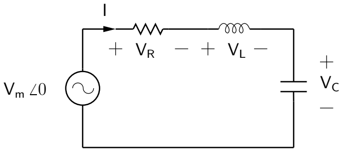
EE101: Resonance in *RLC* circuits

M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay

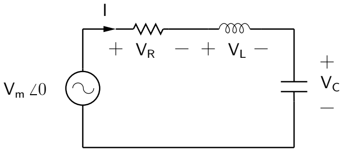
Resonance in series RLC circuits



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{, where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

Resonance in series RLC circuits

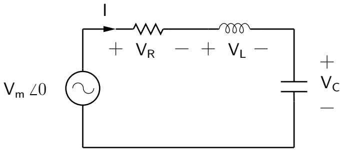


$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{, where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* As ω is varied, both I_m and θ change.

Resonance in series RLC circuits

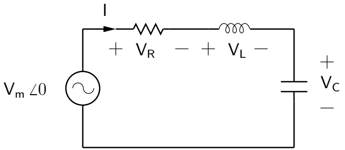


$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{ , where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \, , \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right] .$$

* As ω is varied, both I_m and θ change.
* When $\omega L = 1/\omega C$, I_m reaches its maximum value, $I_m^{max} = V_m/R$, and θ becomes 0, i.e., the current $\mathbf{I}$ is *in phase* with the applied voltage.

Resonance in series RLC circuits



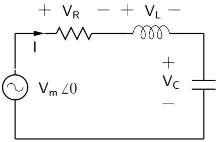
$$\mathbf{I} = \frac{V_m\angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m\angle\theta\,,\ \text{where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,,\quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* As ω is varied, both I_m and θ change.
* When $\omega L = 1/\omega C$, I_m reaches its maximum value, $I_m^{max} = V_m/R$, and θ becomes 0, i.e., the current $\mathbf{I}$ is *in phase* with the applied voltage.
* The above condition is called "resonance," and the corresponding frequency is called the "resonance frequency" (ω_0).

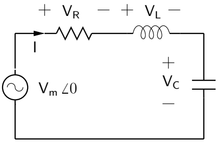
$$\boxed{\omega_0 = 1/\sqrt{LC}}$$

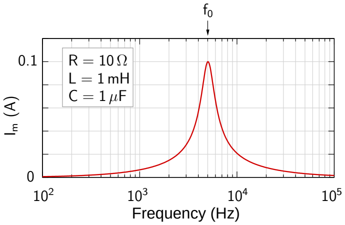
Resonance in series *RLC* circuits



$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]\,.$$

Resonance in series RLC circuits

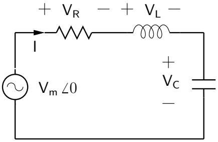


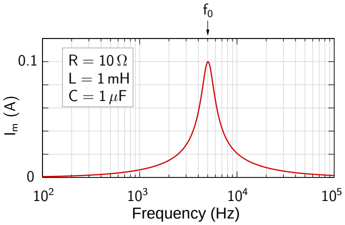


$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* As ω deviates from ω_0, I_m decreases.

Resonance in series RLC circuits

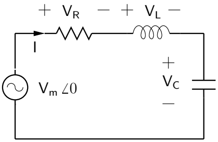


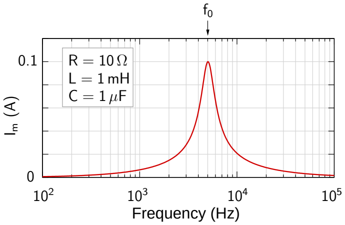


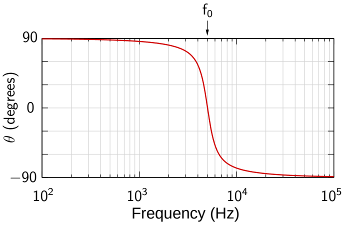
$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* As ω deviates from ω_0, I_m decreases.
* As $\omega \to 0$, the term $1/\omega C$ dominates, and $\theta \to \pi/2$.

Resonance in series RLC circuits



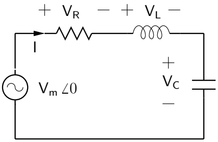


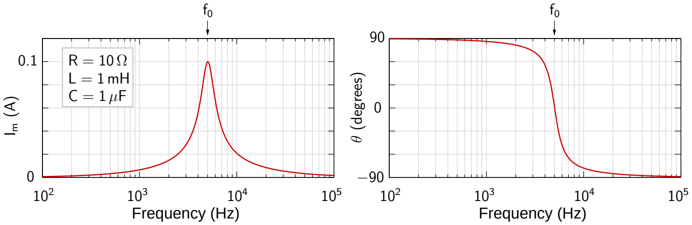


$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* As ω deviates from ω_0, I_m decreases.
* As $\omega \rightarrow 0$, the term $1/\omega C$ dominates, and $\theta \rightarrow \pi/2$.
* As $\omega \rightarrow \infty$, the term ωL dominates, and $\theta \rightarrow -\pi/2$.

Resonance in series *RLC* circuits



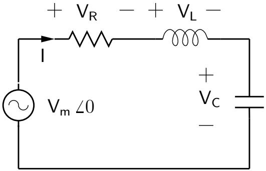


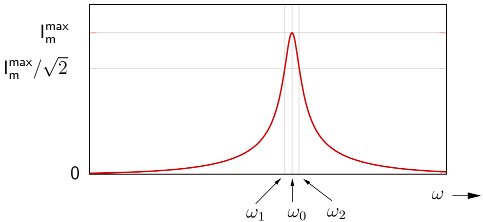
$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

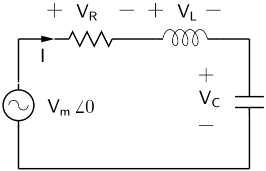
* As ω deviates from ω_0, I_m decreases.
* As $\omega \to 0$, the term $1/\omega C$ dominates, and $\theta \to \pi/2$.
* As $\omega \to \infty$, the term ωL dominates, and $\theta \to -\pi/2$.

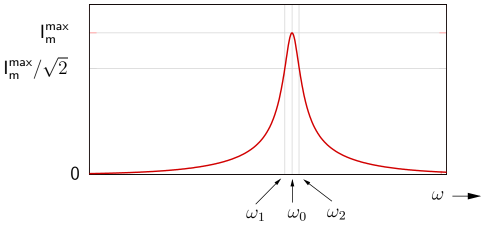
(SEQUEL file: `ee101_reso_rlc_1.sqproj`)

Resonance in series *RLC* circuits

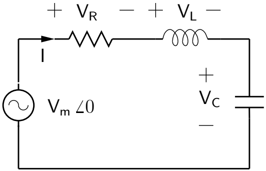


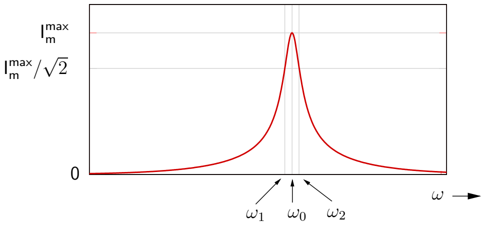




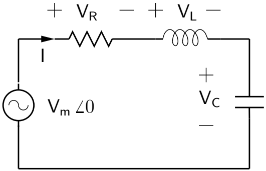


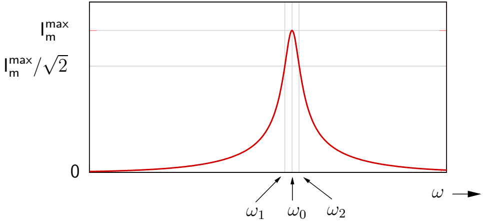
* The maximum power that can be absorbed by the resistor is $P_{\text{max}} = \dfrac{1}{2}\,(I_m^{\text{max}})^2\,R = \dfrac{1}{2}\,V_m^2/R.$





* The maximum power that can be absorbed by the resistor is $P_{\text{max}} = \frac{1}{2}\,(I_m^{\text{max}})^2\;R = \frac{1}{2}\,V_m^2/R$.
* Define ω_1 and ω_2 (see figure) as frequencies at which $I_m = I_m^{\text{max}}/\sqrt{2}$, i.e., the power absorbed by R is $P_{\text{max}}/2$.



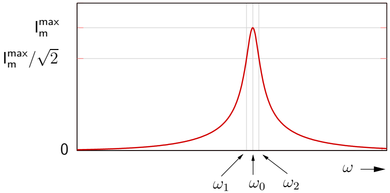


* The maximum power that can be absorbed by the resistor is $P_{max} = \frac{1}{2}\,(I_m^{max})^2\;R = \frac{1}{2}\,V_m^2/R.$
* Define ω_1 and ω_2 (see figure) as frequencies at which $I_m = I_m^{max}/\sqrt{2}$, i.e., the power absorbed by R is $P_{max}/2$.
* The *bandwidth* of a resonant circuit is defined as $B = \omega_2 - \omega_1$, and the *quality factor* as $Q = \omega_0/B$. Quality is a measure of the sharpness of the I_m versus frequency relationship.

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \,.$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R\,.$

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}\,.$

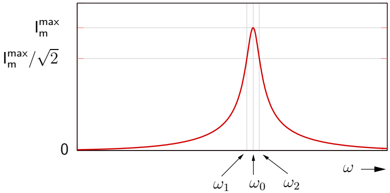


$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \,.$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}$.

$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2} \,.$$

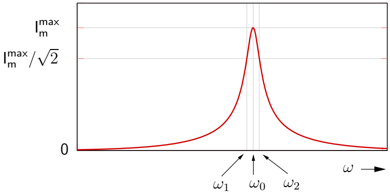


Resonance in series RLC circuits

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,.$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}$.



$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2}\,.$$

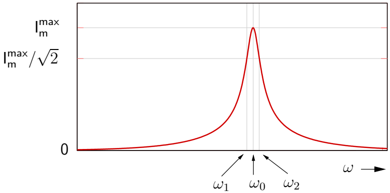
$$2\,R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C)\,.$$

Resonance in series RLC circuits

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \,.$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}$.



$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2} \,.$$

$$2\,R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C)\,.$$

Solving for ω (and discarding negative solutions), we get

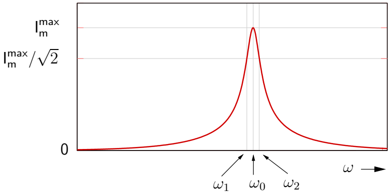
$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \,.$$

Resonance in series RLC circuits

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,.$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R\,.$

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}\,.$



$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2}\,.$$

$$2\,R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C)\,.$$

Solving for ω (and discarding negative solutions), we get

$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\,.$$

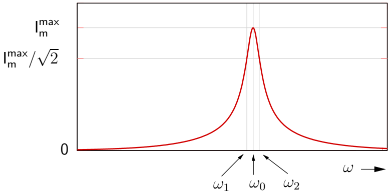
* Bandwidth $B = \omega_2 - \omega_1 = R/L\,.$

Resonance in series RLC circuits

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,.$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R\,.$

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}\,.$



$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2}\,.$$

$$2\,R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C)\,.$$

Solving for ω (and discarding negative solutions), we get

$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\,.$$

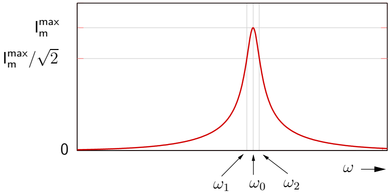
- Bandwidth $B = \omega_2 - \omega_1 = R/L\,.$
- Quality $Q = \omega_0/B = \omega_0 L/R\,.$

Resonance in series RLC circuits

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \, .$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}$.



$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2} \, .$$

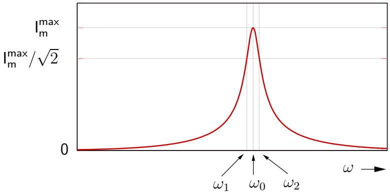
$2\,R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C)$.

Solving for ω (and discarding negative solutions), we get

$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \, .$$

- * Bandwidth $B = \omega_2 - \omega_1 = R/L$.
- * Quality $Q = \omega_0/B = \omega_0 L/R$.
- * Show that, at resonance (i.e., $\omega = \omega_0$), $|\mathbf{V}_L| = |\mathbf{V}_C| = Q\,V_m$.

Resonance in series RLC circuits



$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} .$$

For $\omega = \omega_0$, $I_m = I_m^{max} = V_m/R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{max}/\sqrt{2}$.

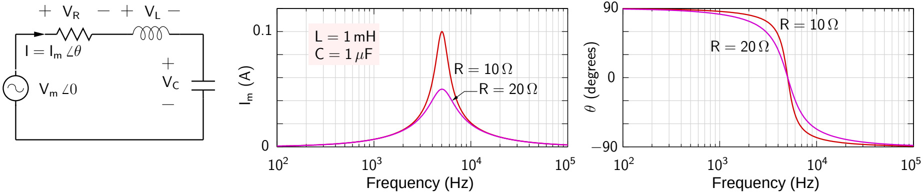
$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2} .$$

$2\,R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C)$.

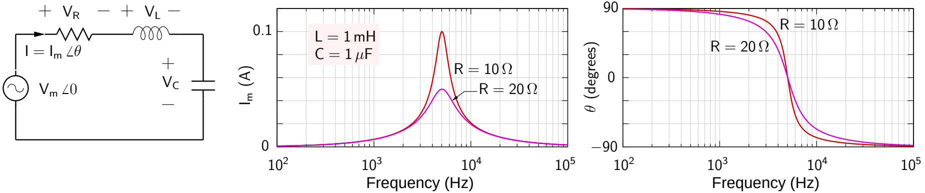
Solving for ω (and discarding negative solutions), we get

$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} .$$

* Bandwidth $B = \omega_2 - \omega_1 = R/L$.
* Quality $Q = \omega_0/B = \omega_0 L/R$.
* Show that, at resonance (i.e., $\omega = \omega_0$), $|\mathbf{V}_L| = |\mathbf{V}_C| = Q\,V_m$.
* Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$.

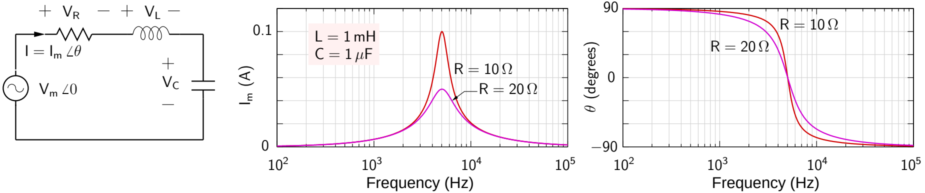


As R is increased,



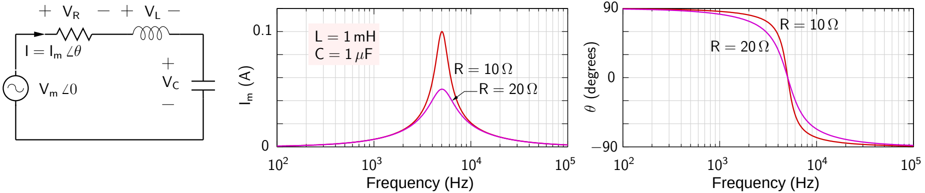
As R is increased,

* The quality factor $Q = \omega_0 L/R$ decreases, i.e., I_m versus ω curve becomes broader.



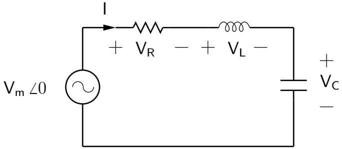
As R is increased,

* The quality factor $Q = \omega_0 L/R$ decreases, i.e., I_m versus ω curve becomes broader.
* The maximum current (at $\omega = \omega_0$) decreases (since $I_m^{max} = V_m/R$).



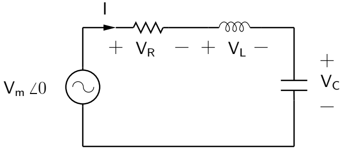
As R is increased,

* The quality factor $Q = \omega_0 L/R$ decreases, i.e., I_m versus ω curve becomes broader.
* The maximum current (at $\omega = \omega_0$) decreases (since $I_m^{max} = V_m/R$).
* The resonance frequency ($\omega_0 = 1/\sqrt{LC}$) is not affected.



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{ , where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \, , \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right] .$$

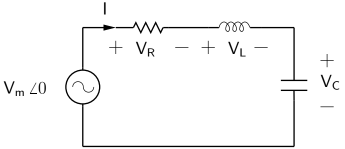


$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{ , where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.

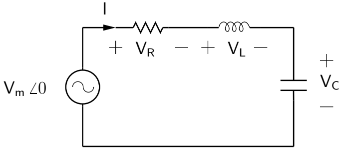
Resonance in series RLC circuits



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{ , where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

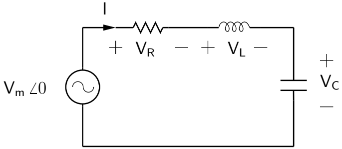
* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.
* For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.



$$\mathbf{I} = \frac{V_m\angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle\theta\,, \text{ where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.
* For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.
* For $\omega > \omega_0$, $\omega L > 1/\omega C$, the net impedance is inductive, and the current lags the applied voltage.

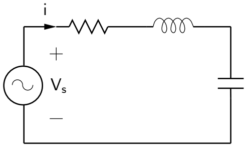


$$\mathbf{I} = \frac{V_m\angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m\angle\theta\,,\ \text{where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,,\quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.
* For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.
* For $\omega > \omega_0$, $\omega L > 1/\omega C$, the net impedance is inductive, and the current lags the applied voltage.
* Let us look at an example (next slide).

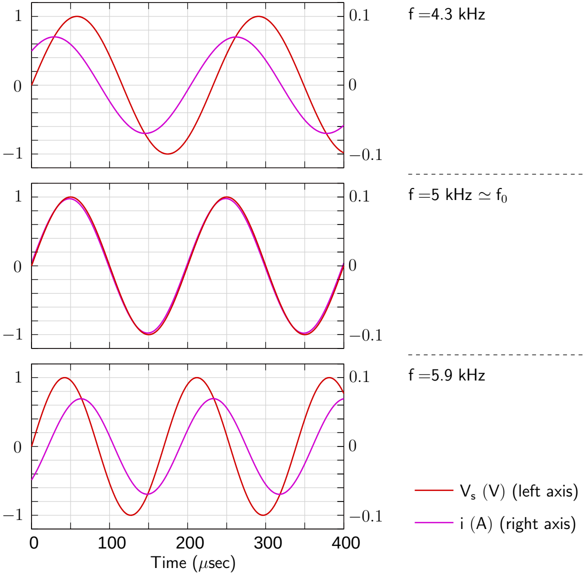
Resonance in series *RLC* circuits



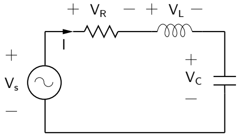
R = 10 Ω

L = 1 mH

C = 1 μF



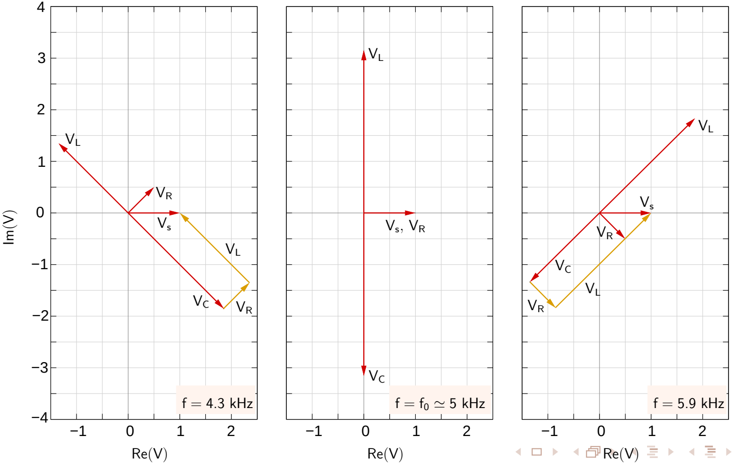
Resonance in series *RLC* circuits: phasor diagrams

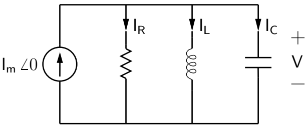


$R = 10\,\Omega$

$L = 1$ mH

$C = 1\ \mu$F



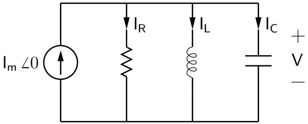


$I_m \angle 0 = \mathbf{Y}\,\mathbf{V}$, where $\mathbf{Y} = G + j\omega C + 1/j\omega L \;\; (G = 1/R)$.

$$\mathbf{V} = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta \text{, where}$$

$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right].$$

Resonance in parallel RLC circuits

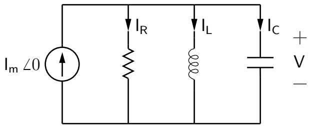


$I_m \angle 0 = \mathbf{Y}\,\mathbf{V}$, where $\mathbf{Y} = G + j\omega C + 1/j\omega L \;\; (G = 1/R)$.

$$\mathbf{V} = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta \text{ , where}$$

$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}} \, , \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right] .$$

* As ω is varied, both V_m and θ change.



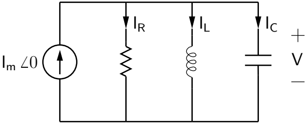
$I_m \angle 0 = \mathbf{Y}\,\mathbf{V}$, where $\mathbf{Y} = G + j\omega C + 1/j\omega L \;\; (G = 1/R)$.

$$\mathbf{V} = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta\,, \text{ where}$$

$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right].$$

* As ω is varied, both V_m and θ change.
* When $\omega C = 1/\omega L$, V_m reaches its maximum value, $V_m^{max} = I_m/G = I_m R$, and θ becomes 0, i.e., the voltage **V** is *in phase* with the source current.

Resonance in parallel RLC circuits



$$I_m \angle 0 = \mathbf{Y}\,\mathbf{V}, \text{ where } \mathbf{Y} = G + j\omega C + 1/j\omega L \ \ (G = 1/R)\,.$$

$$\mathbf{V} = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta\,, \text{ where}$$

$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right].$$

* As ω is varied, both V_m and θ change.
* When $\omega C = 1/\omega L$, V_m reaches its maximum value, $V_m^{max} = I_m/G = I_m R$, and θ becomes 0, i.e., the voltage **V** is *in phase* with the source current.
* The above condition is called "resonance," and the corresponding frequency is called the "resonance frequency" (ω_0).

$$\boxed{\omega_0 = 1/\sqrt{LC}}$$

Resonance in parallel RLC circuits

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right]$.

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right]$.

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right].$

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right].$

* The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
  $L \leftrightarrow C$.

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right].$

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right].$

* The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
  $L \leftrightarrow C$.
* Thus, our results for series RLC circuits can be easily extended to parallel RLC circuits.

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right].$

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right].$

* The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
  $L \leftrightarrow C$.
* Thus, our results for series RLC circuits can be easily extended to parallel RLC circuits.
* Show that $\omega_{1,2} = \mp\dfrac{1}{2RC} + \sqrt{\left(\dfrac{1}{2RC}\right)^2 + \dfrac{1}{LC}}$
  $\Rightarrow$ Bandwidth $B = 1/RC$.

Resonance in parallel RLC circuits

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right]$.

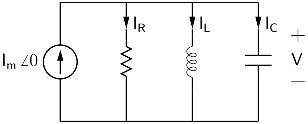
Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right]$.

* The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
  $L \leftrightarrow C$.
* Thus, our results for series RLC circuits can be easily extended to parallel RLC circuits.
* Show that $\omega_{1,2} = \mp\dfrac{1}{2RC} + \sqrt{\left(\dfrac{1}{2RC}\right)^2 + \dfrac{1}{LC}}$
  $\Rightarrow$ Bandwidth $B = 1/RC$.
* Show that, at resonance (i.e., $\omega = \omega_0$), $|\mathbf{I}_L| = |\mathbf{I}_C| = Q\, I_m$.

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right]$.

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right]$.

* The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
  $L \leftrightarrow C$.
* Thus, our results for series RLC circuits can be easily extended to parallel RLC circuits.
* Show that $\omega_{1,2} = \mp\dfrac{1}{2RC} + \sqrt{\left(\dfrac{1}{2RC}\right)^2 + \dfrac{1}{LC}}$
  $\Rightarrow$ Bandwidth $B = 1/RC$.
* Show that, at resonance (i.e., $\omega = \omega_0$), $|\mathbf{I}_L| = |\mathbf{I}_C| = Q\, I_m$.
* Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$.



$I_m = 50$ mA
$R = 2\,\text{k}\Omega$
$L = 40$ mH
$C = 0.25\ \mu$F

* Calculate ω_0, f_0, B, Q.
* Calculate $\mathbf{I}_R$, $\mathbf{I}_L$, $\mathbf{I}_C$ at $\omega = \omega_0, \omega_1, \omega_2$.
* Verify graphically that $\mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C = \mathbf{I}_s$ in each case.
* Plot the power absorbed by R as a function of frequency for $f_0/10 < f < 10\,f_0$.