EE 325: Probability and Random Processes (Spring 2014)
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Assignment 3: 20 points — Due Date: March 21, 2014

1. [5 points] Suppose observations Y_i, $i = 1, 2, \ldots, N$ are Poisson distributed with parameter λ. Assume that the Y_i's are independent.
   (a) Derive the ML estimator for λ.
   (b) Find the mean and variance of the ML estimate.

2. [5 points] Suppose we observe a sequence of random variables $Y_1, Y_2, \ldots, Y_n$ given by
$$Y_k = \theta s_k + N_k, \quad k = 1, 2, \ldots, n$$
   where the N_k's are independent zero-mean Gaussian random variables with variance σ^2. The sequence $s_1, \ldots, s_n$ is a known signal sequence and θ is an unknown parameter.
   (a) Find the maximum likelihood estimate $\hat{\theta}_{ML}(\mathbf{Y})$ of the parameter θ.
   (b) Find the mean and variance of $\hat{\theta}_{ML}(\mathbf{Y})$.

3. [5 points] Specify a method to generate a random variable with Rayleigh distribution which is a continuous random variable with probability distribution function given by
$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\frac{x^2}{2\sigma^2}} & \text{otherwise} \end{cases}$$
   where σ is a known parameter.

4. [5 points] Let F and G be the distribution functions of random variables X and Y respectively. Generate a random variable whose distribution function is $\alpha F + (1-\alpha) G$ for a fixed α such that $0 \leq \alpha \leq 1$?