EE 325: Probability and Random Processes

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Quiz 3 : **16 points** (75 min) April 12, 2013

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1. [**4 points**] Suppose the input X and output Y to a channel are related by $Y = \rho X + N$ where N is a zero-mean Gaussian random variable with variance σ^2 and ρ is a random variable independent of the noise. Assume that X is equally likely to be $\pm A$. Our goal is to decide on the value of X given the observation Y.

   (a) If ρ is the constant 1, what is the optimal decision rule and the resulting decision error probability?

   (b) If ρ takes values ± 1 with equal probability, what is the optimal decision rule and the resulting decision error probability?

2. [**2 points**] Find the maximum likelihood decision rule for the following 3-ary hypothesis testing problem where $\mu = \sqrt{2}\sigma$.

$$\begin{aligned} H_1 &: Y \sim N(-\mu, \sigma^2) \\ H_2 &: Y \sim N(0, e^2\sigma^2) \\ H_3 &: Y \sim N(\mu, \sigma^2) \end{aligned}$$

   Hint: Sketch the density functions keeping in mind that the variances are unequal.

3. [**4 points**] Suppose we observe $Y_i,\ i = 1, 2, \ldots, M$ such that

$$Y_i \sim N(\mu, \sigma^2)$$

   where the Y_i's are independent.

   (a) If μ is **unknown** and σ is **known**, derive the maximum likelihood estimator of μ.

   (b) If μ is **known** and σ is **unknown**, derive the maximum likelihood estimator of σ^2.

4. [**2 points**] A random variable has generating function $G(s) = \frac{3s}{4-s}$. Find its mean and variance.

5. [**2 points**] Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed (iid) random variables with common generating function $G_X(s)$. Let N be a positive integer-valued random variable which is independent of the X_i's and has generating function $G_N(s)$. If $S = X_1 + X_2 + \cdots + X_N$, **derive** the generating function of S in terms of G_X and G_N.

6. [**2 points**] Suppose X and Y are independent random variables with characteristic functions

$$\begin{aligned} \phi_X(t) &= \exp\left(i5t - 5t^2\right) \\ \phi_Y(t) &= \exp\left(i6t - 4t^2\right) \end{aligned}$$

   respectively. Find the characteristic function of $3X + 4Y + 5$.