EE 325: Probability and Random Processes (Spring 2014)
Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay

Practice Problem Set 1 January 15, 2014

1. Consider the collection $\mathcal{F} = \left\{ A \subset \mathbb{N} \middle| A \text{ is a finite set} \right\}$. Which of the following statements are true?
   (a) If $A, B \in \mathcal{F}$, then $A \bigcup B \in \mathcal{F}$.
   (b) If $A_1, A_2, A_3, \cdots \in \mathcal{F}$, then $\bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$.
   (c) $\mathcal{F}$ is a σ-field.

2. Repeat the previous question with $\mathcal{F} = \left\{ A \subset \mathbb{N} \middle| \text{Either } A \text{ or } A^c \text{ is finite} \right\}$.

3. Suppose $\Omega = \{1, 2, 3\}$ and $\mathcal{F} = 2^{\Omega}$. Find necessary and sufficient conditions on real numbers x, y, z such that there exists a countably additive probability measure P satisying $x = P(\{1, 2\})$, $y = P(\{2, 3\})$ and $z = P(\{1, 3\})$.

4. Let $\mathcal{B} = \left\{ b_1 b_2 b_3 \cdots \middle| b_i = 0 \text{ or } 1 \text{ for } i \in \mathbb{N} \right\}$ be the set of all infinite binary strings. Show that $\mathcal{B}$ is uncountable.

5. Determine if each of the following binary relations on $\mathbb{Z}$ are equivalence relations. If yes, determine the equivalence class of the integer 1.
   (a) $x \sim y \iff |y - x| \leq 5$
   (b) $x \sim y \iff |y| \leq 5$ and $|x| \leq 5$
   (c) $x \sim y \iff |x| \geq |y|$

6. Let $\Omega = [0, 1]$ and $\mathcal{F} = 2^{\Omega}$. Consider $P : \mathcal{F} \to [0, 1]$ such that $P(\phi) = 0$, $P(\Omega) = 1$ and $P(A) = 0$ for every nonempty proper subset A of Ω. Is P a valid probability measure? If yes, it is defined for all subsets of Ω. Why is this not a contradiction?