EE 325: Probability and Random Processes

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Spring 2013

Quiz 1 : **16 points** (75 min) February 8, 2013

Each question is worth 2 points.

1. Using the countable additivity property of a probability measure P, prove that for any increasing sequence of events $A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots$ the following equality holds.
$$P\left(\lim_{i\to\infty} A_i\right) = \lim_{i\to\infty} P(A_i)$$

2. We have five coins: two of them have heads on both sides, one of them has tails on both sides and two of them are fair coins. A coin is picked at random and tossed twice. What is the probability of heads showing up in both tosses?

3. Box 1 contains 3 white and 2 black balls. Box 2 contains 4 white and 6 black balls. A box is selected at random and a ball is chosen at random from it. If the chosen ball is black, what is the probability that box 1 was selected?

4. Let A and B be independent events. Show that
   (a) A and B^c are independent
   (b) A^c and B^c are independent

5. Consider a shuffled deck of 52 cards.
   (a) What is the probability that the top 13 cards are all spades?
   (b) What is the probability that the top 13 cards are all spades and the bottom 13 cards are all hearts?

6. A coin shows heads with probability p and tails with probability $1-p$. It is tossed twice where the outcome of the first toss is independent of the outcome of the second toss. Let X be the total number of heads which appear in the two tosses. What is the probability mass function of X? What is the distribution function of X?

7. Let X and Y be random variables having a joint distribution function $F(x,y) = P(X \leq x, Y \leq y)$. If $a < b$ and $c < d$, express $P(a < X \leq b, c < Y \leq d)$ in terms of F.

8. Consider the Monty Hall problem with four doors. One of the doors has a car behind it and the other three have goats. The car is equally likely to be behind any of the four doors. A contestant picks a door at random. The game show host then reveals two other doors which do not have the car. If the contestant always switches from his currently chosen door to the door which is not open, what is the probability that he wins the car?