EE 325: Probability and Random Processes

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1. For a sample space Ω, prove the following statements.
   (a) If $A \subseteq \Omega$, $\mathcal{F} = \{\phi, A, A^c, \Omega\}$ is a σ-field.
   (b) 2^Ω is a σ-field.
2. Let $\mathcal{F}$ be a σ-field of subsets of Ω and suppose the $B \in \mathcal{F}$. Show that $\mathcal{G} = \{A \cap B : A \in \mathcal{F}\}$ is a σ-field of subsets of B.
3. For events $A_1, \ldots, A_n$, prove that following.

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \cdots + (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots A_n)$$

$$P\left(\bigcap_{i=1}^{n} A_i\right) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cup A_j) + \sum_{i<j<k} P(A_i \cup A_j \cup A_k) - \cdots + (-1)^{n+1} P(A_1 \cup A_2 \cup \cdots A_n)$$

4. Let A_r, $r \geq 1$, be events such that $P(A_r) = 1$ for all r. Show that $P\left(\bigcap_{r=1}^{\infty} A_r\right) = 1$.