

EE 325: Probability and Random Processes

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Quiz 2 : **16 points** (75 min)

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Each question is worth 2 points.

1. Let  $X$  be uniformly distributed in the interval  $[a, b]$ . Find the mean and variance of  $X$ .
2. Let  $X$  and  $Y$  be independent random variables with common probability distribution function  $F$  and probability density function  $f$ . Find the probability distribution functions and probability density functions of the following in terms of  $F$  and  $f$ .

(a)  $\max(X, Y)$

(b)  $\min(X, Y)$

3. Let  $X$  be uniformly distributed on  $[0, \frac{\pi}{2}]$ . Find the probability density function of  $Y = \sin X$ .
4. If  $U$  is uniformly distributed on  $[0, 1]$ , what are the probability mass and probability distribution functions of  $X = \lfloor nU \rfloor + 1$  where  $n$  is a fixed positive integer and  $\lfloor y \rfloor$  is the largest integer less than or equal to  $y$ ?
5. Specify a method to generate a random variable with Rayleigh distribution which is a continuous random variable with probability distribution function given by

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\frac{x^2}{2\sigma^2}} & \text{otherwise} \end{cases}$$

where  $\sigma$  is a known parameter.

6. Let  $X$  and  $Y$  have joint probability density function  $f(x, y) = 2e^{-x-y}$ ,  $0 < x < y < \infty$ . Find the expected values of  $X$  and  $Y$ .
7. If  $X$  and  $Y$  are independent standard Gaussian random variables, derive the density function of  $X + Y$ .
8. A point's location in the two-dimensional plane is given by the ordered pair  $(X, Y)$  where  $X$  and  $Y$  are independent Gaussian random variables with mean  $A$  and variance  $\sigma^2$ . What is the probability that the point **does not** lie in the first quadrant?

