Sequential Equivalence Checking - II

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EE 709: Testing & Verification of VLSI Circuits

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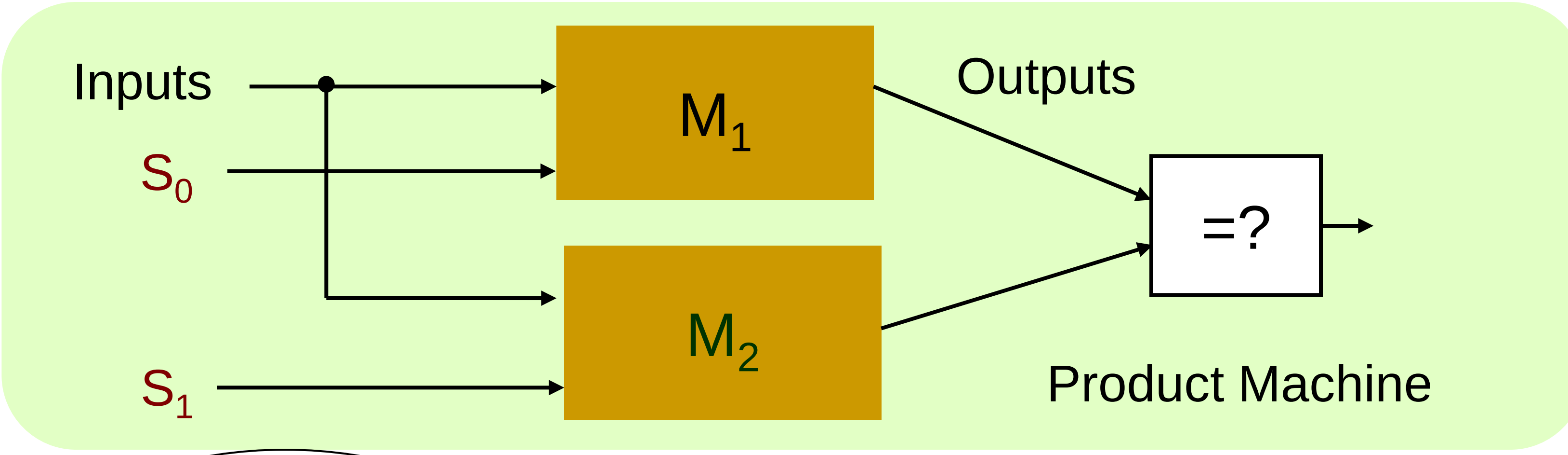
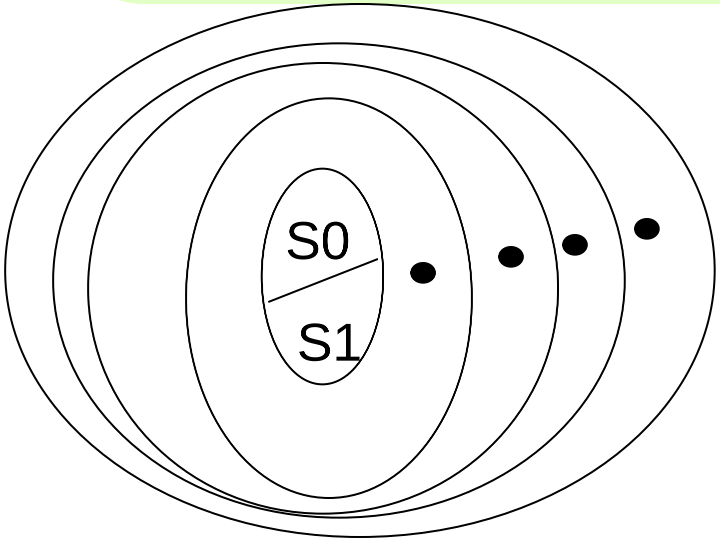
Reachability-Based Equivalence Checking

Approach 3: Symbolic Traversal Based Reachability Analysis



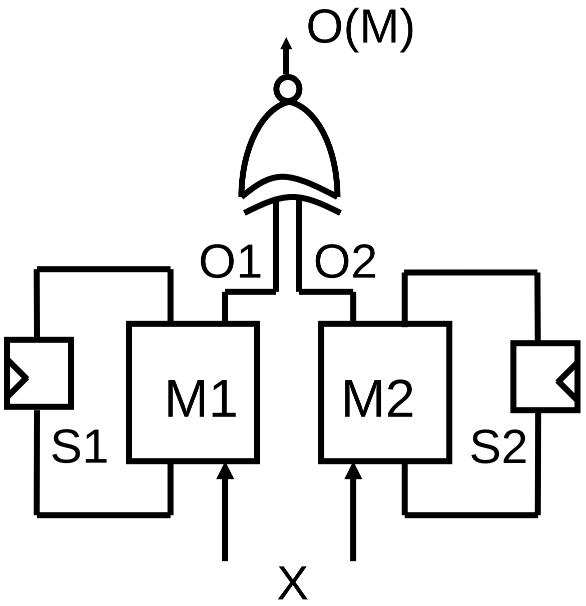


- Build product machine of M_1 and M_2
- Traverse state-space of product machine starting from reset states S_0, S_1
- Test equivalence of outputs in each state
- Can use any state-space traversal technique

Sequential Verification

- Symbolic FSM traversal of the product machine

- Given two FSMs: $M_1(X, S_1, \delta_1, \lambda_1, O_1)$, $M_2(X, S_2, \delta_2, \lambda_2, O_2)$
- Create a product FSM: $M = M_1 \mathbf{x}\, M_2$
  - traverse the states of M and check its output for each transition
  - the output O(M) =1, if outputs $O_1 = O_2$
  - if all outputs of M are 1, M_1 and M_2 are *equivalent*
  - otherwise, an *error state* is reached
  - *error trace* is produced to show: $M_1 \neq M_2$

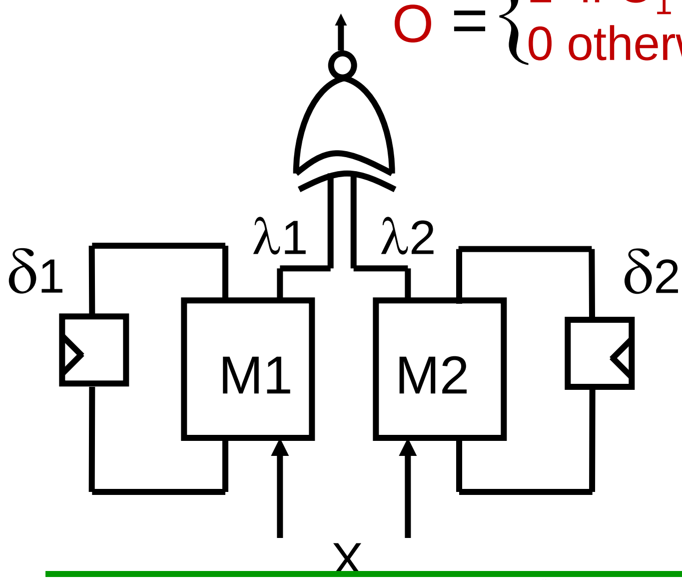


Product Machine - Construction

- Define the product machine $M(X, S, S^0, \delta, \lambda, O)$
  - states, $S = S_1 \times S_2$
  - next state function, $\delta(s, x) : (S_1 \times S_2) \times X \rightarrow (S_1 \times S_2)$
  - output function, $\lambda(s, x) : (S_1 \times S_2) \times X \rightarrow \{0,1\}$

$$O = \begin{cases} 1 & \text{if } O_1 = O_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{\lambda(s,x) = \lambda_1(s_1,x) \overline{\oplus} \lambda_2(s_2,x)}$$



- Error trace (*distinguishing sequence*) that leads to an error state
  - sequence of inputs which produces 1 at the output of M
  - produces a state in M for which M1 and M2 give different outputs

FSM Traversal - Algorithm

- Traverse the product machine $M(X,S,\delta, \lambda,O)$
  - start at an initial state S_0
  - iteratively compute symbolic image $Img(S_0,R)$ (set of *next states*):

$$Img(S_0,R) = \exists_x \exists_s S_0(s) \bullet R(x,s,t)$$

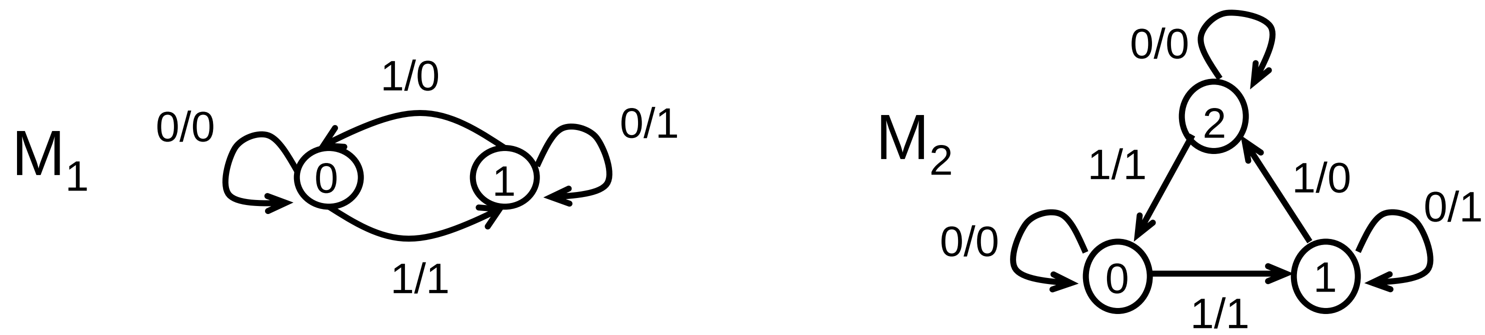
$$R = \prod_i R_i = \prod_i (t_i \equiv \delta_i(s,x))$$

    until an *error state* is reached

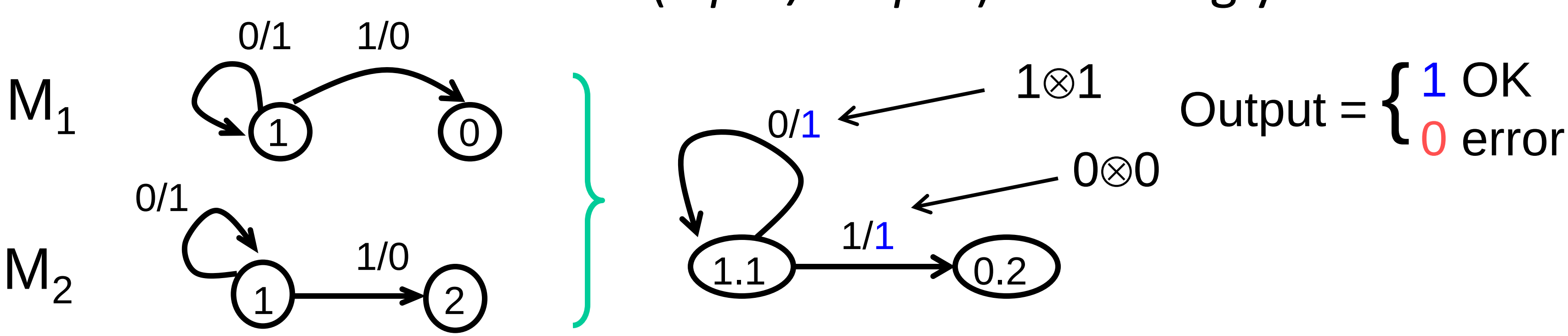
  - transition relation R_i for each next state variable t_i can be computed as $t_i = (t \otimes \delta(s,x))$

    (this is an alternative way to compute transition relation, when design is specified at gate level)

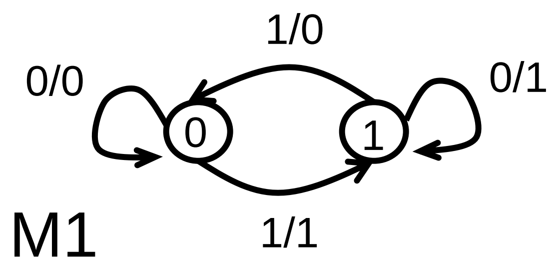
Construction of the Product FSM

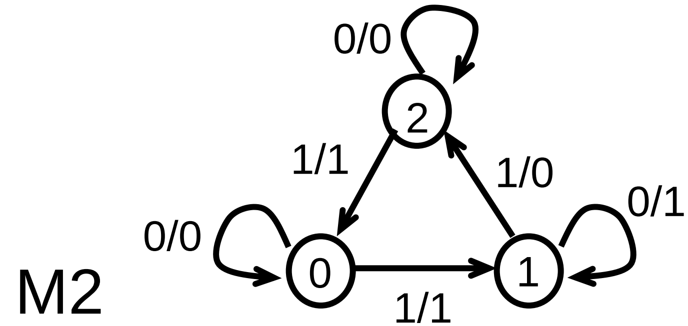


- For each pair of states, $s_1 \in M_1$, $s_2 \in M_2$
  - ➢ create a combined state s = (s_1. s_2) of M
  - ➢ create transitions out of this state to other states of M
  - ➢ label the transitions (*input/output*) accordingly



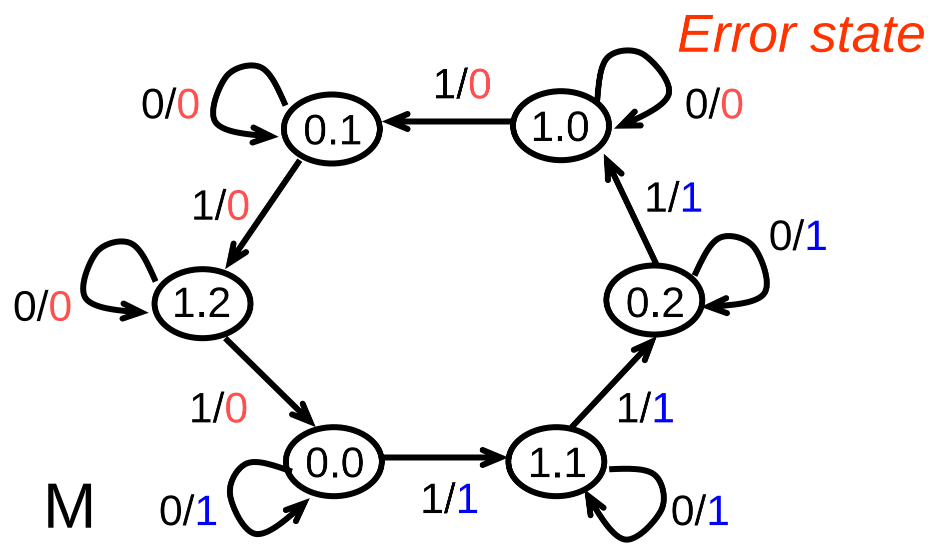
FSM Traversal in Action





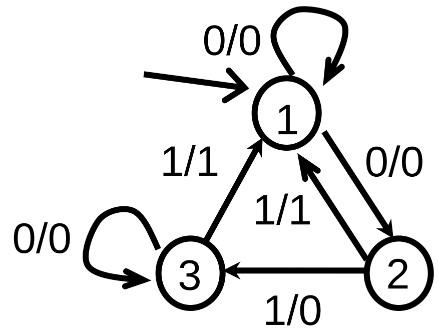
Initiall states: s_1=0, s_2=0, s=(0.0)

State reached	*Out(M)* *x=0*	*x=1*
• *New* ^{0} = (0.0)	1	1
• *New* ^{1} = (1.1)	1	1
• *New* ^{2} = (0.2)	1	1
• *New* ^{3} = (1.0)	0	0

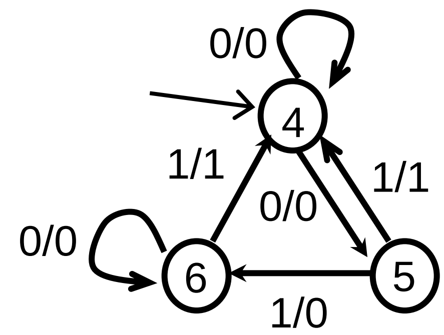


- STOP - backtrack to initial state to get *error trace: x*={1,1,1,0}

FSM Traversal in Action



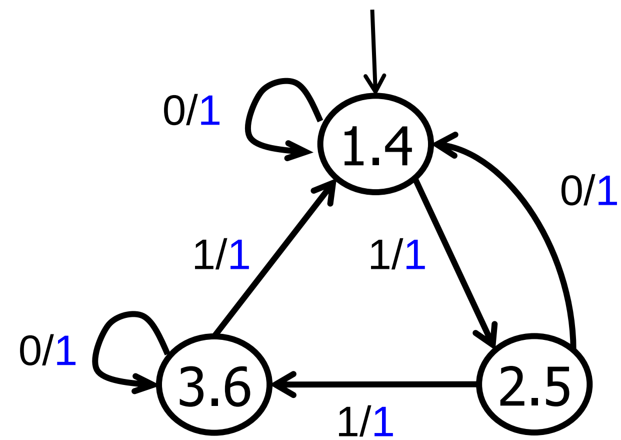
M_1



M_2

Initiall states: s_1=1, s_2=4, s=(1.4)

State reached	*Out(M)* *x=0*	*x=1*
• *New* ^{0} = (1.4)	1	1
• *New* ^{1} = (2.5)	1	1
• *New* ^{2} = (3.6)	1	1

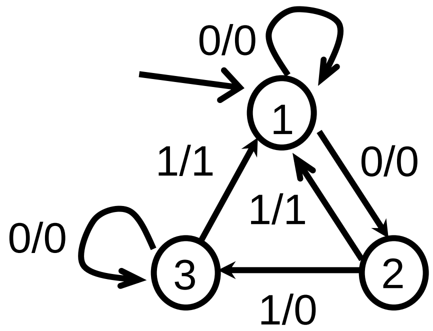


$M = M_1 x M_2$

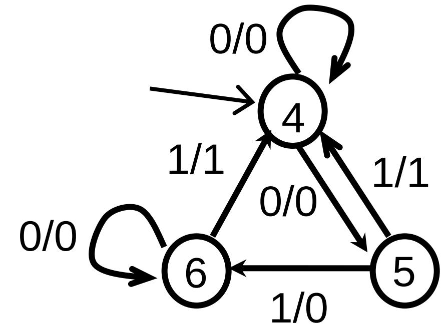
- STOP: No new reachable state

FSM Traversal in Action

M$_1$



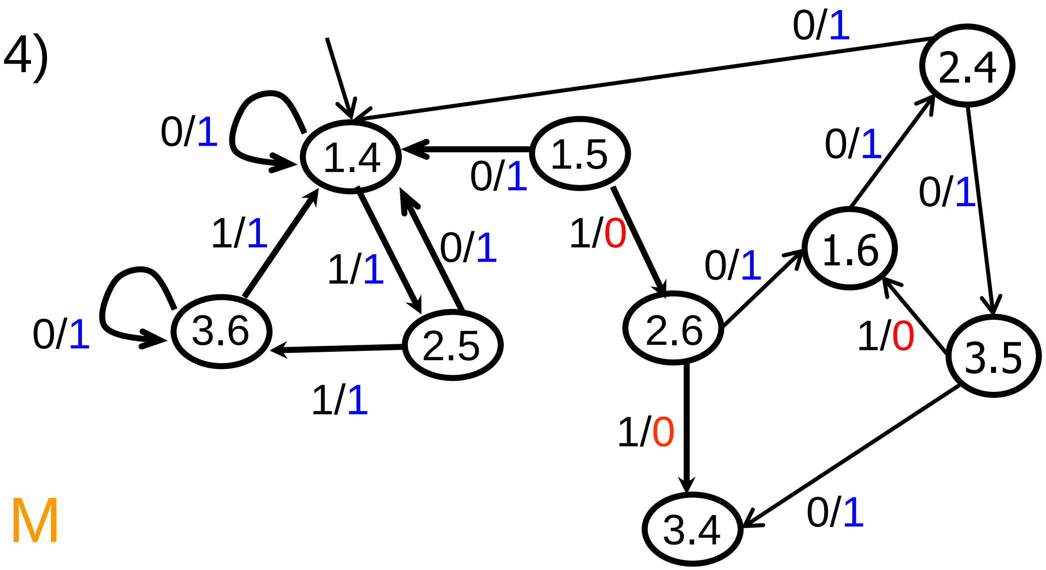
M$_2$



Initiall states: s_1=1, s_2=4, s=(1.4)

State reached	*Out(M)* *x=0*	*x=1*

- *New* ^{0} = (1.4) 1 1
- *New* ^{1} = (2.5) 1 1
- *New* ^{2} = (3.6) 1 1



M

- Erroneous states are not reachable

How to Represent States ?

- Not practical to represent individual states
- Represent s set of states symbolically
- OBDD encodes boolean functions
  - Code elements S
- Represent a subset T as boolean function f_T
- $f_T = \{0,1\}^n \rightarrow \{0.1\}$

How to Represent States ?

- Computation uses symbolic BFS approach to all reachable states by shortest path
- Key step is image computation
  - Img($\delta(s,x)$, C(s))
- BFS allows to deal multiple states simultaneously
- BDD is used to represent TF
- Let $t_i = \delta_i(s,x)$ $i = 1,2,\ldots n$
- C(s) is a symbolic state set

How to Represent TF ?

- Given a deterministic transition function (s,x) the corresponding transition relation is defined by
  - $T(s,x,t) = \Pi\ (t_i = \delta_i(s,x))$
- T(s,x,t) = 1 denotes a set of encoded tripples (s,x,t) , each representing a transition in the FST of a given FSM
- Straight forward to compute image
- Need new boolean operation
  - Existential Abstraction
  - $\exists_{xi}.f = f_{xi} + f_{x'i}$
  - f_{xi}- smallest (fewest minterm) function that contains all minterms of f and independent of xi

Thank you