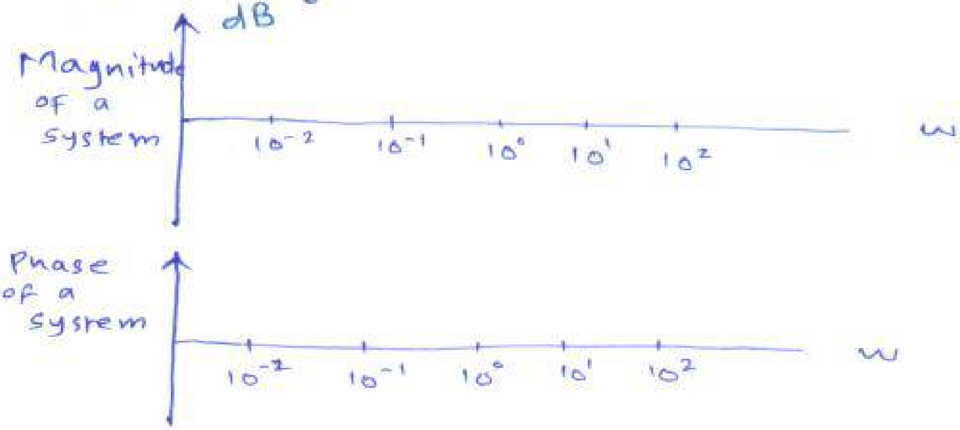
In 1930, Hendrik Bode found the way of plotting frequency response of a system using asymptotic approximation.

Bode plot contains magnitude plot and Phase plot.

We are interested in large variation in frequency 'w'.



In Electrical circuits, we deal with voltage gains or current gains. The Bode's magnitude plot deals with the ratio of powers.

The large deviation in gain is captured by using magnitude in log domain.

Bode's magnitude uses deciBel (dB) as a unit which is $\frac{1}{10th}$ of Bel.

$\therefore$ Ratio of power is converted to log using 10 log (Power ratio)

Power $\propto V^2$ and $\propto I^2$

$\therefore$ The log domain representation of amplitude & gain will need

$20 \log_{10}$ [Amplitude gain]

Let's consider a s-domain system tansfer function : $H(s)$

$$V_i(s) \longrightarrow \boxed{H(s)} \longrightarrow V_o(s)$$

Replace s by jw :

$$H(jw) = \frac{V_o(jw)}{V_i(jw)}$$

$\therefore$ Magnitude of $H(jw)$ in dB =

$$20 \log |H(jw)|$$

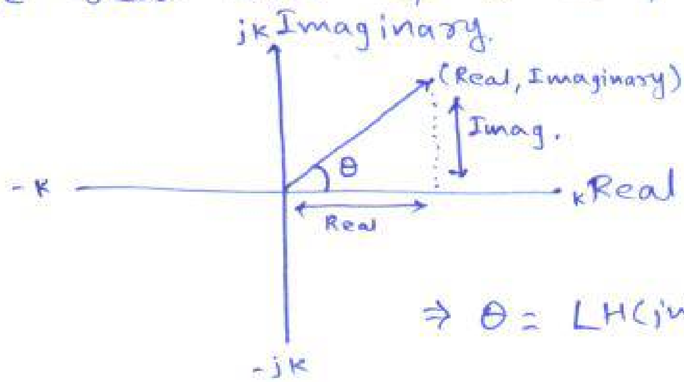
Phase of $H(jw)$ is calculated in degrees or radians.

Basics required for Bode plot :

① Pole : Value of 's' which contributes to $H(s) \rightarrow \infty$ i.e. denominator of $H(s) \rightarrow 0$.

② Zero : Value of 's' which contributes to $H(s) \rightarrow 0$.

$\angle H(jw)$ is always calculated with positive real axis of a complex plain :



$$\Rightarrow \theta = \angle H(jw) = \tan^{-1}\left(\frac{\text{Imag.}}{\text{Real}}\right)$$

Here, $H(jw) = \text{Real} + j\,(\text{Imag.})$

Let's start with examples :

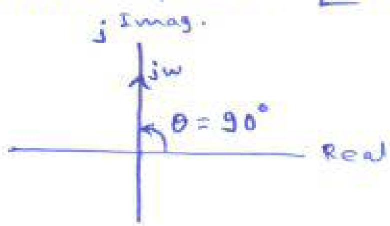
- $H(s) = s \Rightarrow H(s) = 0$ @ $s = 0$.

$\therefore$ There is a zero at origin

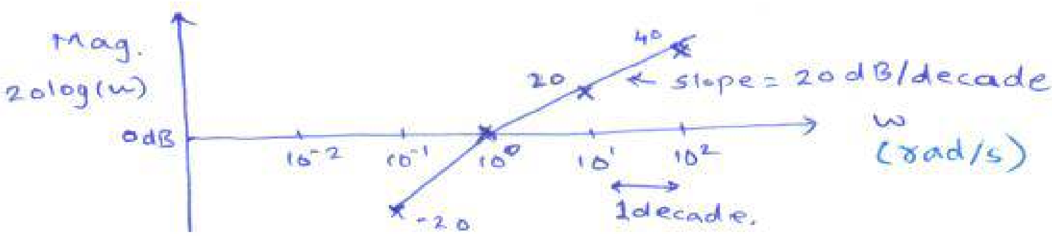
Magnitude Plot : $|H(j\omega)| = |j\omega| = \omega$

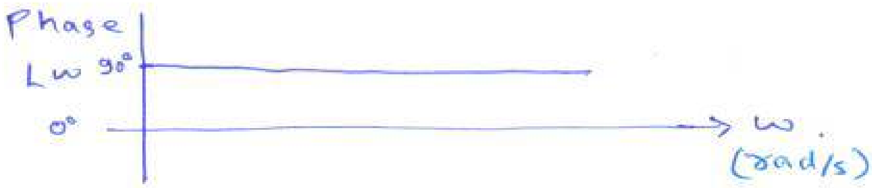
In dB $\Rightarrow 20\log_{10}(\omega)$

Phase plot : $\angle H(j\omega) = \angle j\omega$



$= \tan^{-1}\left(\frac{\omega}{0}\right) = 90°$



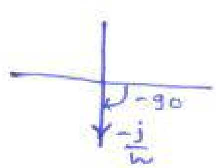


$\therefore$ Zero introduces 20dB/dec. line in magnitude plot and adds constant phase of $90°$.

Similarly, you can find Bode plot for a pole at origin i.e., $H(s) = \frac{1}{s}$

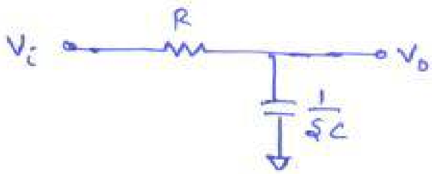
Hint : $20\log\left(\frac{1}{\omega}\right) = -20\log(\omega) \Rightarrow$ slope of -20dB/dec.

and



$\Rightarrow \angle H(j\omega) = -90°$

Let's consider following circuit:



$\Rightarrow H(s) = \frac{V_o(s)}{V_i(s)}$

Solving $\Rightarrow H(s) = \frac{1}{1 + sRC}$

$H(s) \to \infty$ when $s \to -\frac{1}{RC}$

$\therefore$ There is a pole at $1/RC$.

For simplicity, consider $\frac{1}{RC} = 10$.

$\therefore H(s) = \frac{1}{1 + s/10} \Rightarrow H(j\omega) = \frac{1}{1 + j\omega/10}$

As said earlier, Bode plot deals with asymptotic approximations. Let's use the following ~~appr~~approximations:

① $\omega << 10 \Rightarrow H(j\omega) \to 1$

Magnitude $\Rightarrow 20\log(1) = 0\,dB$

Phase $\Rightarrow \tan^{-1}\left(\frac{0}{1}\right) = 0$ deg.

$H(j\omega)$ approaches these values for ω ten times smaller than pole freq.

② $\omega = 10 \Rightarrow H(j\omega) = \frac{1}{1+j}$

Magnitude : $20\log\left(\frac{1}{\sqrt{2}}\right) = -3\,dB \to$ Near $0\,dB$

Phase : $-\tan^{-1}(1) = -45°$.

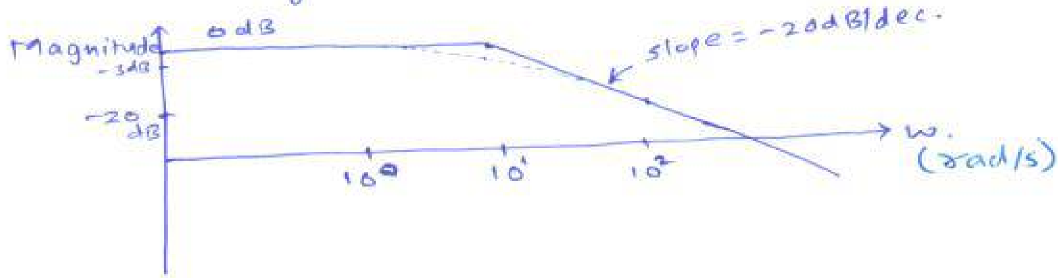
③ $\omega >> 10 \Rightarrow H(j\omega) \to \frac{10}{j\omega} = \frac{-10j}{\omega}$

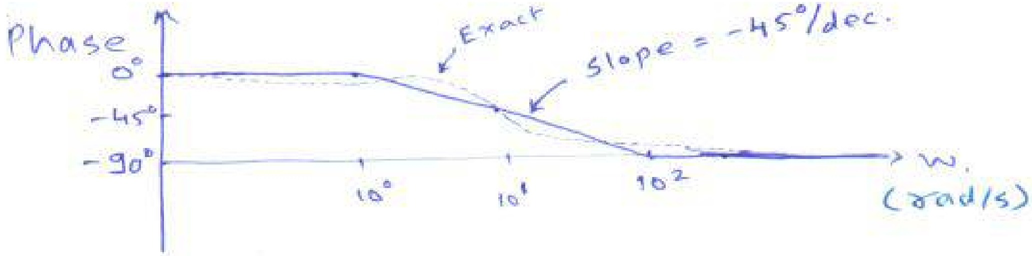
$\Rightarrow$ Mag. drops with slope of $-20\,dB/dec$.

& Phase is $-90°$.

Phase approaches $-90°$ for ~~phase~~ ω ten times larger than a pole freq.

Combining three calculations and drawing asymptotes:

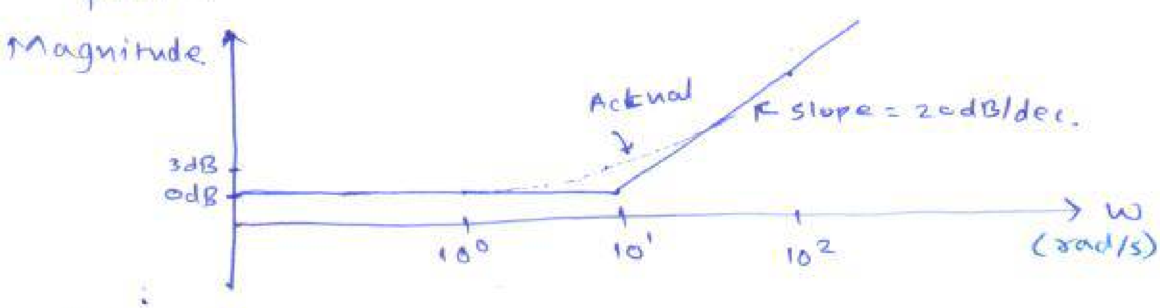


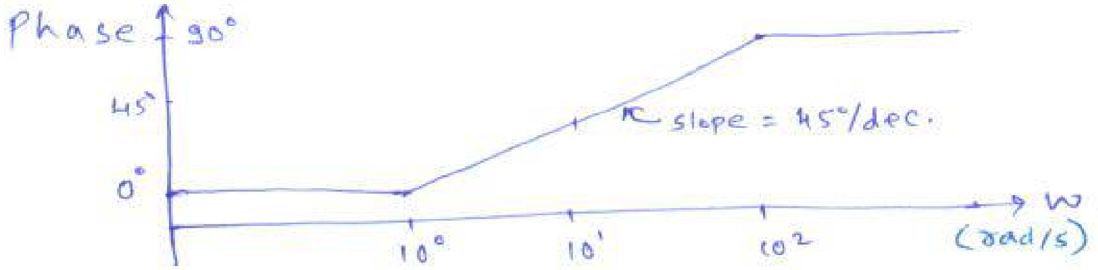


Insights: ① Pole drops the gain with a slope of -20dB/dec. beyond pole Freq.

② Pole degrades the phase by -90° with a degraddtion slope -45°/dec. around pole frequency.

Similarly, you can do same math with $H(s) = 1 + s/10$ get the following plot:



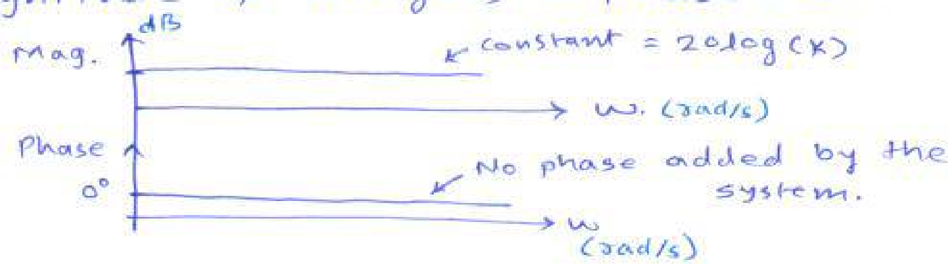


Insights: ① For a frequencies more than a zero freq., gain raises with a slope of +20dB/dec.

② Zero adds a phase of 90° for frequencies higher than a zero freq. The slope is 45°/dec. around a zero freq.

Let's look at a bode plot of a constant numer. 'K'.

Magnitude ⇒ $20\log(K)$ & Phase = 0°.



∴ Constant term shifts magnitude plot by $20\log(K)$ and adds no phase.

Using this understanding, we can now draw a Bode plot of any systems with combination of poles and zeros.

* Combining different terms:

Let $H(s) = H_1(s) \cdot H_2(s)$

Magnitude: $|H(j\omega)| = |H_1(j\omega)| \cdot |H_2(j\omega)|$

$\Rightarrow$ In dB $= 20\log(H_1) + 20\log(H_2)$

Phase: H_1 & H_2 are complex numbers

∴ writing $H_1 = K_1 \angle \alpha = K_1 e^{j\alpha}$ and

$H_2 = K_2 \angle \beta = K_2 e^{j\beta}$.

$$\therefore H_1 \cdot H_2 = K_1 \cdot K_2 \, e^{j(\alpha+\beta)} = K_1 \cdot K_2 \angle(\alpha+\beta)$$

$$\therefore \angle H = \angle H_1 + \angle H_2$$

Let's consider $H(s) = \frac{10}{(1+ s/10)(1+ s/10^3)}$