Bode Plots

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  Interesting facts:

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  - Bell considered the telephone an intrusion and refused to put one in his office.
* The unit Bel turned out to be too large in practice $\rightarrow$ deciBel (i.e., one tenth of a Bel).

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* The gain of a voltage-to-voltage amplifier is often expressed in dB. In that case, the ratio V_o^2/V_i^2 is considered (since $P \propto V^2$ or $P \propto I^2$ for a resistor).

  A_V in dB $= 10\,\log_{10}|V_o/V_i|^2 = 20\,\log_{10}|V_o/V_i|$,

* "dBm" is a related unit used to describe voltages with a reference of 1 mV.

  For example, 2.2 V: $20\,\log_{10}\left(\dfrac{2.2\,\text{V}}{1\,\text{mV}}\right) = 66.85\,\text{dBm}$.

Example

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Let $\widehat{V}_i$ and $\widehat{V}_o$ be the input and output amplitudes.

If $\widehat{V}_i = 2.5\,\text{mV}$ and $A_V = 36.3\,\text{dB}$, compute $\widehat{V}_o$ in dBm and mV.

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Method 1:

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$$\widehat{V}_o\ \text{in dBm} = 20\ \log_{10}\left(\frac{162.5\,\text{mV}}{1\,\text{mV}}\right) = 44.2\,\text{dBm}.$$

dB in audio measurements

* When sound intensity is specified in dB, the reference pressure is $P_{\text{ref}} = 20\,\mu Pa$ (our hearing threshold). If the pressure corresponding to the sound being measured is P, we say that it is $20\,\log\left(P/P_{\text{ref}}\right)$ dB.

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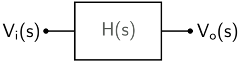
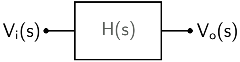
Industrial area	75 dB
Commercial area	65 dB
Residential area	55 dB
Silence zone	50 dB

Bode plots





* The transfer function of a circuit such as an amplifier or a filter is given by,
  $H(s) = V_o(s)/V_i(s), \quad s = j\omega.$
  e.g., $H(s) = \dfrac{K}{1+s\tau} = \dfrac{K}{1+j\omega\tau}$

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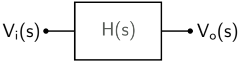

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* $H(j\omega)$ is a complex number, and a complete description of $H(j\omega)$ involves
  (a) a plot of $|H(j\omega)|$ versus ω (Bode magnitude plot),
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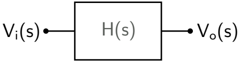

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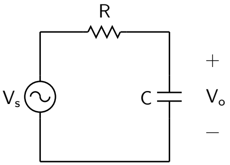
* Bode gave simple rules which allow construction of the above plots in an approximate (asymptotic) manner.

A simple transfer function



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

$$\to H(s) = \frac{1}{1{+}s\,RC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

$$\omega_0 = \frac{1}{RC}\,.$$

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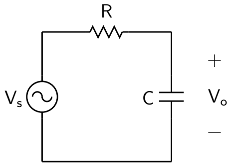

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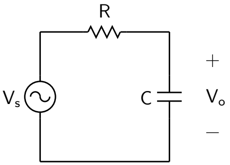

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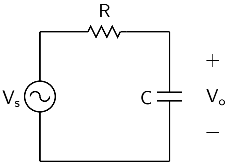

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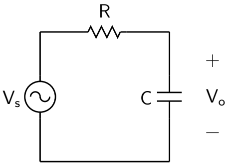
  For $\omega \gg \omega_0$, $\dfrac{\omega}{\omega_0} \gg 1$, $H(j\omega) \approx \dfrac{1}{j\,\dfrac{\omega}{\omega_0}}$, and $|H(j\omega)| \rightarrow \dfrac{1}{\omega}$.



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* The circuit behaves like a low-pass filter.
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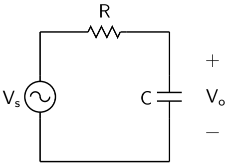
  For $\omega \gg \omega_0$, $\dfrac{\omega}{\omega_0} \gg 1$, $H(j\omega) \approx \dfrac{1}{j\,\dfrac{\omega}{\omega_0}}$, and $|H(j\omega)| \rightarrow \dfrac{1}{\omega}$.
* The magnitude and phase of $H(j\omega)$ are given by,
  $$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right).$$

A simple transfer function



$$V_o = \frac{(1/sC)}{R + (1/sC)}\, V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1+sRC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

$$\omega_0 = \frac{1}{RC}\,.$$

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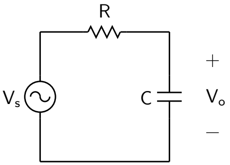
- ∗ We are generally interested in a large variation in ω (several orders), and its effect on $|H|$ and $\angle H$.

A simple transfer function



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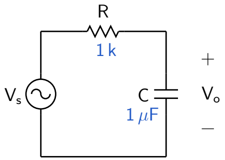
  $$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right).$$

* We are generally interested in a large variation in ω (several orders), and its effect on $|H|$ and $\angle H$.

* The magnitude ($|H|$) varies by orders of magnitude as well.

  The phase ($\angle H$) varies from 0 (for $\omega \ll \omega_0$) to $-\pi/2$ (for $\omega \gg \omega_0$).

A simple transfer function: magnitude



$$V_o = \frac{(1/sC)}{R + (1/sC)} V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1+sRC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

$$\omega_0 = \frac{1}{RC} = 10^3\ \text{rad/s}\,.$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

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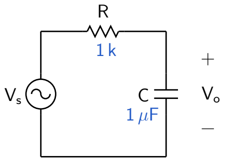

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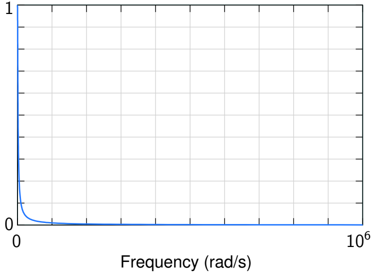
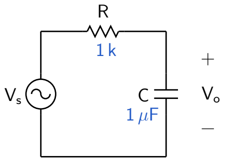


A simple transfer function: magnitude



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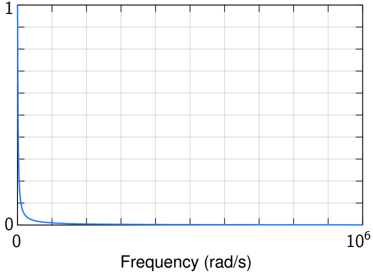

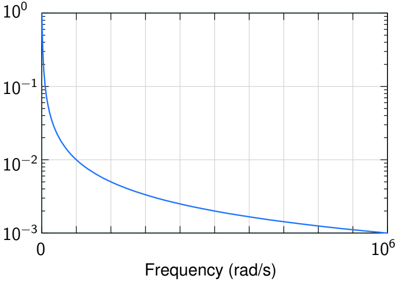

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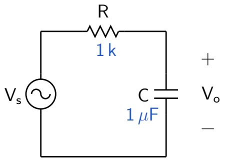

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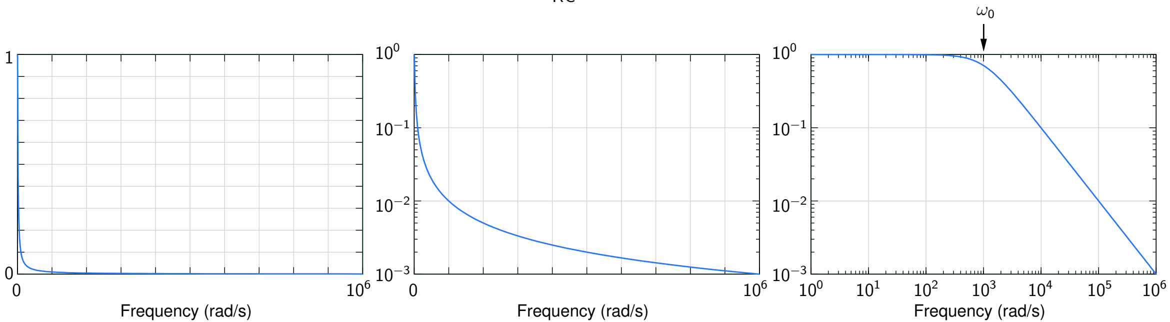
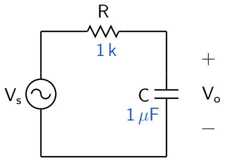


A simple transfer function: magnitude



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

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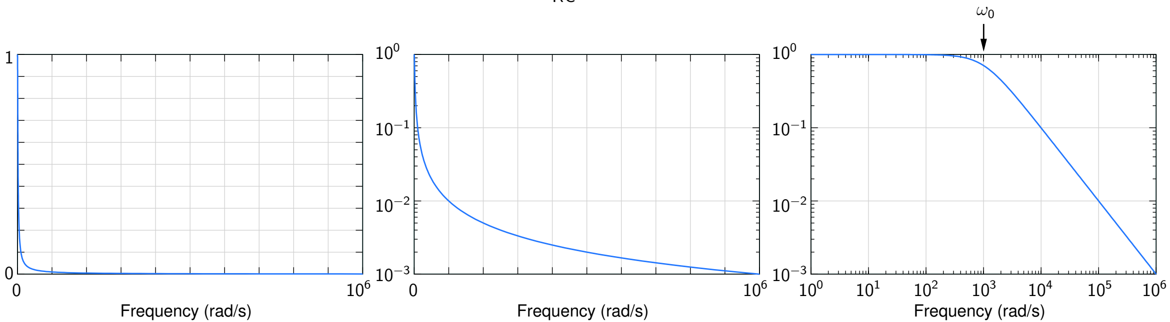
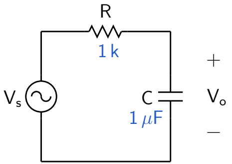


Since ω and $|H(j\,\omega)|$ vary by several orders of magnitude, a linear ω- or $|H|$-axis is not appropriate $\rightarrow$ $\log|H|$ is plotted against $\log\omega$.

A simple transfer function: magnitude



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\, V_s\,,$$

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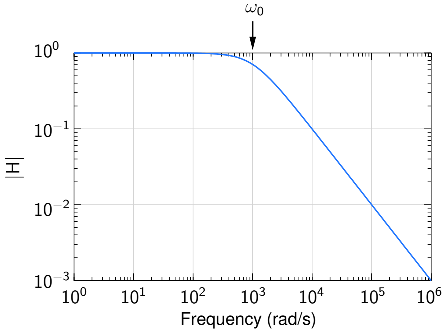
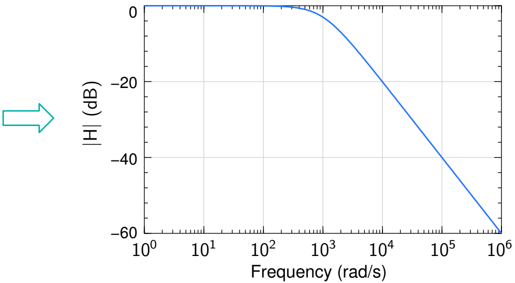
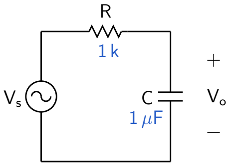




A simple transfer function: magnitude



$$V_o = \frac{(1/sC)}{R + (1/sC)}\,V_s\,,$$

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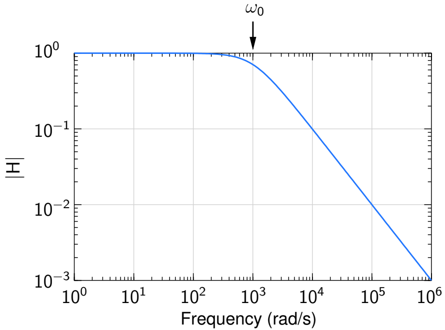
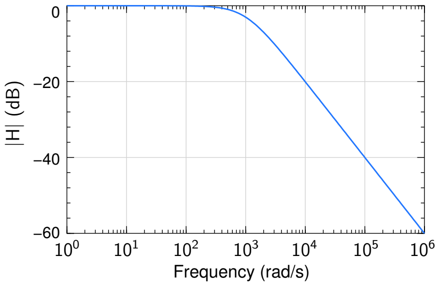
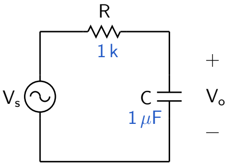




Note that the *shape* of the plot does not change.

$|H|$ (dB) $= 20\,\log\,|H|$ is simply a scaled version of $\log\,|H|$.

A simple transfer function: magnitude



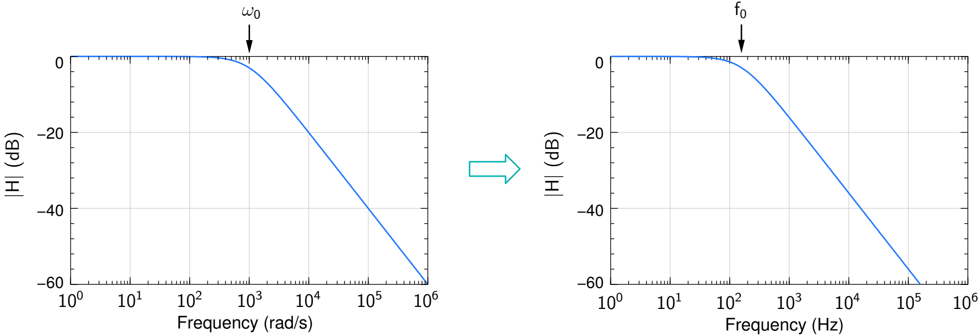
$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1{+}s\,RC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

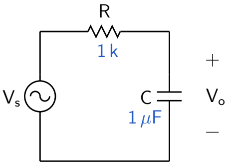
$$\omega_0 = \frac{1}{RC} = 10^3\ \text{rad/s},\ f_0 = 159\ \text{Hz}\,.$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

$$\angle(H(j\omega)) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$



A simple transfer function: magnitude



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\, V_s\,,$$

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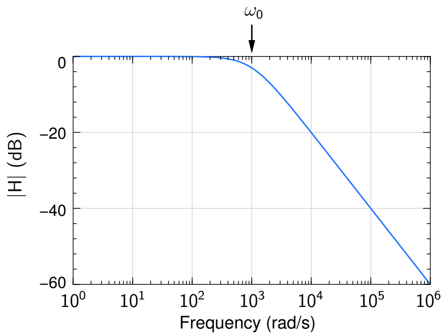
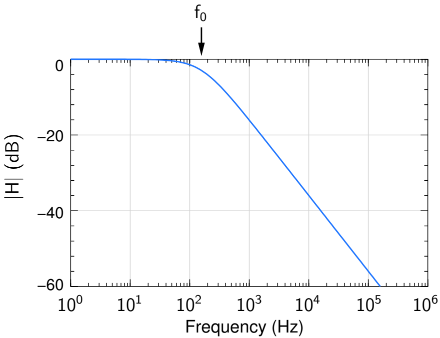
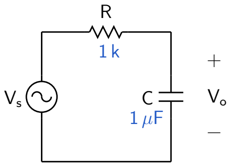




Since $\omega = 2\pi\, f$, $\log \omega = \log(2\,\pi) + \log f$ which causes a shift in the x direction, but the *shape* of the plot does not change.

A simple transfer function: phase



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

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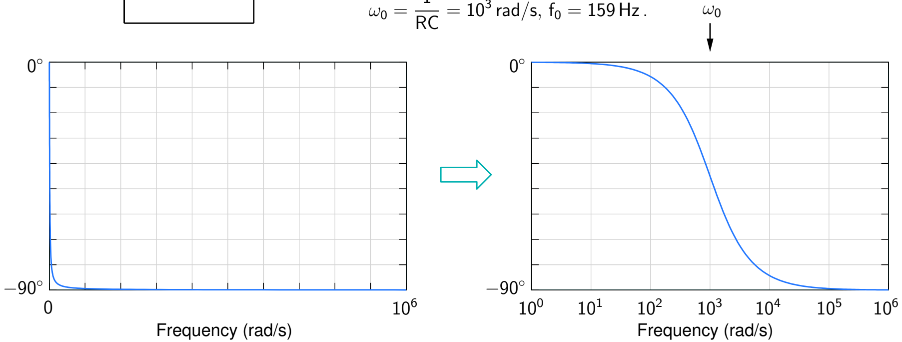
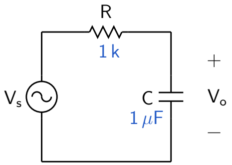


A simple transfer function: phase



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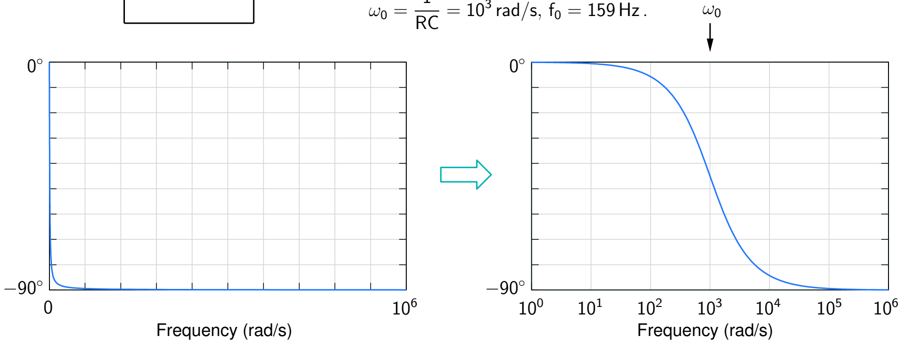
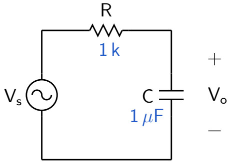


* Since $\angle H = -\tan^{-1}(\omega/\omega_0)$ varies in a limited range (0° to -90° in this example), a linear axis is appropriate for $\angle H$.



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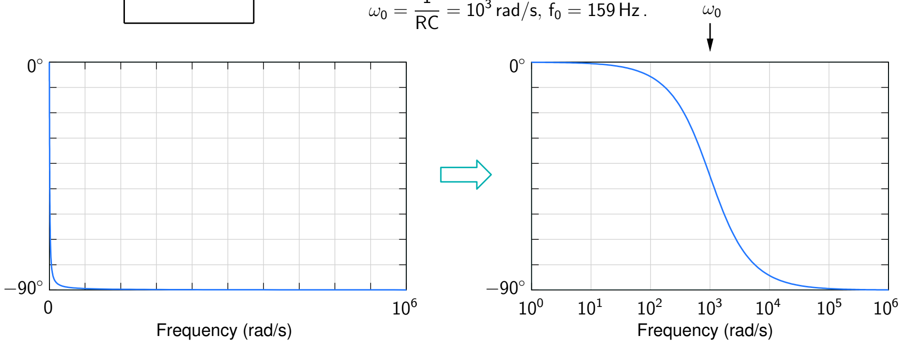

* Since $\angle H = -\tan^{-1}(\omega/\omega_0)$ varies in a limited range (0° to −90° in this example), a linear axis is appropriate for $\angle H$.
* As in the magnitude plot, we use a log axis for ω, since we are interested in a wide range of ω.

Consider $H(s) = \dfrac{K\,(1+s/z_1)(1+s/z_2)\cdots(1+s/z_M)}{(1+s/p_1)(1+s/p_2)\cdots(1+s/p_N)}$.

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$-z_1$, $-z_2, \cdots$ are called the "zeros" of $H(s)$.

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(In addition, there could be terms like s, $s^2, \cdots$ in the numerator.)

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We will assume, for simplicity, that the zeros and poles are real and distinct.

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Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function of ω.

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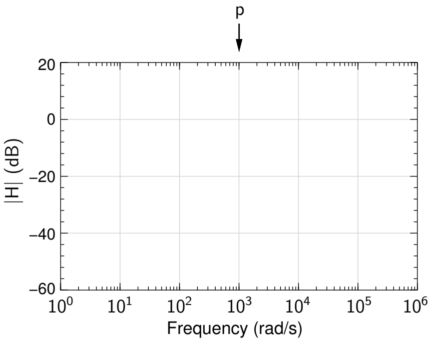
We will assume, for simplicity, that the zeros and poles are real and distinct.

Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function of ω.

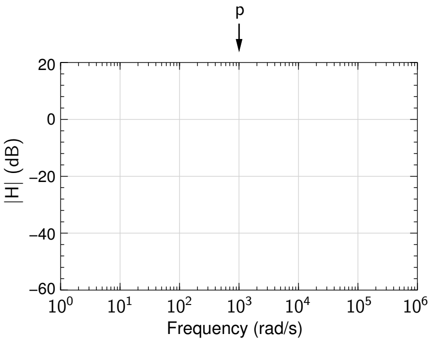
(b) combining the various contributions to obtain $|H|$ and $\angle H$ versus ω.

Contribution of a pole: magnitude



Consider $H(s) = \dfrac{1}{1+s/p} \rightarrow H(j\omega) = \dfrac{1}{1+j\,(\omega/p)}$, $|H(j\omega)| = \dfrac{1}{\sqrt{1+(\omega/p)^2}}$.

In this example, $p = 10^3$ rad/s.



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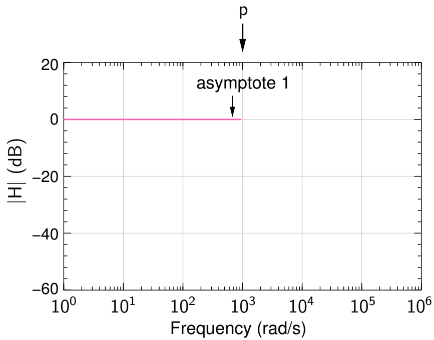
Asymptote 1:

$\omega \ll p$: $|H| \rightarrow 1,\ 20 \log |H| = 0\,\text{dB}.$

Contribution of a pole: magnitude



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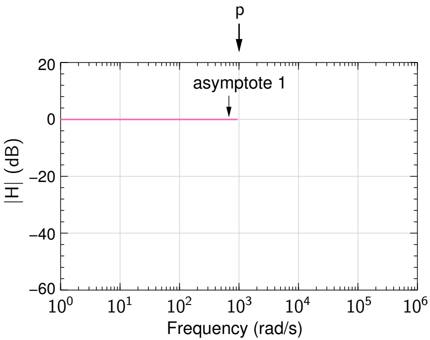

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In this example, $p = 10^3$ rad/s.

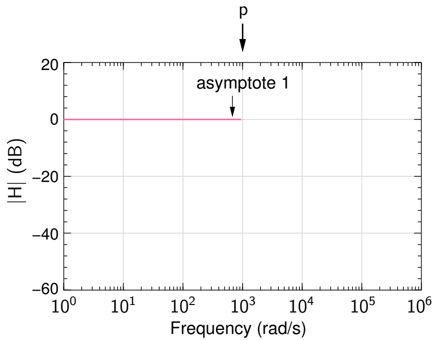
Asymptote 1:

$\omega \ll p$: $|H| \rightarrow 1,\ 20\log|H| = 0\,\text{dB}$.

Asymptote 2:

$$\omega \gg p\text{: } |H| \rightarrow \frac{1}{\omega/p} = \frac{p}{\omega} \rightarrow |H| = 20\ \log\ p - 20\ \log\omega\ \text{(dB)}$$

Contribution of a pole: magnitude



Consider $H(s) = \dfrac{1}{1+s/p} \rightarrow H(j\omega) = \dfrac{1}{1+j\,(\omega/p)}$, $|H(j\omega)| = \dfrac{1}{\sqrt{1+(\omega/p)^2}}$.

In this example, $p = 10^3$ rad/s.

Asymptote 1:

$\omega \ll p$: $|H| \rightarrow 1,\;\; 20\log|H| = 0\,\text{dB}$.

Asymptote 2:

$\omega \gg p$: $|H| \rightarrow \dfrac{1}{\omega/p} = \dfrac{p}{\omega} \rightarrow |H| = 20\,\log\,p - 20\,\log\omega$ (dB)

Consider two values of ω: ω_1 and $10\,\omega_1$.

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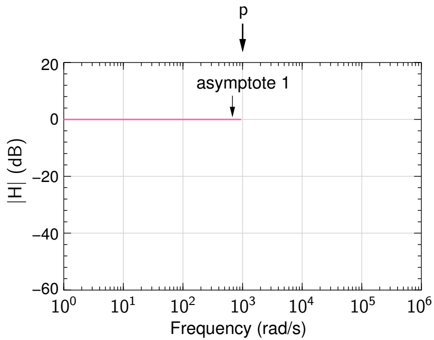

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$$|H|_1 - |H|_2 = -20\ \log\ \frac{\omega_1}{10\,\omega_1} = 20\ \text{dB}.$$

$\rightarrow$ $|H|$ versus ω has a slope of $-20\,$dB/decade.

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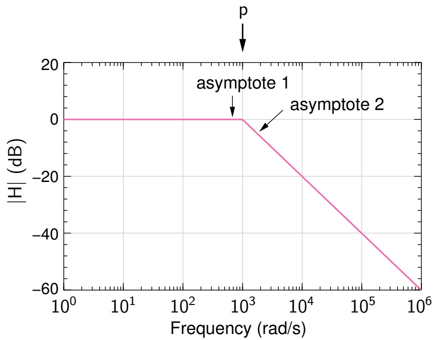

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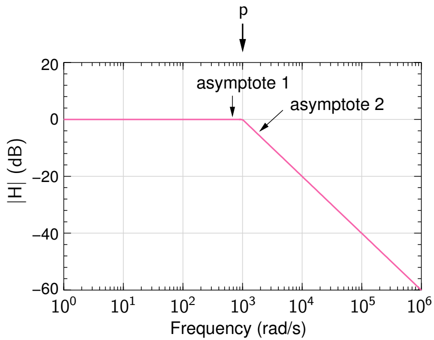

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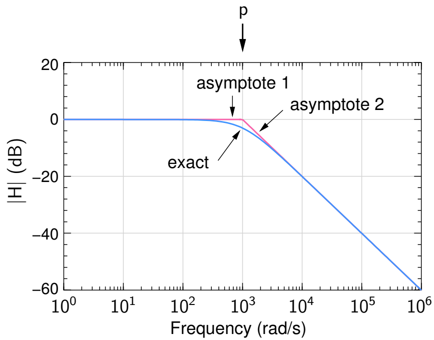

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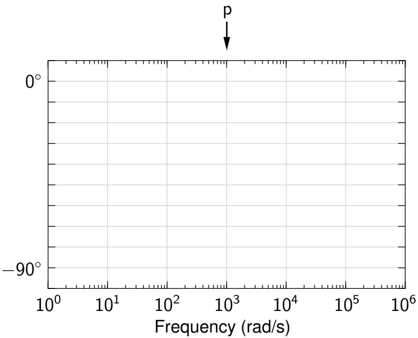
Contribution of a pole: phase



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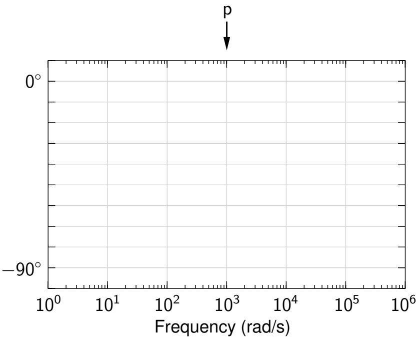

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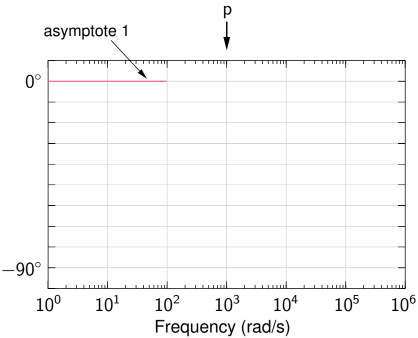

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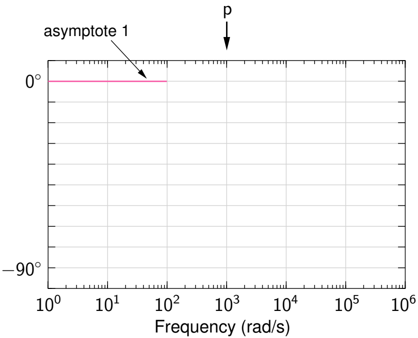

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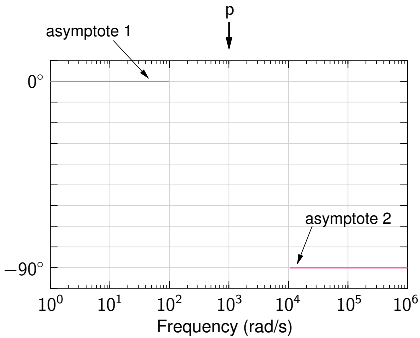

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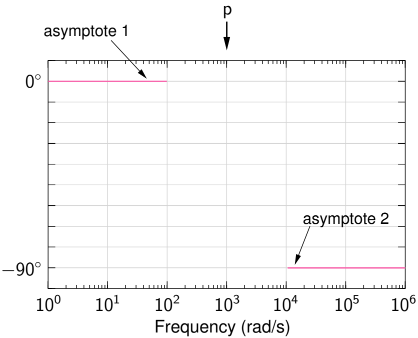

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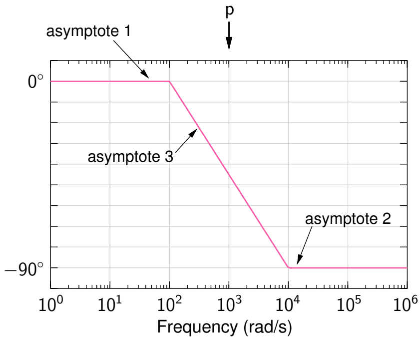
Asymptote 3:

For $p/10 < \omega < 10\,p$, $\angle H$ is assumed to vary linearly with $\log \omega$
$\rightarrow$ at $\omega = p$, $\angle H = -\pi/4$
(which is also the actual value of $\angle H$).

Contribution of a pole: phase



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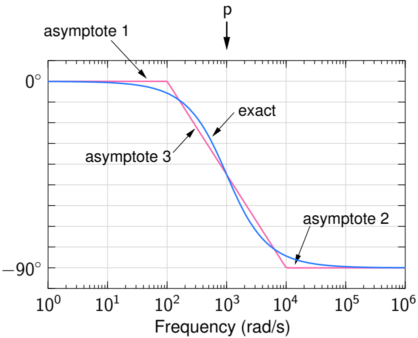

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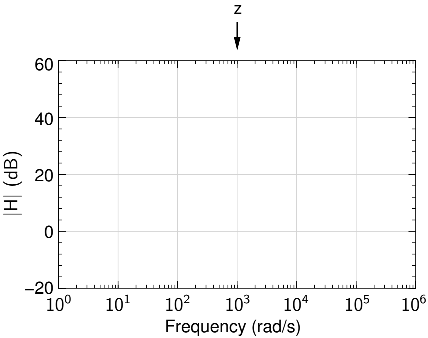
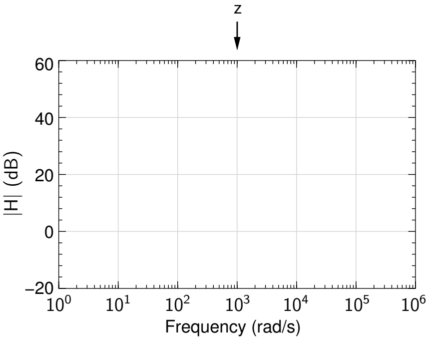


Consider $H(s) = 1 + s/z \rightarrow H(j\omega) = 1 + j\,(\omega/z)\,, |H(j\omega)| = \sqrt{1 + (\omega/z)^2}\,.$

In this example, $z = 10^3$ rad/s.

Contribution of a zero: magnitude



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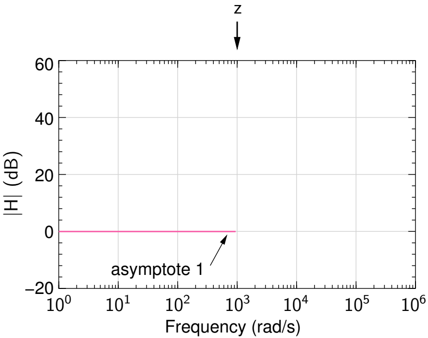

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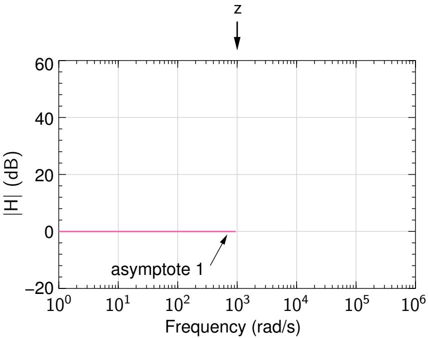

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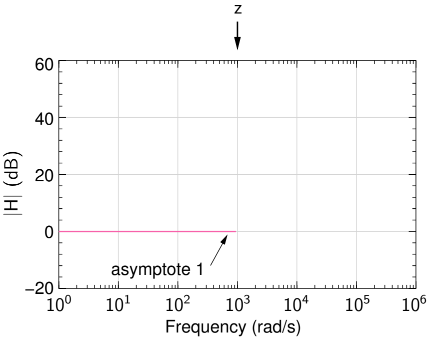

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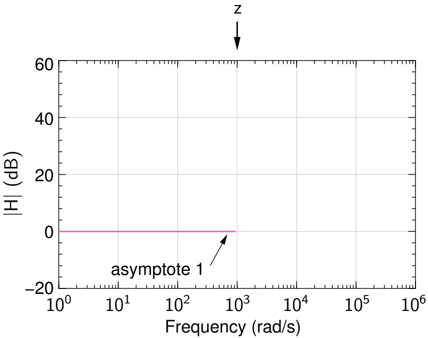

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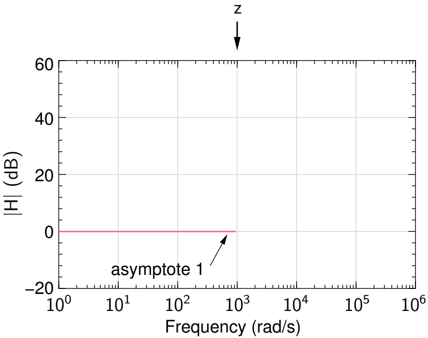

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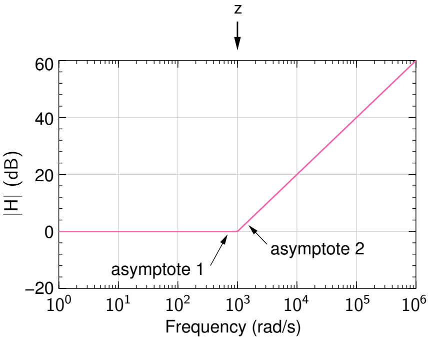
Contribution of a zero: magnitude



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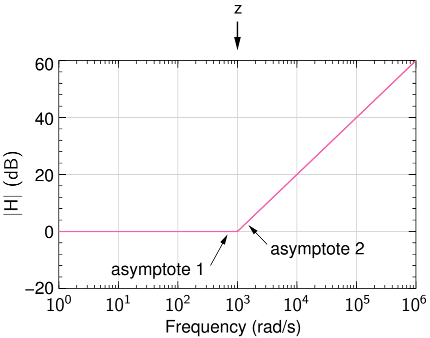

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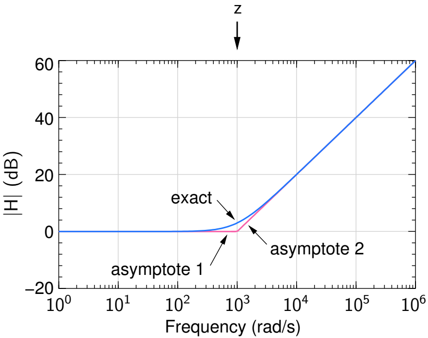

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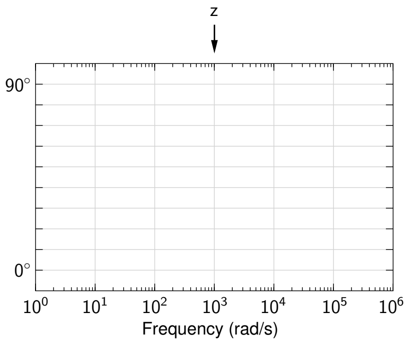
Contribution of a zero: phase



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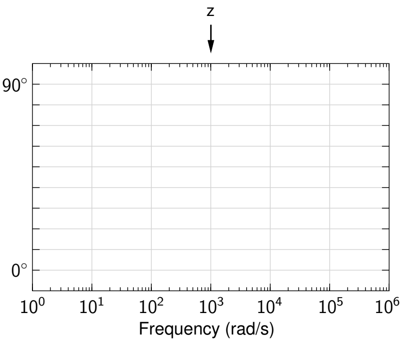
Contribution of a zero: phase



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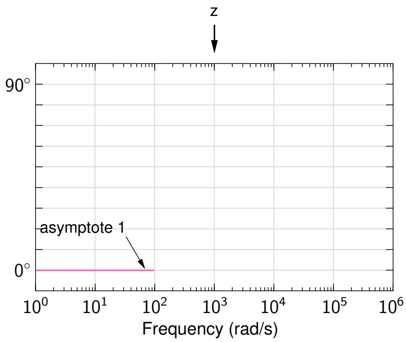
Asymptote 1:

$\omega \ll z$ (say, $\omega < z/10$): $\angle H = 0$.

Contribution of a zero: phase



Consider $H(s) = 1 + s/z = 1 + j\,(\omega/z) \rightarrow \angle H = \tan^{-1}\left(\frac{\omega}{z}\right)$

In this example, $z = 10^3$ rad/s.

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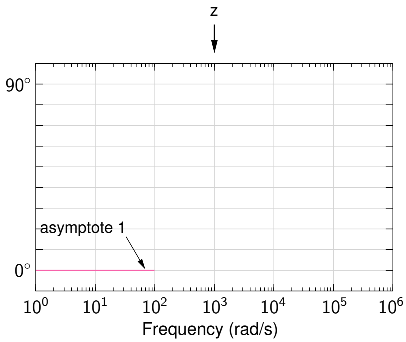

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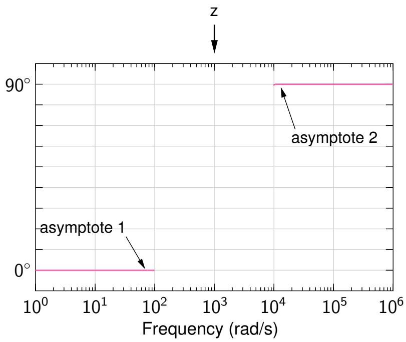

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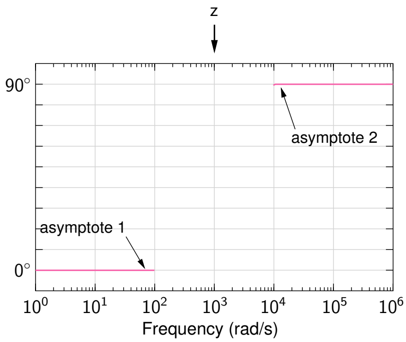

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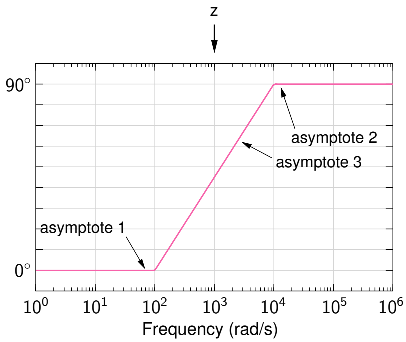

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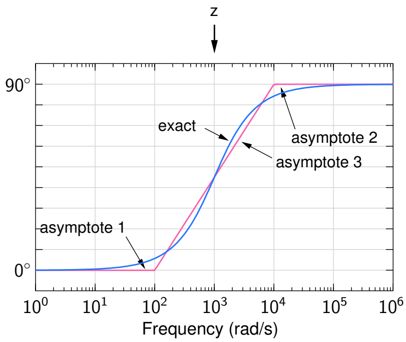

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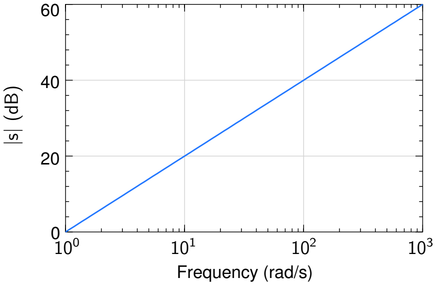

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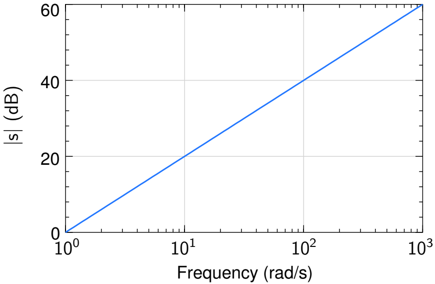

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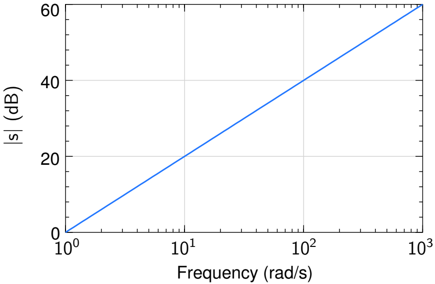

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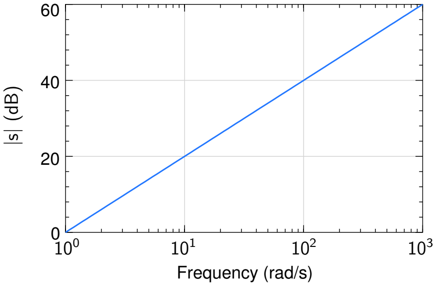

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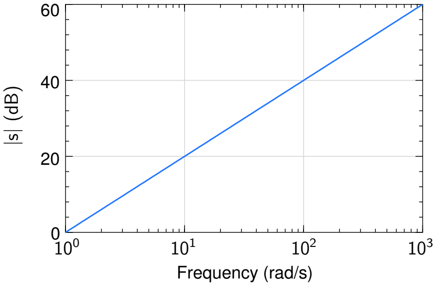

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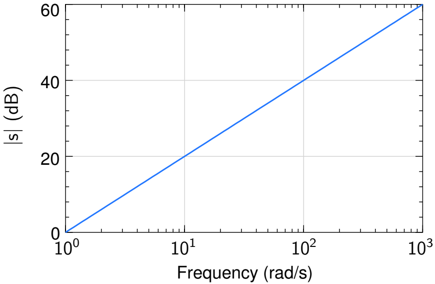

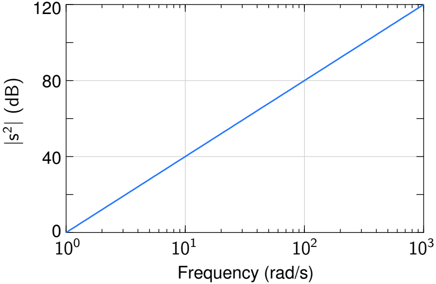

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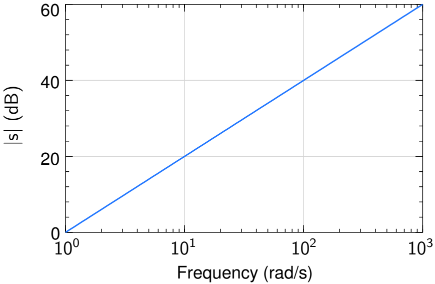

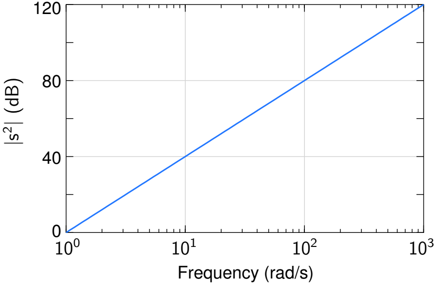

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Consider $H(s) = H_1(s) \times H_2(s)$.

Combining different terms

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<u>Magnitude</u>:

$|H(j\omega)| = |H_1(j\omega)| \times |H_2(j\omega)|$.

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Phase:

$H_1(j\omega)$ and $H_2(j\omega)$ are complex numbers.

At a given ω, let $H_1 = K_1\angle\alpha = K_1\,e^{j\alpha}$, and $H_2 = K_2\angle\beta = K_2\,e^{j\beta}$.

Then, $H_1 H_2 = K_1\,K_2\,e^{j(\alpha+\beta)} = K_1 K_2\angle\,(\alpha+\beta)$.

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In the Bode phase plot, the contributions due to H_1 and H_2 also get added.

The same reasoning applies to more than two terms as well.

Combining different terms: example

Consider $H(s) = \dfrac{10\,s}{(1+s/10^2)\,(1+s/10^5)}$.

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Let $H(s) = H_1(s)\,H_2(s)\,H_3(s)\,H_4(s)$, where

$H_1(s) = 10$,

$H_2(s) = s$,

$H_3(s) = \dfrac{1}{1+s/p_1}$, $p_1 = 10^2$ rad/s,

$H_4(s) = \dfrac{1}{1+s/p_2}$, $p_2 = 10^5$ rad/s.

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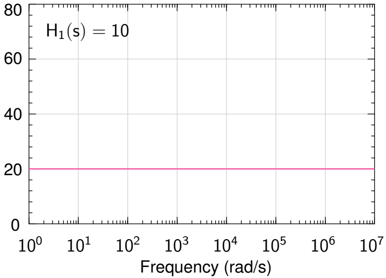
We can now plot the magnitude and phase of H_1, H_2, H_3, H_4 *individually* versus ω and then simply add them to obtain $|H|$ and $\angle H$.

Magnitude plot ($|H|$ in dB)



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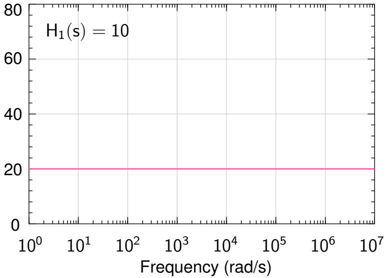

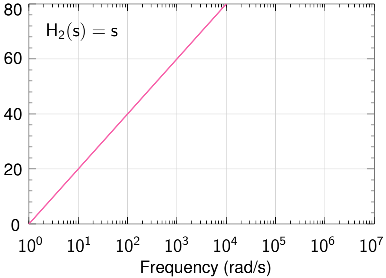

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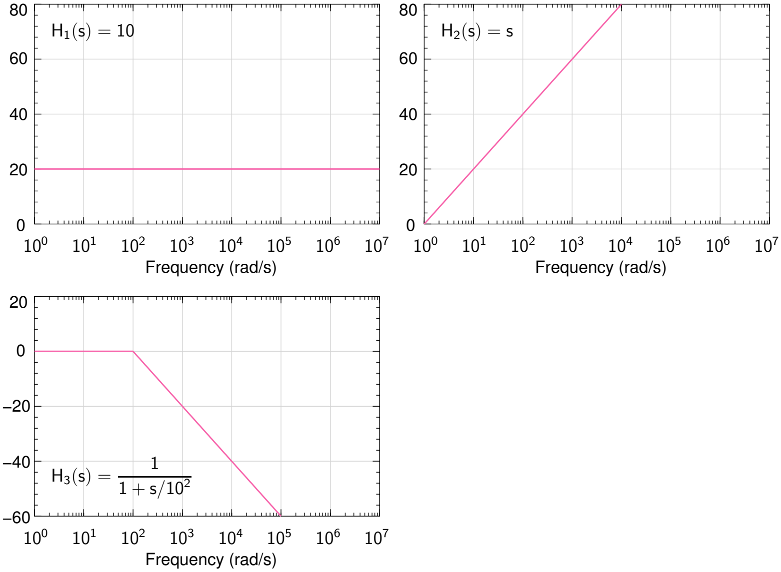

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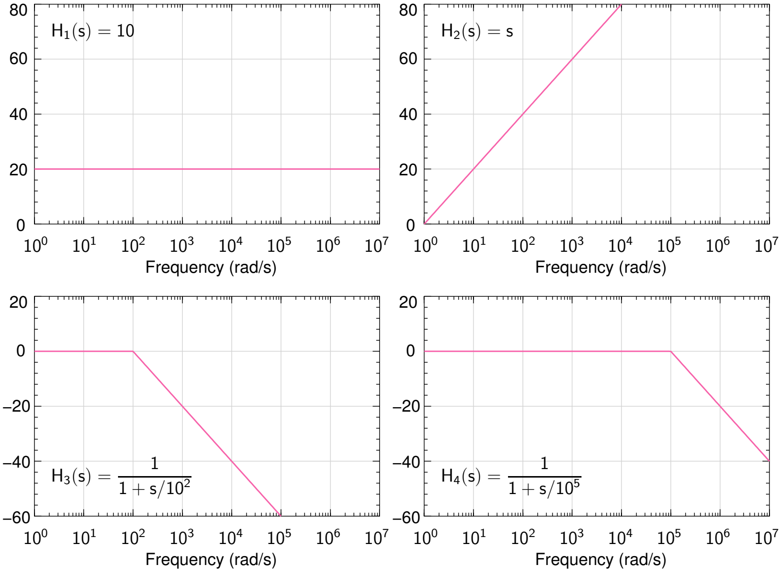

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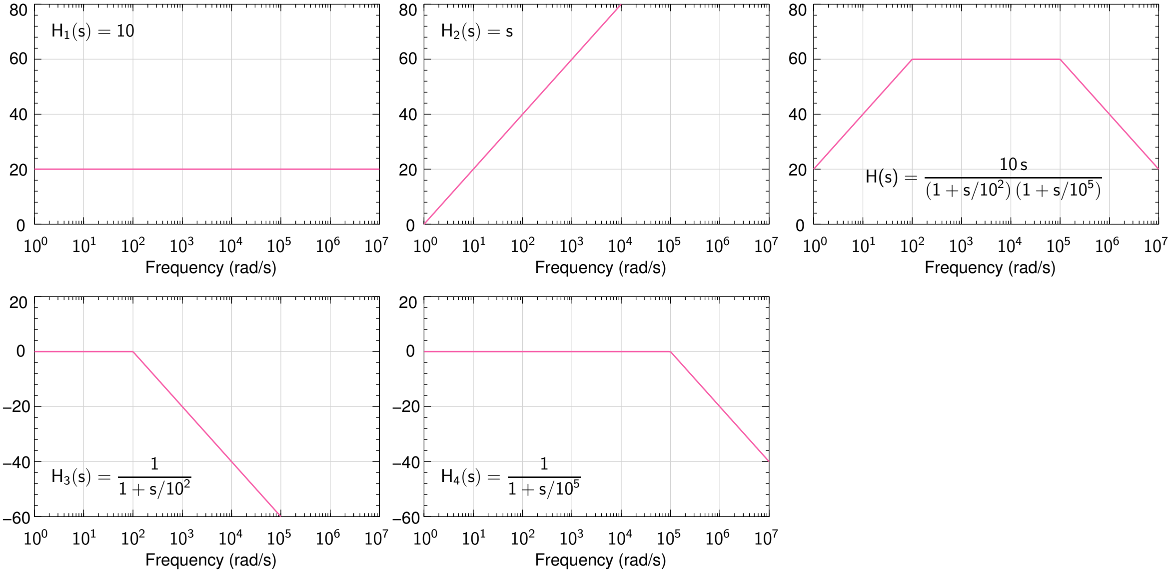

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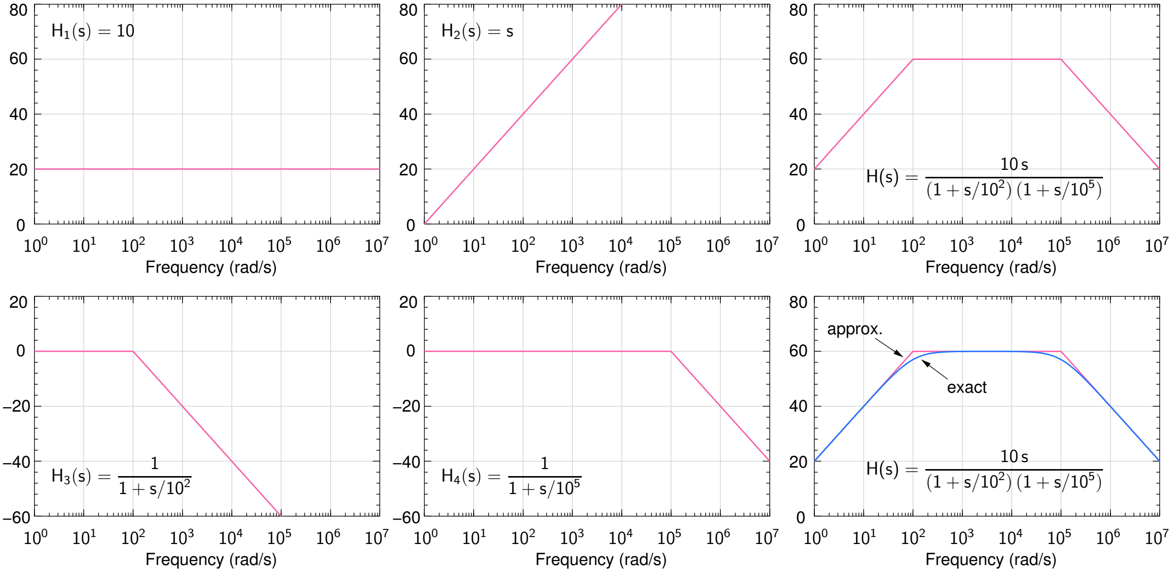
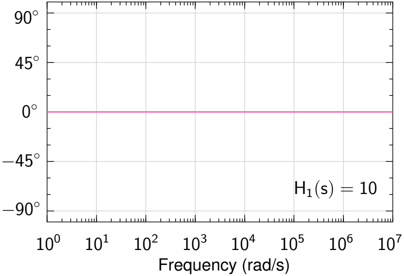
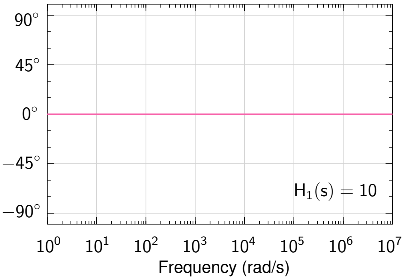


Phase plot



90°
45°
0°
−45°
−90°
$H_1(s) = 10$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



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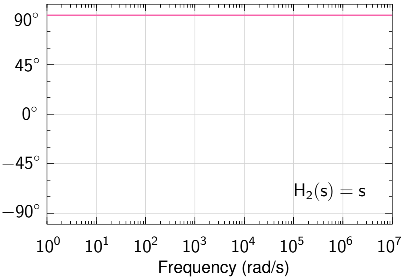
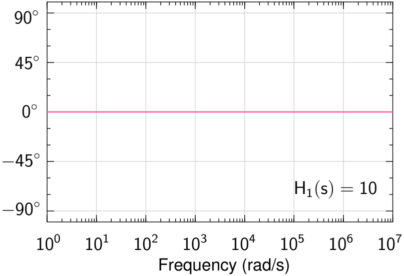


90°
45°
0°
−45°
−90°
$H_2(s) = s$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



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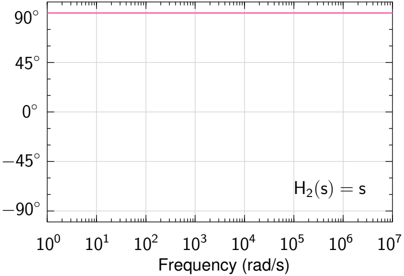

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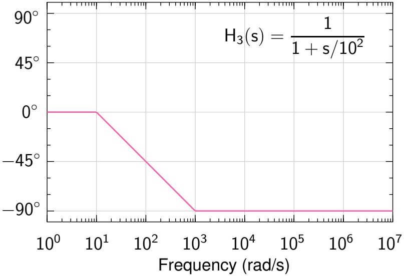
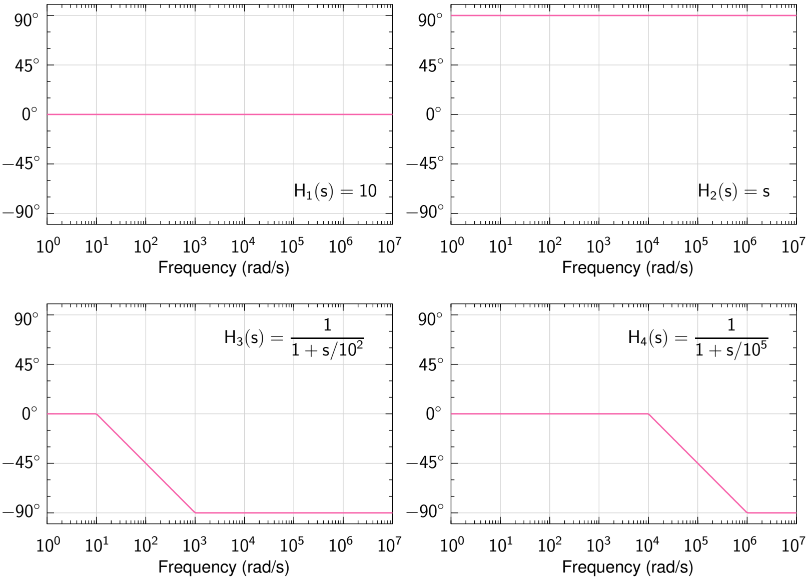
−45°
−90°
$H_2(s) = s$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)



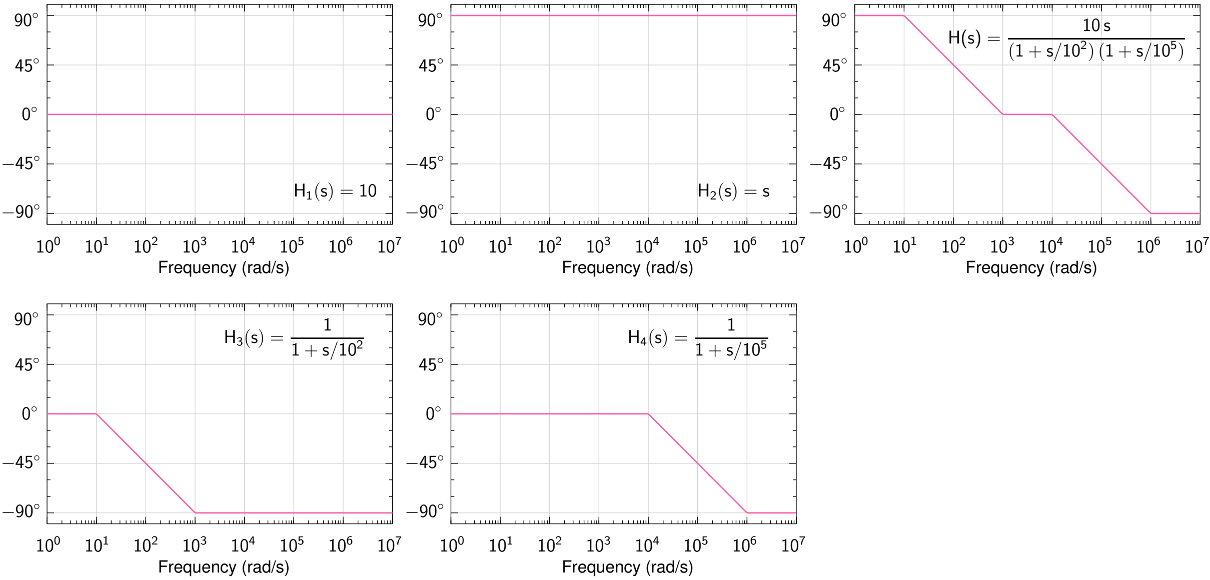
90°
45°
0°
−45°
−90°
$H_3(s) = \dfrac{1}{1 + s/10^2}$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



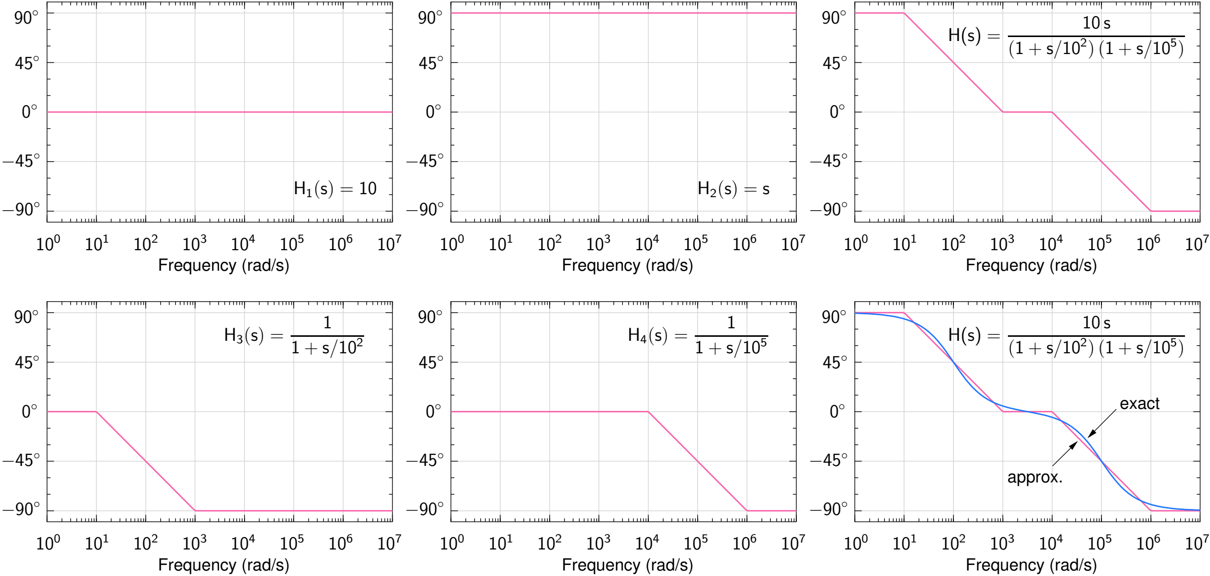
90°
45°
0°
−45°
−90°
$H_1(s) = 10$
$H_2(s) = s$
$H_3(s) = \frac{1}{1 + s/10^2}$
$H_4(s) = \frac{1}{1 + s/10^5}$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



$H_1(s) = 10$
$H_2(s) = s$
$H(s) = \frac{10\,s}{(1 + s/10^2)\,(1 + s/10^5)}$
$H_3(s) = \frac{1}{1 + s/10^2}$
$H_4(s) = \frac{1}{1 + s/10^5}$
90°
45°
0°
-45°
-90°
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



90°
45°
0°
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10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)
$H_1(s) = 10$
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exact
approx.

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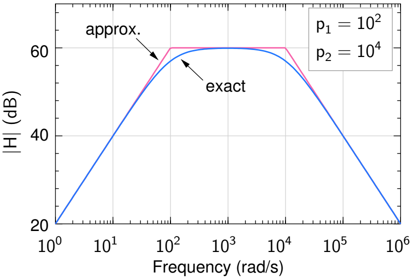
* When the poles and zeros are not sufficiently separated, the Bode approximation should be used only for a rough estimate, follwed by a numerical calculation. However, even in such cases, it does give a good idea of the *asymptotic* magnitude and phase plots, which is valuable in amplifier design.

How good are the approximations?

Consider $H(s) = \dfrac{10\, s}{(1 + s/p_1)\,(1 + s/p_2)}$.



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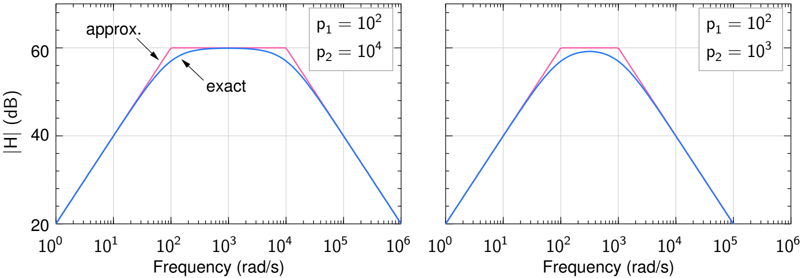

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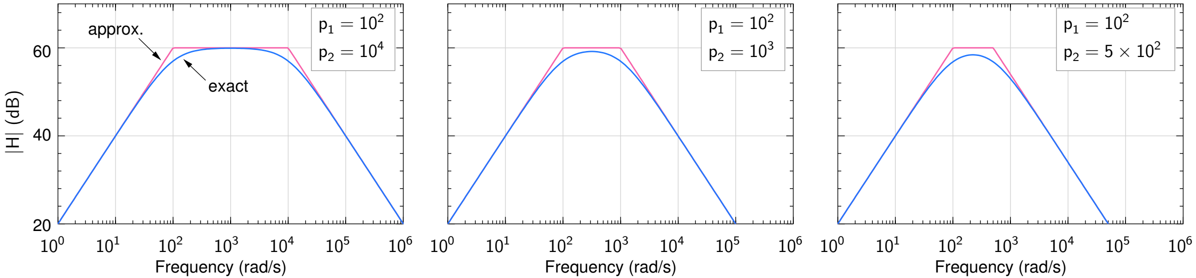

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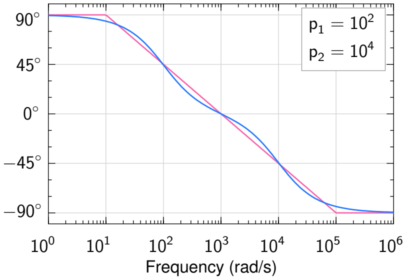

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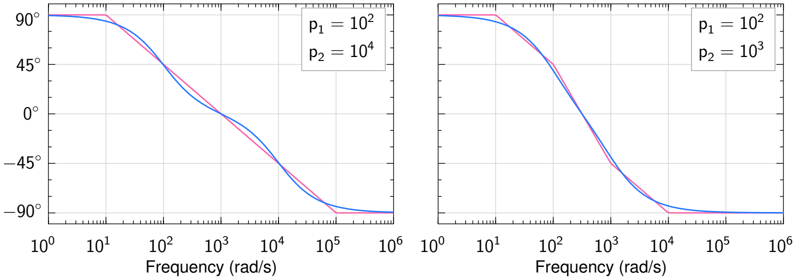

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