EE101: Resonance in RLC circuits

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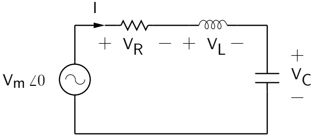
Resonance in series *RLC* circuits



$$\mathbf{I} = \frac{V_m\,\angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m\,\angle\theta\,,\ \text{where}$$

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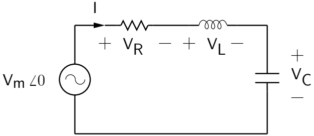

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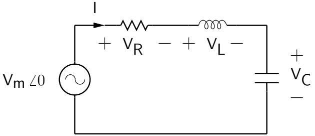

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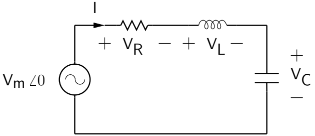

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- ∗ When $\omega L = 1/\omega C$, I_m reaches its maximum value, $I_m^{max} = V_m/R$, and θ becomes 0, i.e., the current **I** is *in phase* with the applied voltage.
- ∗ The above condition is called "resonance," and the corresponding frequency is called the "resonance frequency" (ω_0).

$$\boxed{\omega_0 = 1/\sqrt{LC}}$$

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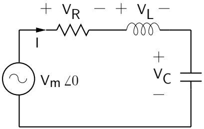

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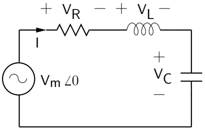

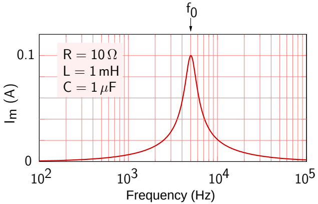

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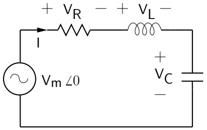

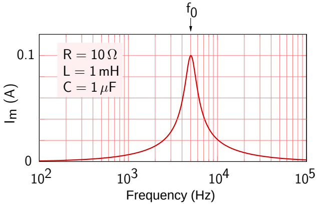

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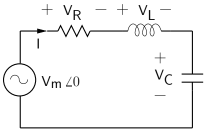

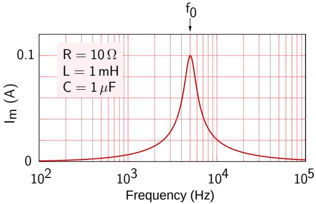

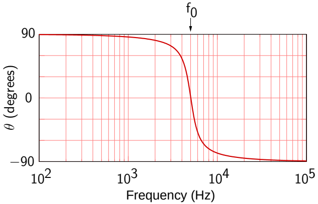

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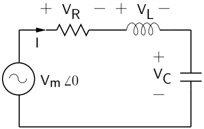

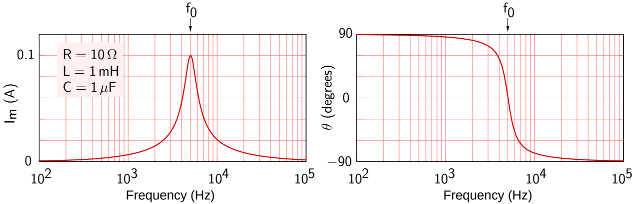

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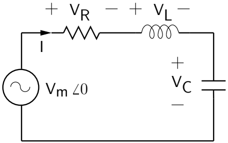

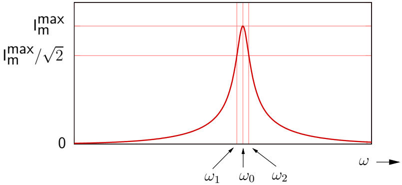

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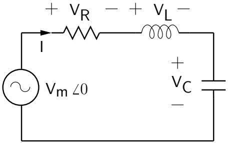

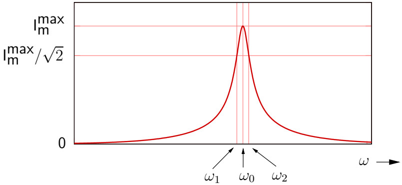

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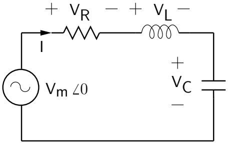

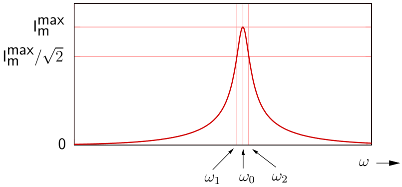

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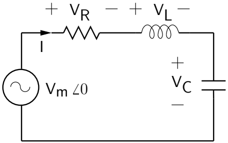

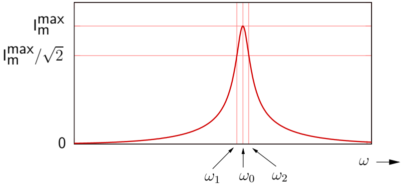

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- * Define ω_1 and ω_2 (see figure) as frequencies at which $I_m = I_m^{max}/\sqrt{2}$, i.e., the power absorbed by R is $P^{max}/2$.
- * The *bandwidth* of a resonant circuit is defined as $B = \omega_2 - \omega_1$, and the *quality factor* as $Q = \omega_0/B$. Quality is a measure of the sharpness of the I_m versus frequency relationship.

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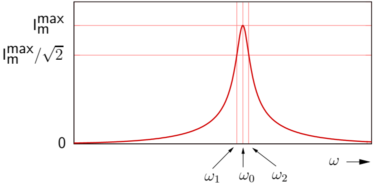

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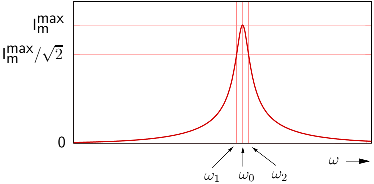

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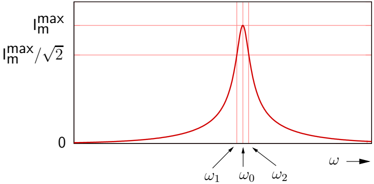

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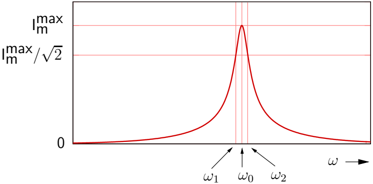

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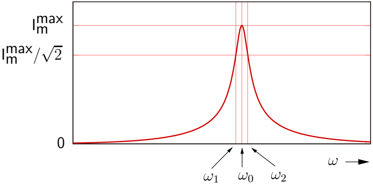

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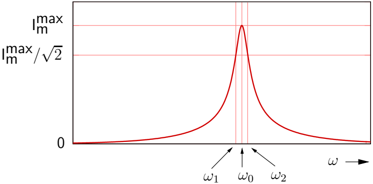

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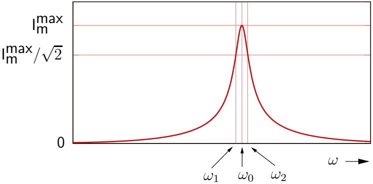

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- $*$ Bandwidth $B = \omega_2 - \omega_1 = R/L$.
- $*$ Quality $Q = \omega_0/B = \omega_0 L/R$.
- $*$ Show that, at resonance (i.e., $\omega = \omega_0$), $|\mathbf{V}_L| = |\mathbf{V}_C| = Q\,V_m$.

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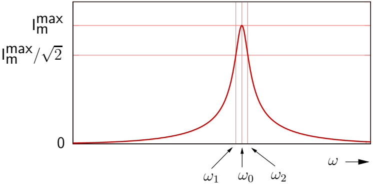

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- Show that $\omega_0 = \sqrt{\omega_1\omega_2}\,.$

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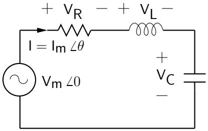

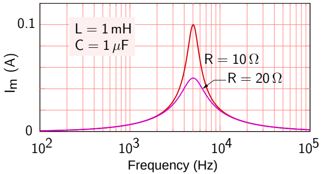

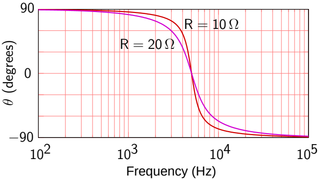

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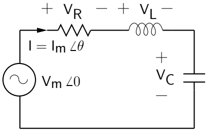

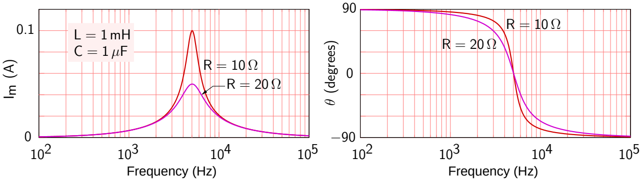

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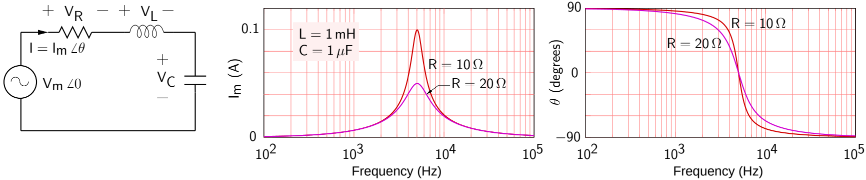

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- The quality factor $Q = \omega_0 L/R$ decreases, i.e., I_m versus ω curve becomes broader.
- The maximum current (at $\omega = \omega_0$) decreases (since $I_m^{max} = V_m/R$).

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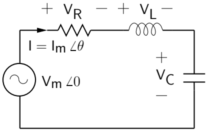
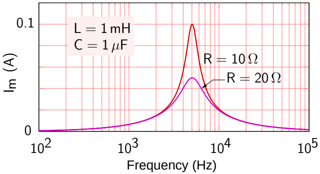
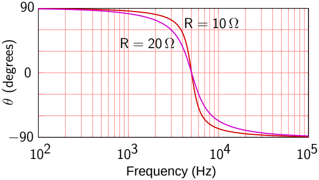


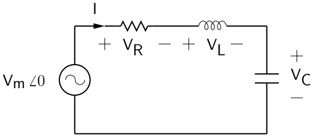




As R is increased,

- The quality factor $Q = \omega_0 L/R$ decreases, i.e., I_m versus ω curve becomes broader.
- The maximum current (at $\omega = \omega_0$) decreases (since $I_m^{max} = V_m/R$).
- The resonance frequency ($\omega_0 = 1/\sqrt{LC}$) is not affected.

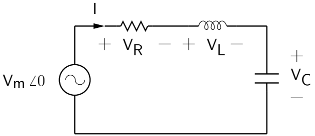
Resonance in series RLC circuits



$$\mathbf{I} = \frac{V_m\angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m\angle\theta\,, \text{ where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

Resonance in series RLC circuits

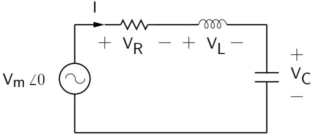


$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{ , where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \, , \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right] .$$

- For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.

Resonance in series *RLC* circuits



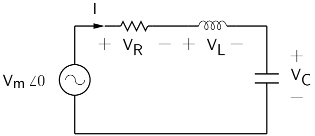
$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{ , where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \, , \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right] .$$

* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.

* For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.

Resonance in series RLC circuits

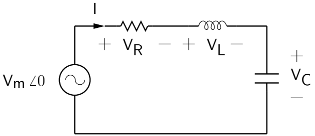


$$\mathbf{I} = \frac{V_m\,\angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m\,\angle\theta\,,\ \text{where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].$$

- * For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.
- * For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.
- * For $\omega > \omega_0$, $\omega L > 1/\omega C$, the net impedance is inductive, and the current lags the applied voltage.

Resonance in series *RLC* circuits

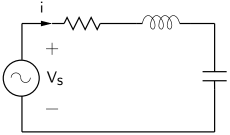


$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta \text{ , where}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} , \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right] .$$

- * For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.
- * For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.
- * For $\omega > \omega_0$, $\omega L > 1/\omega C$, the net impedance is inductive, and the current lags the applied voltage.
- * Let us look at an example (next slide).

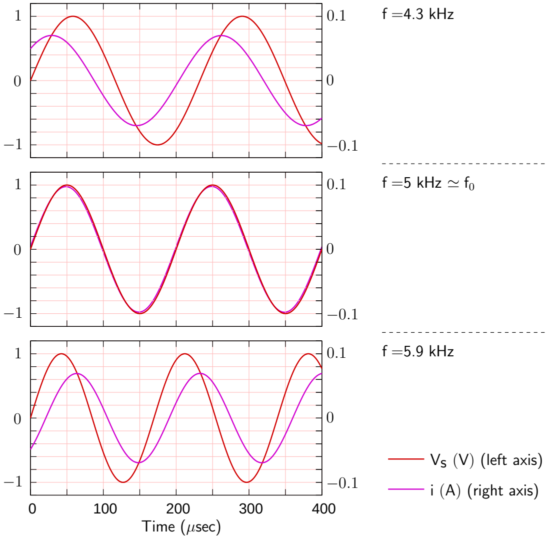
Resonance in series *RLC* circuits



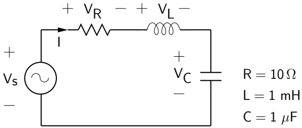
$R = 10\,\Omega$

$L = 1$ mH

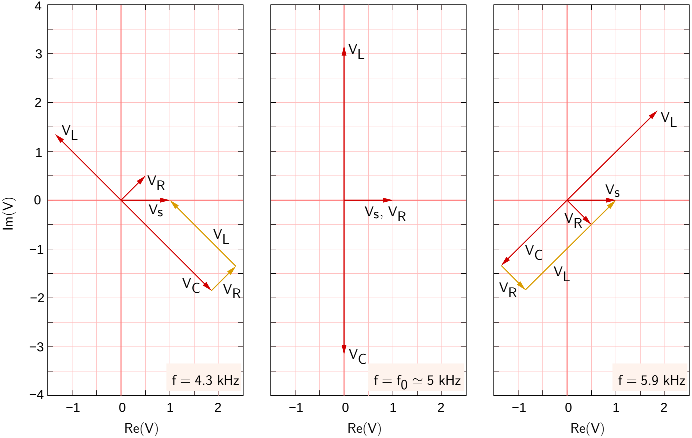
$C = 1\ \mu$F



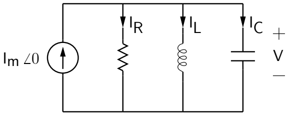
Resonance in series *RLC* circuits: phasor diagrams



R = 10 Ω
L = 1 mH
C = 1 μF



Resonance in parallel *RLC* circuits

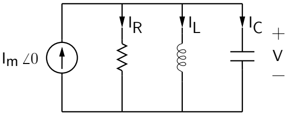


$$I_m\angle 0 = \mathbf{Y}\,\mathbf{V}, \text{ where } \mathbf{Y} = G + j\omega C + 1/j\omega L \;\; (G = 1/R)\,.$$

$$\mathbf{V} = \frac{I_m\angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m\angle\theta\,, \text{ where}$$

$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right]\,.$$

Resonance in parallel *RLC* circuits



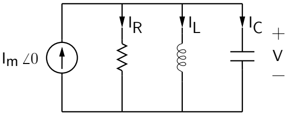
$I_m \angle 0 = \mathbf{Y}\,\mathbf{V}$, where $\mathbf{Y} = G + j\omega C + 1/j\omega L \;\; (G = 1/R)$.

$$\mathbf{V} = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta \text{, where}$$

$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right].$$

- As ω is varied, both V_m and θ change.

Resonance in parallel *RLC* circuits



$I_m \angle 0 = \mathbf{Y}\,\mathbf{V}$, where $\mathbf{Y} = G + j\omega C + 1/j\omega L \;\; (G = 1/R)$.

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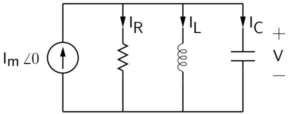
$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right].$$

- As ω is varied, both V_m and θ change.
- When $\omega C = 1/\omega L$, V_m reaches its maximum value, $V_m^{max} = I_m/G = I_m R$, and θ becomes 0, i.e., the voltage **V** is *in phase* with the source current.

Resonance in parallel RLC circuits



$I_m \angle 0 = \mathbf{Y}\,\mathbf{V}$, where $\mathbf{Y} = G + j\omega C + 1/j\omega L \;\; (G = 1/R)$.

$$\mathbf{V} = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta \text{, where}$$

$$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right].$$

- As ω is varied, both V_m and θ change.
- When $\omega C = 1/\omega L$, V_m reaches its maximum value, $V_m^{max} = I_m/G = I_m R$, and θ becomes 0, i.e., the voltage $\mathbf{V}$ is *in phase* with the source current.
- The above condition is called "resonance," and the corresponding frequency is called the "resonance frequency" (ω_0).

$$\boxed{\omega_0 = 1/\sqrt{LC}}$$

Resonance in parallel *RLC* circuits

Series *RLC* circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right].$

Parallel *RLC* circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right].$

Resonance in parallel RLC circuits

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,,\quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right]\,.$

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,,\quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right]\,.$

* The two situations are identical if we make the following substitutions:
$\mathbf{I} \leftrightarrow \mathbf{V}$,
$R \leftrightarrow 1/R$,
$L \leftrightarrow C$.

Resonance in parallel *RLC* circuits

Series *RLC* circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right]\,.$

Parallel *RLC* circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right]\,.$

- The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
  $L \leftrightarrow C$.
- Thus, our results for series *RLC* circuits can be easily extended to parallel *RLC* circuits.

Resonance in parallel RLC circuits

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right]$.

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right]$.

- The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
  $L \leftrightarrow C$.
- Thus, our results for series RLC circuits can be easily extended to parallel RLC circuits.
- Show that $\omega_{1,2} = \mp\dfrac{1}{2RC} + \sqrt{\left(\dfrac{1}{2RC}\right)^2 + \dfrac{1}{LC}}$
  $\Rightarrow$ Bandwidth $B = 1/RC$.

Resonance in parallel *RLC* circuits

Series *RLC* circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right].$

Parallel *RLC* circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right].$

- The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
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- Thus, our results for series *RLC* circuits can be easily extended to parallel *RLC* circuits.
- Show that $\omega_{1,2} = \mp\dfrac{1}{2RC} + \sqrt{\left(\dfrac{1}{2RC}\right)^2 + \dfrac{1}{LC}}$
  $\Rightarrow$ Bandwidth $B = 1/RC$.
- Show that, at resonance (i.e., $\omega = \omega_0$), $|\mathbf{I}_L| = |\mathbf{I}_C| = Q\, I_m$.

Resonance in parallel RLC circuits

Series RLC circuit: $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega L - 1/\omega C}{R}\right]\,.$

Parallel RLC circuit: $V_m = \dfrac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}\,, \quad \theta = -\tan^{-1}\left[\dfrac{\omega C - 1/\omega L}{G}\right]\,.$

- The two situations are identical if we make the following substitutions:
  $\mathbf{I} \leftrightarrow \mathbf{V}$,
  $R \leftrightarrow 1/R$,
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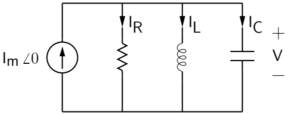
- Thus, our results for series RLC circuits can be easily extended to parallel RLC circuits.
- Show that $\omega_{1,2} = \mp\dfrac{1}{2RC} + \sqrt{\left(\dfrac{1}{2RC}\right)^2 + \dfrac{1}{LC}}$
  $\Rightarrow$ Bandwidth $B = 1/RC$.
- Show that, at resonance (i.e., $\omega = \omega_0$), $|\mathbf{I}_L| = |\mathbf{I}_C| = Q\, I_m$.
- Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$.

Resonance in parallel *RLC* circuits: home work



$I_m = 50$ mA
$R = 2\,k\Omega$
$L = 40$ mH
$C = 0.25\ \mu F$

- Calculate ω_0, f_0, B, Q.
- Calculate $\mathbf{I}_R$, $\mathbf{I}_L$, $\mathbf{I}_C$ at $\omega = \omega_0$, ω_1, ω_2.
- Verify graphically that $\mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C = \mathbf{I}_s$ in each case.
- Plot the power absorbed by R as a function of frequency for $f_0/10 < f < 10\,f_0$.