Conditional Probability

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Conditional Probability

Definition
If $P(B) > 0$ then the conditional probability that A occurs given that B occurs is defined to be

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Examples

- Two fair dice are thrown. Given that the first shows 3, what is the probability that the total exceeds 6?

- A family has two children. What is the probability that both are boys, given that at least one is a boy?

- A family has two children. What is the probability that both are boys, given that the younger is a boy?

- A box has three white balls w_1, w_2, and w_3 and two red balls r_1 and r_2. Two random balls are removed in succession. What is the probability that the first removed ball is white and the second is red?

Law of Total Probability

Theorem
For any events A and B such that $0 < P(B) < 1$,

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c).$$

More generally, let $B_1, B_2, \ldots, B_n$ be a partition of Ω such that $P(B_i) > 0$ for all i. Then

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

Examples

- Box 1 contains 3 white and 2 black balls. Box 2 contains 4 white and 6 black balls. If a box is selected at random and a ball is chosen at random from it, what is the probability that it is white?

- We have two coins; the first is fair and the second has heads on both sides. A coin is picked at random and tossed twice. What is the probability of heads showing up in both tosses?

Bayes' Theorem

Theorem

For any events A and B such that $P(A) > 0$, $P(B) > 0$,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

If $A_1, \ldots, A_n$ is a partition of Ω such that $P(A_i) > 0$ and $P(B) > 0$, then

$$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}.$$

Examples

- Box 1 contains 3 white and 2 black balls. Box 2 contains 4 white and 6 black balls. A box is selected at random and a ball is chosen at random from it. If the chosen ball is white, what is the probability that box 1 was selected?

- We have two coins; the first is fair and the second has heads on both sides. A coin is picked at random and tossed twice. If heads showed up in both tosses, what is the probability that the coin is fair?

Questions?