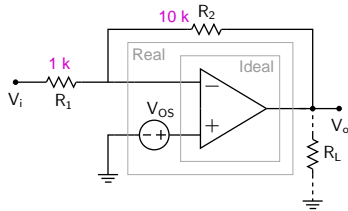
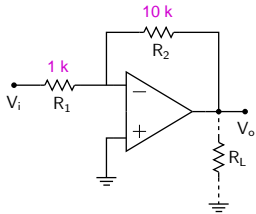
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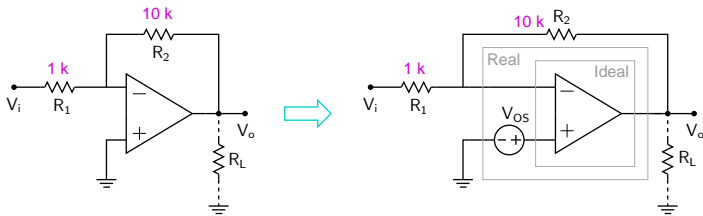
OP-77:  $-50$   $\mu$ V  $< V_{OS} < 50$   $\mu$ V.

# Effect of $V_{OS}$



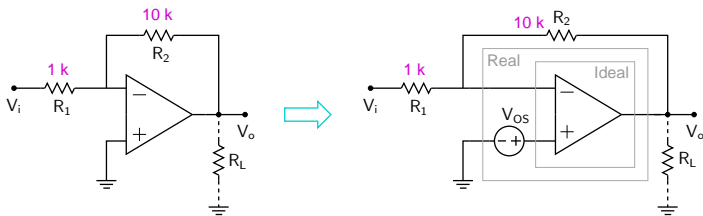


## Effect of $V_{OS}$



By superposition,  $V_o = -\frac{R_2}{R_1} V_i + V_{OS} \left(1 + \frac{R_2}{R_1}\right)$ .

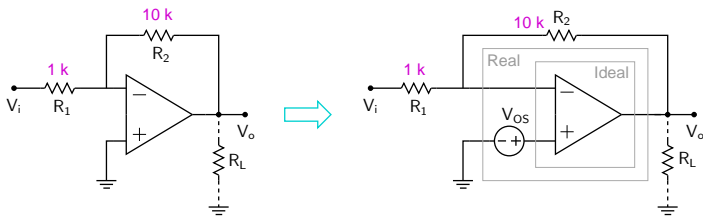
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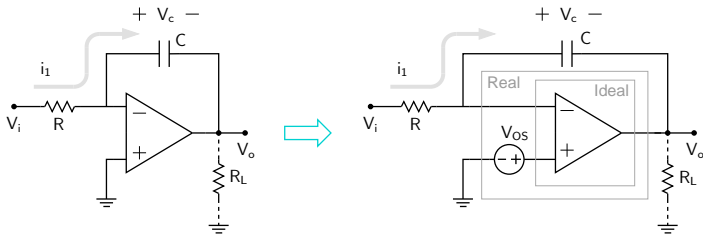


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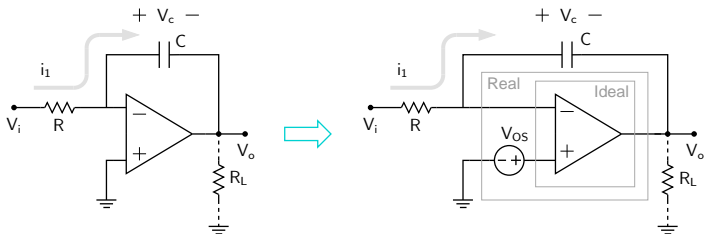
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i.e., a DC shift of  $22\text{ mV}$ .

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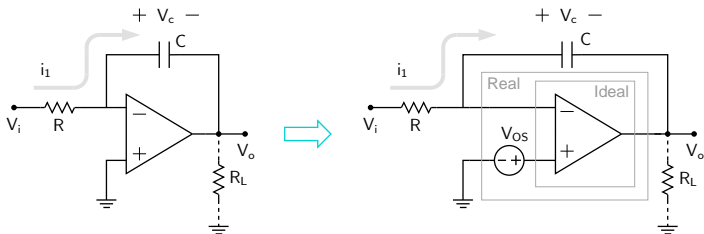


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$$V_- \approx V_+ = V_{OS} \rightarrow i_1 = \frac{1}{R}(V_i - V_{OS}) = C \frac{dV_c}{dt}.$$

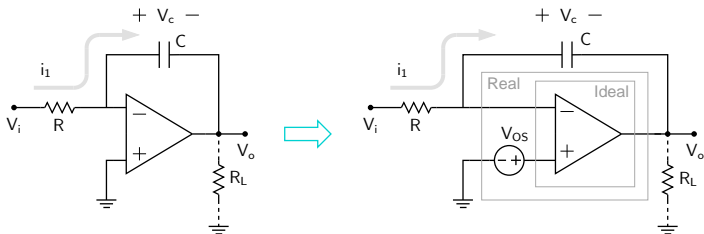
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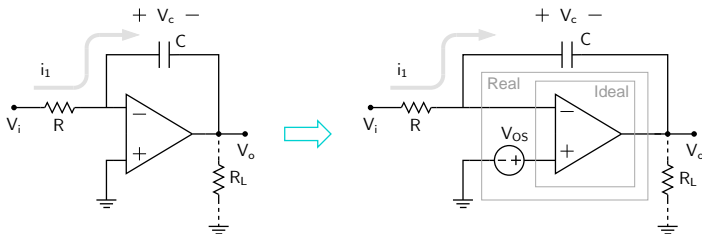
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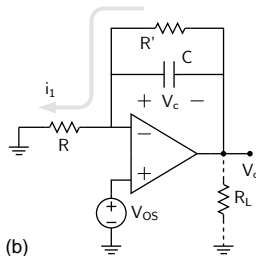
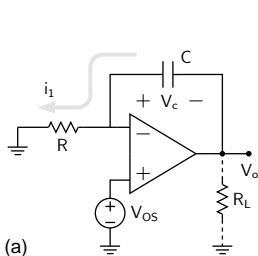
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→ need to address this issue!



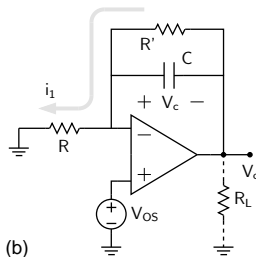
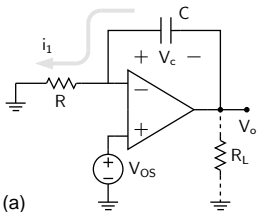
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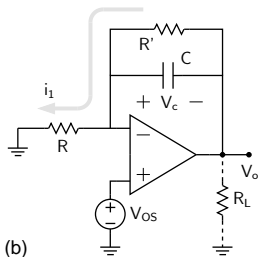
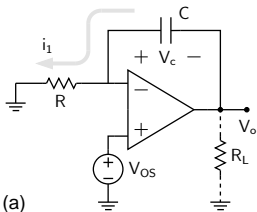
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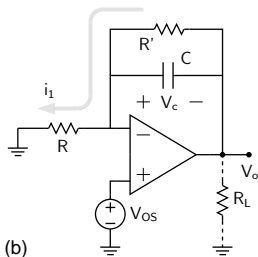
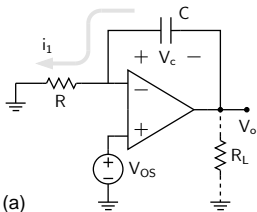
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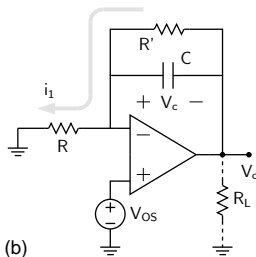
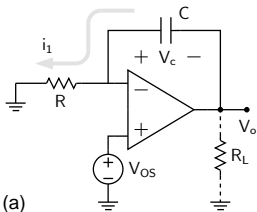
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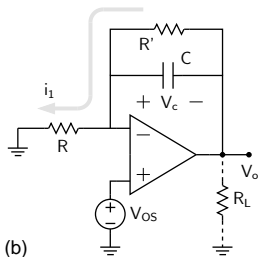
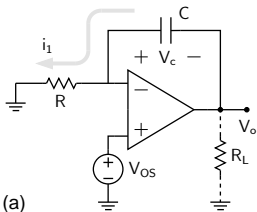
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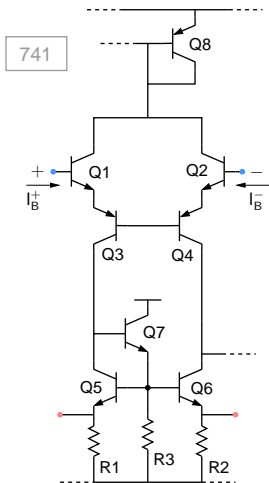
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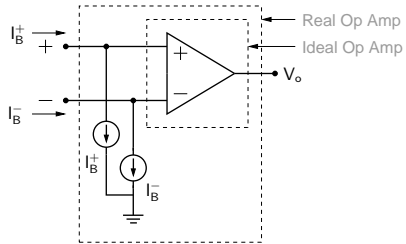
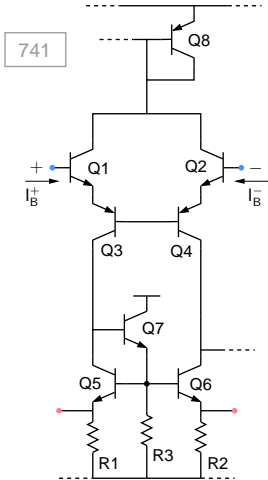
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$\rightarrow R' \gg 1/\omega C$  at the frequency of interest.

# Input bias currents

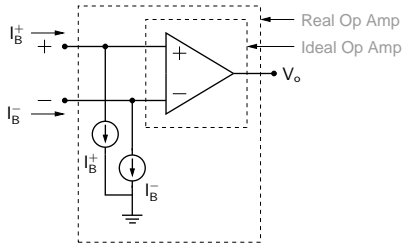
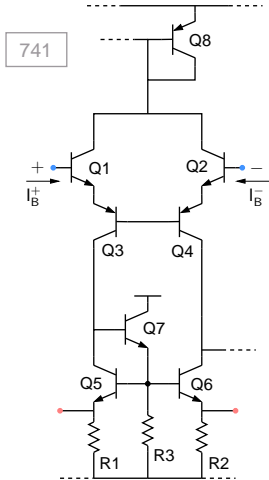


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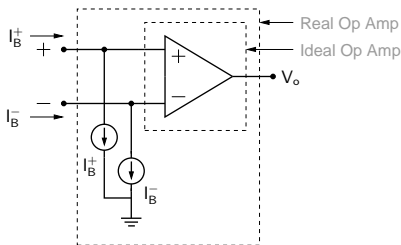
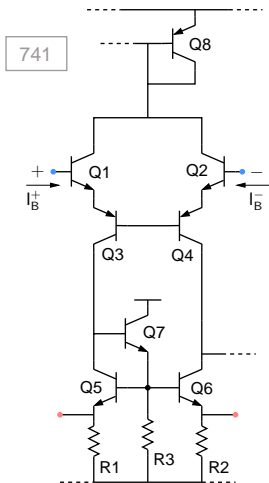


$I_B^+$  and  $I_B^-$  are generally not exactly equal.

$|I_B^+ - I_B^-|$  : "offset current" ( $I_{OS}$ )

$(I_B^+ + I_B^-)/2$  : "bias current" ( $I_B$ ).

# Input bias currents



Op Amp	$I_B$	$I_{os}$	$V_{os}$	
741	80 nA	20 nA	1 mV	BJT input
OP77	1.2 nA	0.3 nA	$10 \mu\text{V}$	BJT input
411	50 pA	25 pA	0.8 mV	FET input

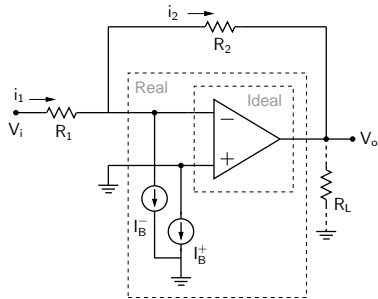
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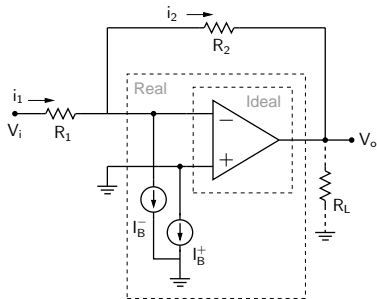
# Effect of bias currents

Inverting amplifier:



## Effect of bias currents

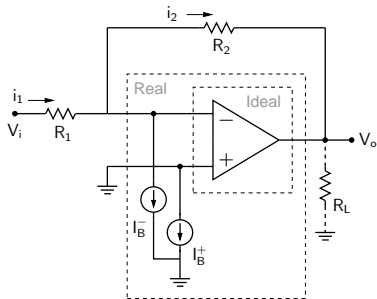
Inverting amplifier:



Assume that the Op Amp is ideal in other respects (i.e.,  $V_{OS} = 0$  V, etc.).

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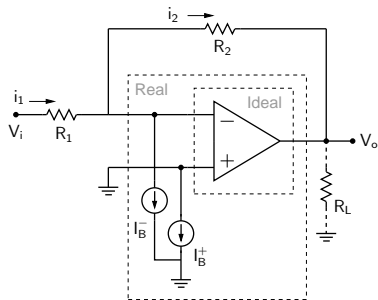


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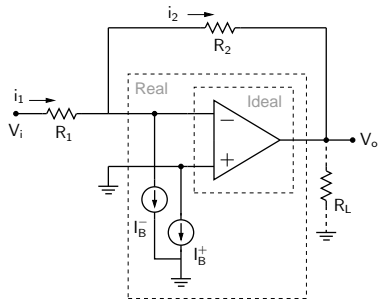
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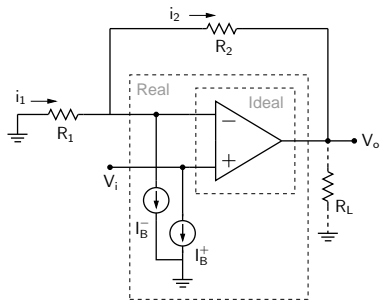
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i.e., the bias current causes a DC shift in  $V_o$ .

$$\text{For } I_B^- = 80\text{ nA}, R_2 = 10\text{ k}, \Delta V_o = 0.8\text{ mV}.$$

## Effect of bias currents

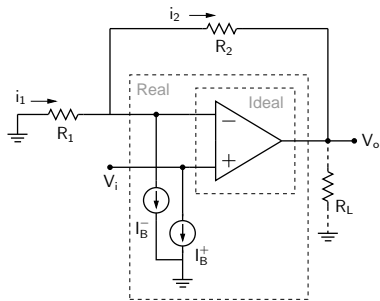
Non-inverting amplifier:





## Effect of bias currents

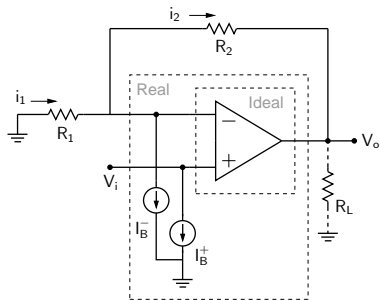
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## Effect of bias currents

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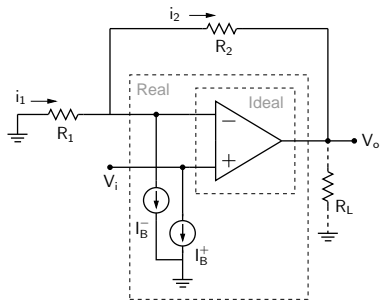


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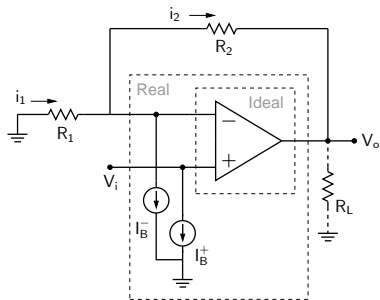
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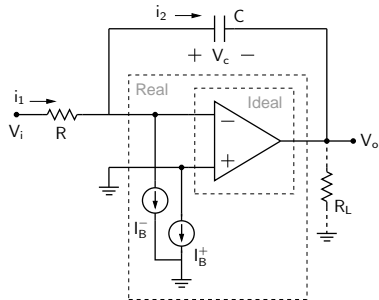
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$$V_o = V_i - i_2 R_2 = V_i - \left(-\frac{V_i}{R_1} - I_B^-\right) R_2 = V_i \left(1 + \frac{R_2}{R_1}\right) + I_B^- R_2.$$

→ Again, a DC shift  $\Delta V_o$ .

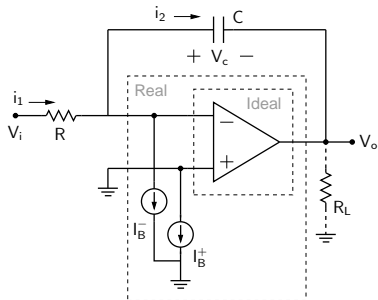
# Effect of bias currents

Integrator:



## Effect of bias currents

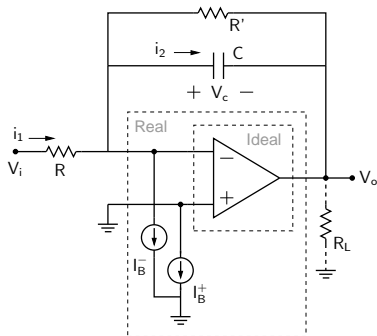
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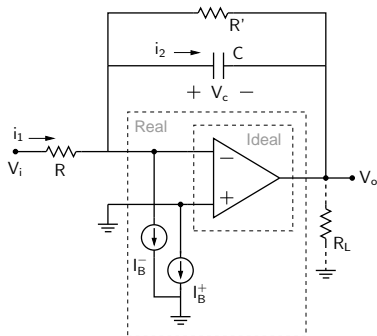


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As we have discussed earlier,  $R'$  should be small enough to have a negligible effect on  $V_o$ . However,  $R'$  must be large enough to ensure that the circuit still functions as an integrator.