EE 325: Probability and Random Processes (Spring 2014)
Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay

Assignment 1: 20 points — Due Date: January 31, 2014

---

1. [5 points] Let A and B be independent events. Show that
   (a) A and B^c are independent
   (b) A^c and B^c are independent

2. [5 points] Consider the Monty Hall problem with four doors. One of the doors has a car behind it and the other three have goats. The car is equally likely to be behind any of the four doors. A contestant picks a door at random. The game show host then reveals one of the other doors which do not have the car. If the contestant always switches from his currently chosen door to one of the two doors which are not open, what is the probability that he wins the car? Assume that both the host and contestant choose randomly when faced with multiple choices for doors.

3. [5 points] Suppose an encoder maps a 0 bit to a binary codeword $\mathbf{v}_0$ of length n and maps a 1 bit to a binary codeword $\mathbf{v}_1$ of length n. The codewords are passed through a binary symmetric channel with crossover probability p. Suppose $\mathbf{r}$ is the received word corresponding to a single transmitted codeword. If $\mathbf{v}_0$ and $\mathbf{v}_1$ share the same prefix[1] of length $k < n$, show that the MAP decoder can ignore the first k bits in the received word $\mathbf{r}$. Assume that the probability of a 0 bit appearing at the input to the encoder is π_0 and the probability of a 1 bit appearing at the input to the encoder is π_1. The maximum a posteriori (MAP) decoding rule is as follows:

   Decide $b = 0$ if $P(0 \text{ sent}|\mathbf{r} \text{ received}) > P(1 \text{ sent}|\mathbf{r} \text{ received})$

   Decide $b = 1$ if $P(0 \text{ sent}|\mathbf{r} \text{ received}) \leq P(1 \text{ sent}|\mathbf{r} \text{ received})$

4. [5 points] If n distinguishable balls are placed in n distinguishable cells, show that the probability that exactly one cell remains empty is $\frac{\binom{n}{2}n!}{n^n}$. *Hint:* Enumerate the events for $n = 3$ to understand the problem better.

---

[1]For instance, codewords 01011 and 01001 share a prefix of length 3