Combinational Equivalence Checking

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EE 709: Testing & Verification of VLSI Circuits

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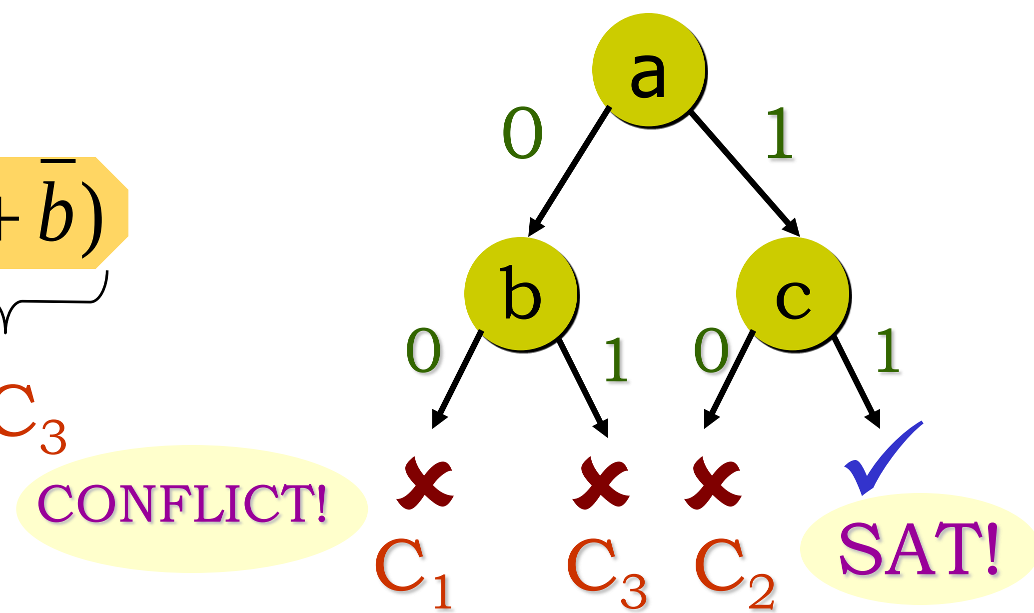
DPLL algorithm for SAT

[Davis, Putnam, Logemann, Loveland 1960,62]

Given : *CNF formula $f(v_1,v_2,..,v_k)$, and an ordering function Next_Variable*

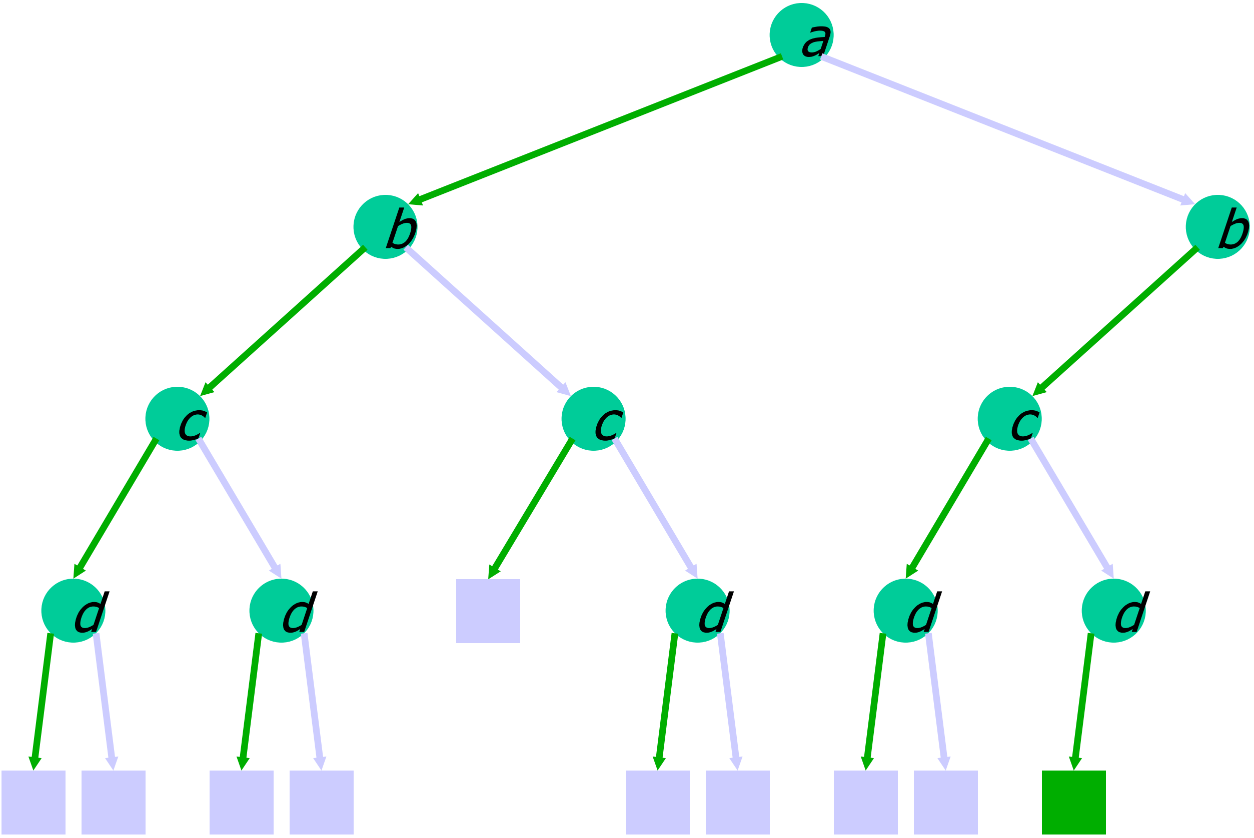
Example :

$$\underbrace{(a+b)}_{C_1}\underbrace{(\bar{a}+c)}_{C_2}\underbrace{(a+\bar{b})}_{C_3}$$



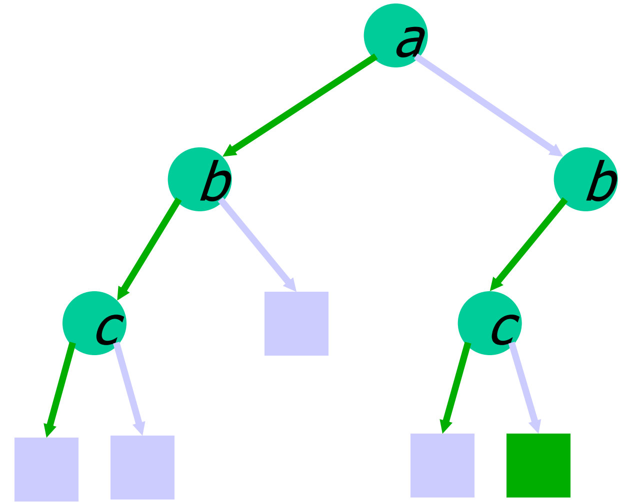
Basic Backtracking Search

1. $(a + b + c)$
2. $(a + b + \neg c)$
3. $(\neg a + b + \neg c)$
4. $(a + c + d)$
5. $(\neg a + c + d)$
6. $(\neg a + c + \neg d)$
7. $(\neg b + \neg c + \neg d)$
8. $(\neg b + \neg c + d)$



Basic Search with Implications

1 $(a + b + c)$
2 $(a + b + \neg c)$
3 $(\neg a + b + \neg c)$
4 $(a + c + d)$
5 $(\neg a + c + d)$
6 $(\neg a + c + \neg d)$
7 $(\neg b + \neg c + \neg d)$
8 $(\neg b + \neg c + d)$



DPLL algorithm: Unit clause rule

Rule: Assign to *true* any single literal clauses.

$$(a + b + c) \quad \Rightarrow \quad c = 1$$

$$\begin{matrix} a & & b \\ \| & & \| \\ 0 & & 0 \end{matrix}$$

Apply Iteratively: *Boolean Constraint Propagation (BCP)*

$$a(\overline{a} + c)(\overline{b} + c)(a + b + \overline{c})(\overline{c} + e)(\overline{d} + e)(c + d + \overline{e})$$

$$\Downarrow$$

$$c(\overline{b} + c)(\overline{c} + e)(\overline{d} + e)(c + d + \overline{e})$$

$$\Downarrow$$

$$e(\overline{d} + e)$$

Pure Literal Rule

- A variable is *pure* if its literals are either all positive or all negative
- Satisfiability of a formula is unaffected by assigning pure variables the values that satisfy all the clauses containing them

$$\varphi = (a + c)(b + c)(b + \neg d)(\neg a + \neg b + d)$$

- Set c to 1; if φ becomes unsatisfiable, then it is also unsatisfiable when c is set to 0.

Resolution (original DP)

- Iteratively apply resolution (consensus) to eliminate one variable each time
  - i.e., consensus between all pairs of clauses containing x and $\neg x$
  - formula satisfiability is preserved
- Stop applying resolution when,
  - Either empty clause is derived $\Rightarrow$ instance is unsatisfiable
  - Or only clauses satisfied or with pure literals are obtained $\Rightarrow$ instance is satisfiable

$\varphi = (a + c)(b + c)(d + c)(\neg a + \neg b + \neg c)$ Eliminate variable c

$\varphi_1 = (a + \neg a + \neg b)(b + \neg a + \neg b)(d + \neg a + \neg b)$
$= (d + \neg a + \neg b)$

Instance is SAT !

Stallmarck's Method (SM) in CNF

- Recursive application of the branch-merge rule to each variable with the goal of identifying common conclusions

$$\varphi = (a + b)(\neg a + c)(\neg b + d)(\neg c + d)$$

Try $a = 0$: $(a = 0) \Rightarrow (b = 1) \Rightarrow (d = 1)$ $C(a = 0) = \{a = 0, b = 1, d = 1\}$

Try $a = 1$: $(a = 1) \Rightarrow (c = 1) \Rightarrow (d = 1)$ $C(a = 1) = \{a = 1, c = 1, d = 1\}$

$C(a = 0) \cap C(a = 1) = \{d = 1\}$

Any assignment to variable a implies $d = 1$.
Hence, $d = 1$ is a necessary assignment **!**

Recursion can be of arbitrary depth

Recursive Learning (RL) in CNF

- Recursive evaluation of clause satisfiability requirements for identifying common assignments

$$\varphi = (a + b)(\neg a + d)\ (\neg b + d)$$

Try $a = 1$: $(a = 1) \Rightarrow (d = 1)$ $C(a = 1) = \{a = 1, d = 1\}$

Try $b = 1$: $(b = 1) \Rightarrow (d = 1)$ $C(b = 1) = \{b = 1, d = 1\}$

$C(a = 1) \cap C(b = 1) = \{d = 1\}$

Every way of satisfying $(a + b)$ implies $d = 1$.
Hence, $d = 1$ is a necessary assignment **!**

Recursion can be of arbitrary depth

SM vs. RL

- Both complete procedures for SAT
- Stallmarck's method:
  - hypothetic reasoning based on variables
- Recursive learning:
  - hypothetic reasoning based on clauses
- Both can be integrated into backtrack search algorithms

Local Search

- Repeat M times:
  - Randomly pick complete assignment
  - Repeat K times (and while exist unsatisfied clauses):
    - Flip variable that will satisfy largest number of unsat clauses

$\varphi = (a + b)(\neg a + c)(\neg b + d)(\neg c + d)$ Pick random assignment

$\varphi = (a + b)(\neg a + c)(\neg b + d)(\neg c + d)$ Flip assignment on d

$\varphi = (a + b)(\neg a + c)(\neg b + d)(\neg c + d)$ Instance is satisfied **!**

Comparison

- Local search is incomplete
  - If instances are known to be SAT, local search can be competitive
- Resolution is in general impractical
- Stallmarck's Method (SM) and Recursive Learning (RL) are in general slow, though robust
  - SM and RL can derive too much unnecessary information
- For most EDA applications backtrack search (DP) is currently the most promising approach !
  - **Augmented with techniques for inferring new clauses/implicates (i.e. learning) !**

Techniques for Backtrack Search

- Conflict analysis
  - Clause/implicate recording
  - Non-chronological backtracking
- Incorporate and extend ideas from:
  - Resolution
  - Recursive learning
  - Stallmarck's method
- Formula simplification & Clause inference [Li,AAAI00]
- Randomization & Restarts [Gomes&Selman,AAAI98]

Clause Recording

- During backtrack search, for each conflict create clause that explains and prevents recurrence of same conflict

$$\varphi = (a + b)(\neg b + c + d)(\neg b + e)(\neg d + \neg e + f)\ldots$$

Assume (decisions) $c = 0$ and $f = 0$

Assign $a = 0$ and imply assignments

A conflict is reached: $(\neg d + \neg e + f)$ is unsat

$(a = 0) \wedge (c = 0) \wedge (f = 0) \Rightarrow (\varphi = 0)$

$(\varphi = 1) \Rightarrow (a = 1) \vee (c = 1) \vee (f = 1)$

$\therefore$ create new clause: $(a + c + f)$

Clause Recording

- Clauses derived from conflicts can also be viewed as the result of applying selective consensus

$$\varphi = (a + b)(\neg b + c + d)\,(\neg b + e)(\neg d + \neg e + f)\ldots$$

consensus

$$(a + c + d) \qquad\qquad (a + e)$$

$$(a + c + \neg e + f)$$

$$(a + c + f)$$

Non-Chronological Backtracking

- During backtrack search, in the presence of conflicts, backtrack to one of the causes of the conflict

$$\varphi = (a + b)(\neg b + c + d)(\neg b + e)(\neg d + \neg e + f)$$
$$(a + c + f)(\neg a + g)(\neg g + b)(\neg h + j)(\neg i + k)\ldots$$

Assume (decisions) $c = 0$, $f = 0$, $h = 0$ and $i = 0$

Assignment $a = 0$ caused conflict $\Rightarrow$ clause $(a + c + f)$ created

$(a + c + f)$ implies $a = 1$

A conflict is again reached: $(\neg d + \neg e + f)$ is unsat

$(a = 1) \wedge (c = 0) \wedge (f = 0) \Rightarrow (\varphi = 0)$

$(\varphi = 1) \Rightarrow (a = 0) \vee (c = 1) \vee (f = 1)$

$\therefore$ create new clause: $(\neg a + c + f)$

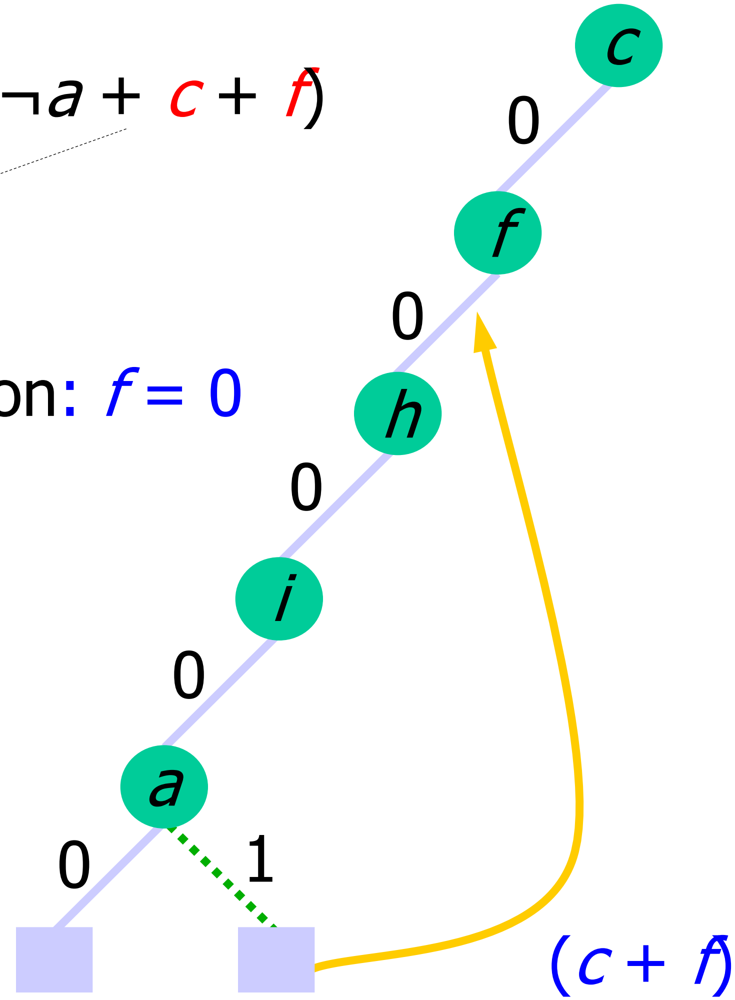
Non-Chronological Backtracking

Created clauses: $(a + c + f)$ and $(\neg a + c + f)$

Apply consensus:

new unsat clause $(c + f)$

$\therefore$ backtrack to most recent decision: $f = 0$



$\therefore$ created clauses/implicates:

$(a + c + f)$,

$(\neg a + c + f)$, and

$(c + f)$

Ideas from other Approaches

- Resolution, Stallmarck's method and recursive learning can be incorporated into backtrack search (DP)
  - create additional clauses/implicates
    - anticipate and prevent conflicting conditions
    - identify necessary assignments
    - allow for non-chronological backtracking

Resolution within DP:

$(a + b + c) \quad (\neg a + b + d)$

consensus

$(b + c + d)$

$(b + c + d)$ Unit clause !

Clause provides explanation for necessary assignment $b = 1$

Stallmarck's Method within DP

$$\varphi = (a + b + e)(\neg a + c + f)(\neg b + d)(\neg c + d + g)$$

Implications:

consensus

$(a = 0) \wedge (e = 0) \Rightarrow (b = 1) \Rightarrow (d = 1)$

$(b + e + c + f)$

$(a = 1) \wedge (f = 0) \Rightarrow (c = 1)$

$(c = 1) \wedge (g = 0) \Rightarrow (d = 1)$

$(d + e + c + f)$

$(e = 0) \wedge (f = 0) \wedge (g = 0) \Rightarrow (d = 1)$

$(e + f + g + d)$

Clausal form:

$(e + f + g + d)$ Unit clause !

Clause provides explanation for necessary assignment $d = 1$

Recursive Learning within DP

$$\varphi = (a + b + c)(\neg a + d + e)(\neg b + d + c)$$

consensus

$$(b + c + e + d)$$

consensus

$$(c + e + d)$$

Implications:

$$(a = 1) \wedge (e = 0) \Rightarrow (d = 1)$$

$$(b = 1) \wedge (c = 0) \Rightarrow (d = 1)$$

$$(c = 0) \wedge ((e = 0) \wedge (c = 0)) \Rightarrow (d = 1)$$

Clausal form:

$(c + e + d)$ Unit clause !

Clause provides explanation for necessary assignment $d = 1$

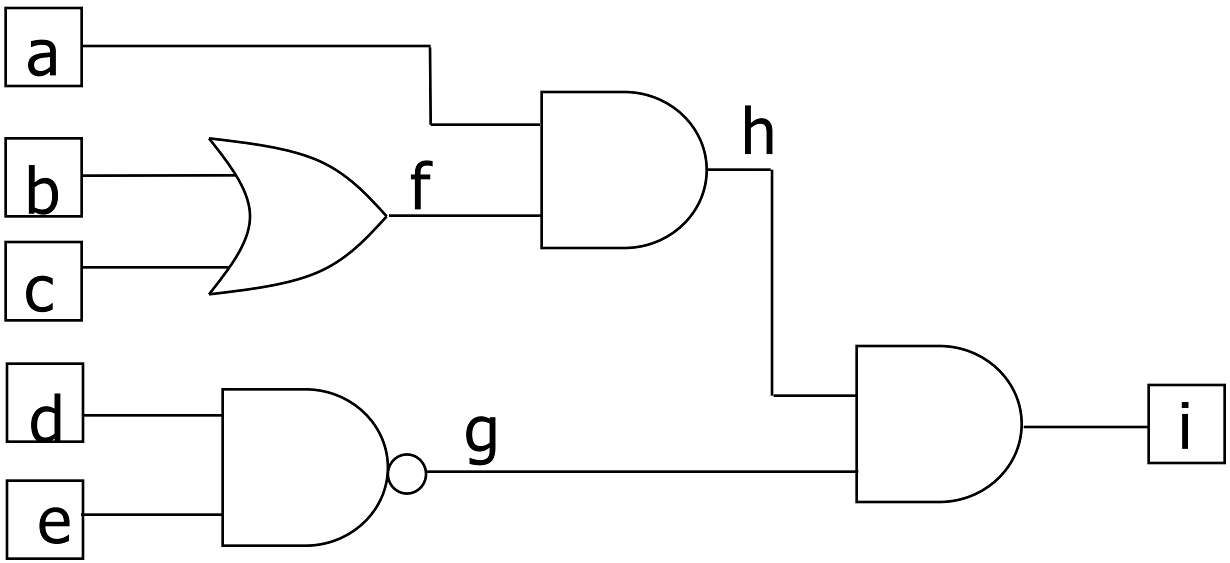
The Power of Consensus

- Most search pruning techniques can be explained as particular ways of applying selective consensus
  - Conflict-based clause recording
  - Non-chronological backtracking
  - Extending Stallmarck's method to backtrack search
  - Extending recursive learning to backtrack search
  - Clause inference conditions
- General consensus is computationally too expensive **!**
- Most techniques indirectly identify which consensus operations to apply **!**
  - To create new clauses/implicates
    - To identify necessary assignments

SAT Solvers Today

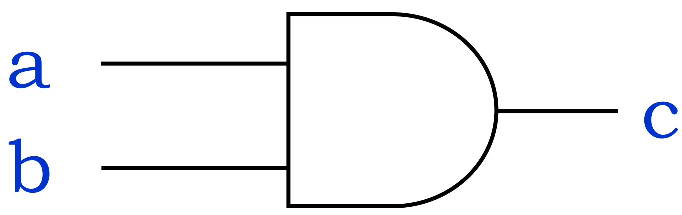
- Capacity:
  - Formulas upto a *million variables* and *3-4 million clauses* can be solved in *few hours*
  - Only for *structured instances* *e.g.* derived from real-world circuits & systems
- Tool offerings:
  - Public domain
    - GRASP : Univ. of Michigan
    - SATO: Univ. of Iowa
    - zChaff: Princeton University
    - BerkMin: Cadence Berkeley Labs.
  - Commercial
    - PROVER: Prover Technologies

Solving circuit problems as SAT

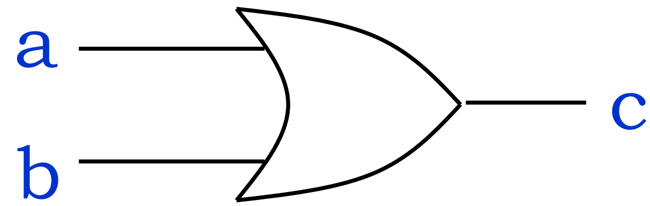


Input Vector Assignment ? → Primary Output 'i' to *1* ?

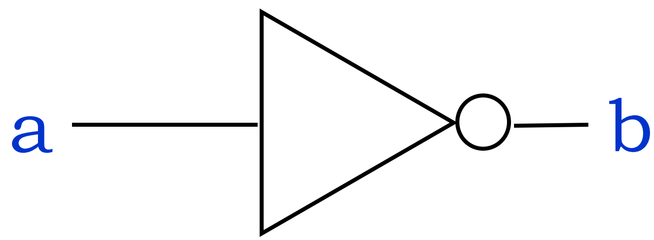
SAT formulas for simple gates



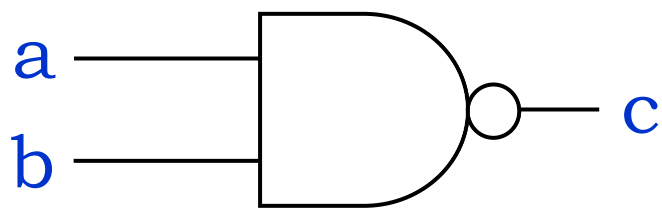
$$(\bar{c}+a)(\bar{c}+b)(c+\bar{a}+\bar{b})$$



$$(c+\bar{a})(c+\bar{b})(\bar{c}+a+b)$$



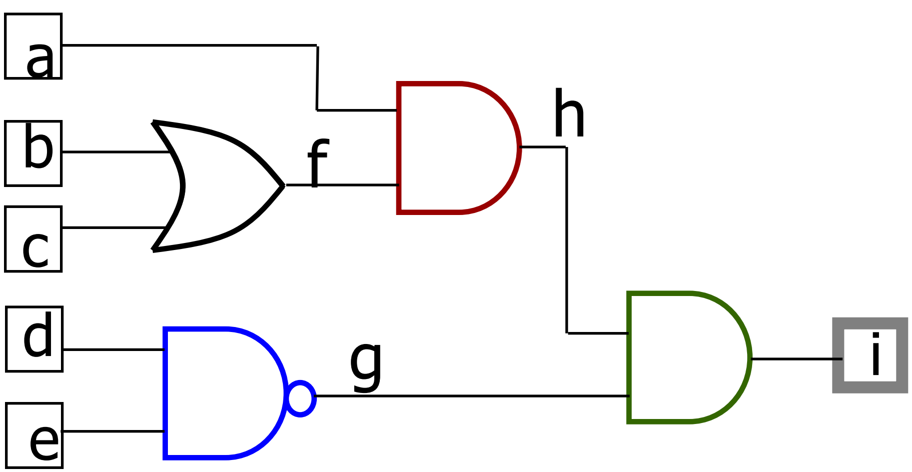
$$(a+b)(\bar{a}+\bar{b})$$



$$(c+a)(c+b)(\bar{c}+\bar{a}+\bar{b})$$

Solving circuit problems as SAT

- Set of clauses representing function of each gate
- Unit literal clause asserting output to *'1'*



$$(\bar{b}+f)(\bar{c}+f)(b+c+\bar{f})$$

$$(d+g)(e+g)(\bar{d}+\bar{e}+\bar{g})$$

$$(a+\bar{h})(f+\bar{h})(\bar{a}+\bar{f}+h)$$

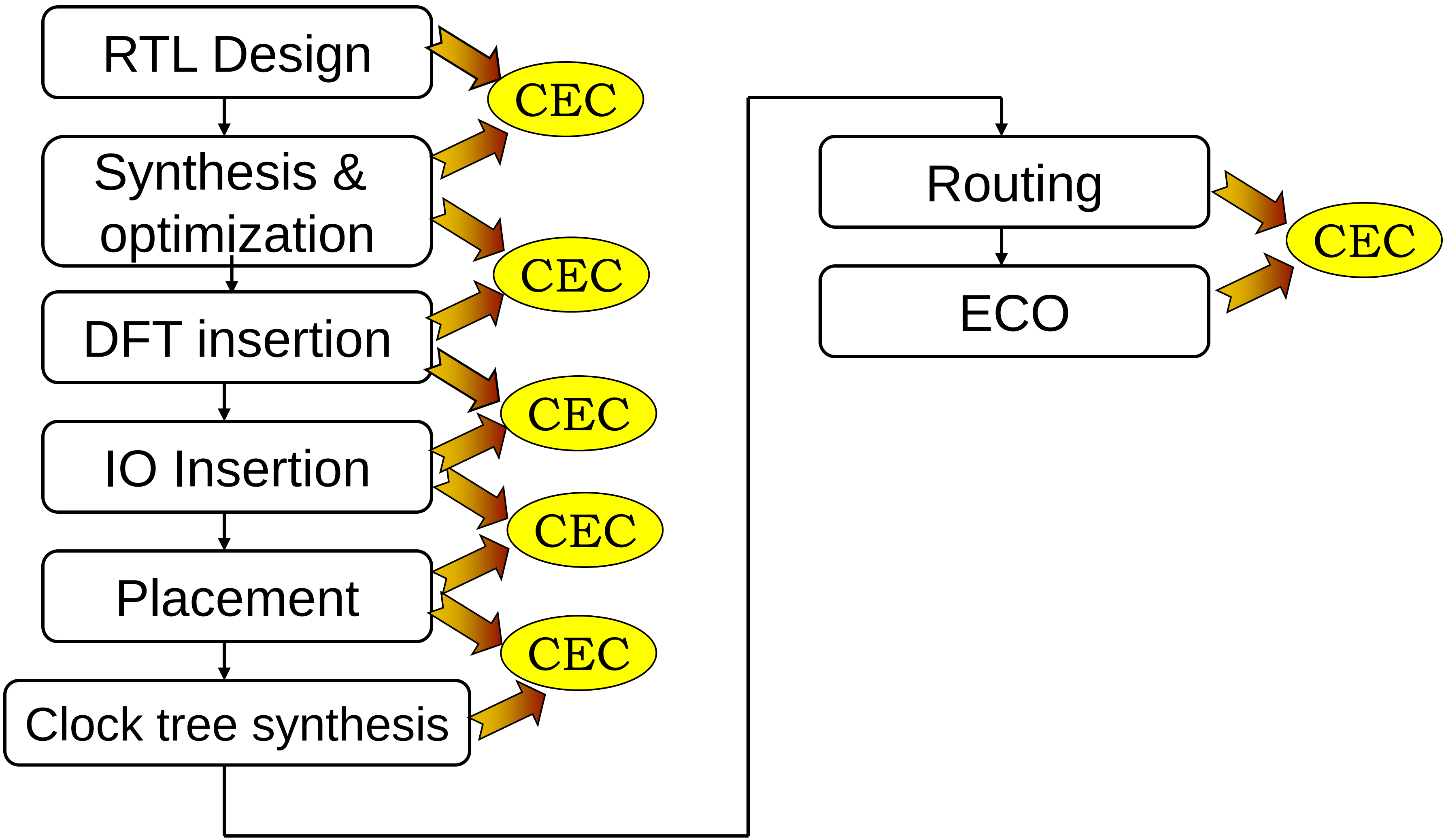
$$(h+\bar{i})(g+\bar{i})(\bar{h}+\bar{g}+i)$$

$$(i)$$

Combinational Equivalence Checking (CEC)

- Currently most practical and pervasive equivalence checking technology
- Nearly full automation possible
- Designs of up to several million gates verified in a few hours or minutes
- Hierarchical verification deployed
- Full chip verification possible
- Key methodology: Convert sequential equivalence checking to a CEC problem!
  - Match Latches & extract comb. portions for EC

CEC in Today's ASIC Design Flow



Major Industrial Offerings of CEC

- Formality *(Synopsys)*
- Conformal Suite *(Verplex, now Cadence)*
- FormalPro *(Mentor Graphics)*
- Typical capabilities of these tools:
  - Can handle circuits of up to several million gates flat in up to a few hours of runtime
  - Comprehensive debug tool to pinpoint error-sources
  - Counter-example display & cross-link of RTL and gate-level netlists for easier debugging
  - Ability to checkpoint verification process and restart from same point later
  - What if capability (unique to FormalPro)

Combinational Equivalence Checking

- Functional Approach
  - transform output functions of combinational circuits into a unique (canonical) representation
  - two circuits are equivalent if their representations are identical
  - efficient canonical representation: BDD
- Structural
  - identify structurally similar internal points
  - prove internal points (cut-points) *equivalent*
  - find implications

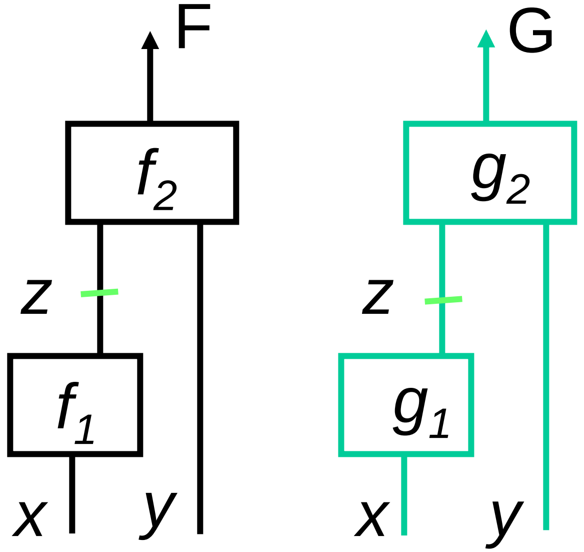
Functional Equivalence

- If BDD can be constructed for each circuit
  - ➢ represent each circuit as *shared* (multi-output) BDD
    - ❖ use the same variable ordering !
  - ➢ BDDs of both circuits must be *identical*

- If BDDs are too large
  - ➢ cannot construct BDD, memory problem
  - ➢ use partitioned BDD method
    - decompose circuit into smaller pieces, each as BDD
    - check equivalence of internal points

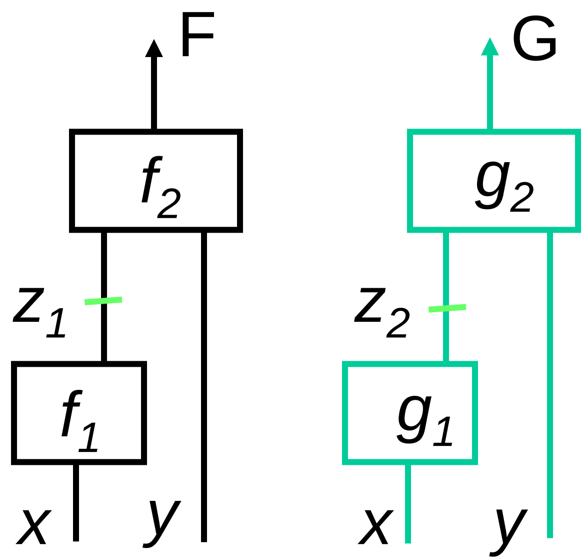
Functional Decomposition

- Decompose each function into *functional* blocks
  - represent each block as a BDD (*partitioned BDD* method)
  - define *cut-points* (*z*)
  - verify equivalence of blocks at cut-points
  - starting at primary inputs



Cut-Points Resolution Problem

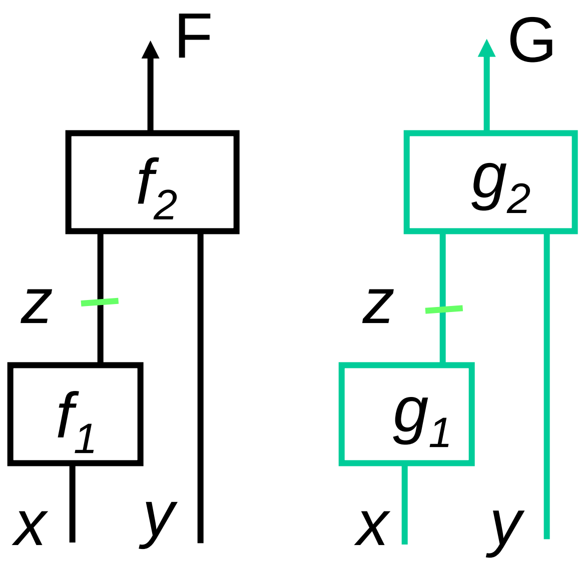
- If *all pairs* of cut-points (z_1,z_2) are equivalent
  - so are the two functions, F,G
- If *intermediate* functions (f_2,g_2) are not equivalent
  - the functions (F,G) may still be equivalent
  - this is called false negative



- Why do we have false negative ?
  - functions are represented in terms of *intermediate* variables
  - to prove/disprove equivalence must represent the functions in terms of *primary inputs* (BDD composition)

Cut-Point Resolution – Theory

- Let $f_1(x)=g_1(x)\ \forall x$
  - if $f_2(z,y) \equiv g_2(z,y),\ \forall z,y$ then $f_2(f_1(x),y) \equiv g_2(f_1(x),y) \Rightarrow F \equiv G$
  - if $f_2(z,y) \neq g_2(z,y),\ \forall z,y \not\Longrightarrow f_2(f_1(x),y) \neq g_2(f_1(x),y) \not\Rightarrow F \neq G$



We *cannot* say if $F \equiv G$ or not

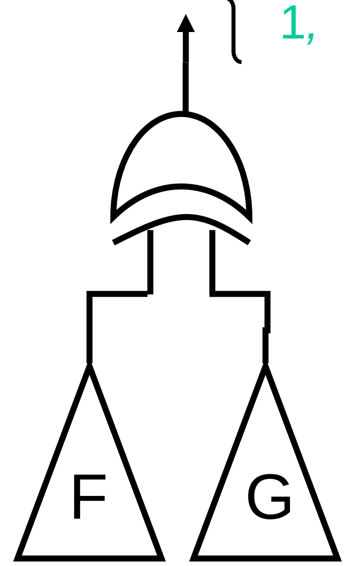
- *False negative*
  - two functions are equivalent, but the verification algorithm declares them as different.

Cut-Point Resolution

- How to verify if negative is *false* or *true* ?

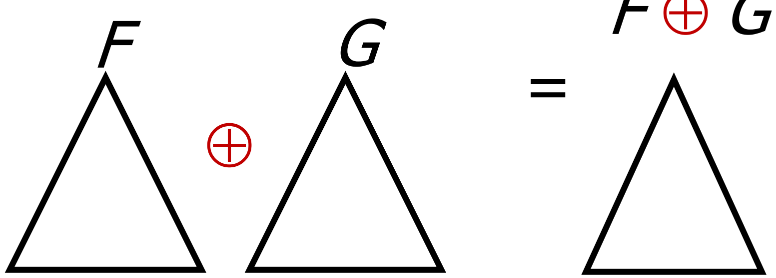
- Procedure 1: create a miter (XOR) between two potentially equivalent nodes/functions
  - perform ATPG test for *stuck-at 0*
  - find test pattern to prove $F \neq G$
  - efiicient for true negative
  - (gives *test vector*, a proof)
  - inefficient when there is no test

$$\begin{cases} 0, & F \equiv G \text{ (false negative)} \\ 1, & F \neq G \text{ (true negative)} \end{cases}$$



Cut-Point Resolution

- Procedure 2: create a BDD for $F \oplus G$
  - ➢ perform satisfiability analysis (SAT) of the BDD
    - if BDD for $F \oplus G = \emptyset$, problem is *not* satisfiable, *false* negative
    - BDD for $F \oplus G \neq \emptyset$, problem is satisfiable, *true* negative



$$F \oplus G = \begin{cases} \emptyset, & F \equiv G \text{ (false negative)} \\ \text{Non-empty}, & F \neq G \end{cases}$$

Note: must compose BDDs until they are equivalent, or expressed in terms of primary inputs

– the SAT solution, if exists, provides a *test vector* (proof of non-equivalence) – as in ATPG

– unlike the ATPG technique, it is effective for false negative (the BDD is empty!)

Thank you