EE 325: Probability and Random Processes (Spring 2014)
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Practice Problem Set 5   April 7, 2014

1. A random variable has generating function $G(s) = \frac{3s}{4-s}$. Find its mean and variance.

2. Suppose X and Y are independent random variables with characteristic functions

$$\phi_X(t) = \exp\left(i5t - 5t^2\right)$$
$$\phi_Y(t) = \exp\left(i6t - 4t^2\right)$$

   respectively. Find the characteristic function of $3X + 4Y + 5$.

3. Let U be uniformly distributed on $[0,1)$. For $m = 1, 2, \ldots$ and $j = 1, 2, \ldots, m$, define

$$Y_{m,j} = \begin{cases} 1 & \text{if } U \in [(j-1)/m, j/m) \\ 0 & \text{otherwise.} \end{cases}$$

   For any m and a given value of U, exactly one of the $Y_{m,j}$'s is 1 and all the others are 0. Here the sample space is $\Omega = [0,1)$ and for $\omega \in \Omega$, $U(\omega) = \omega$. The random variables $Y_{m,j}$ map ω to 0 or 1 depending on the value of $U(\omega)$.

   Construct the sequence of random variables X_n as

$$\begin{aligned}
&X_1 = Y_{1,1}, \\
&X_2 = Y_{2,1}, \quad X_3 = Y_{2,2}, \\
&X_4 = Y_{3,1}, \quad X_5 = Y_{3,2}, \quad X_6 = Y_{3,3}, \\
&X_7 = Y_{4,1}, \quad X_8 = Y_{4,2}, \quad X_9 = Y_{4,3}, \quad X_{10} = Y_{4,4} \\
&\cdots
\end{aligned}$$

   (a) Does the sequence X_n converge to zero almost surely? Explain why or why not.

   (b) Does the sequence X_n converge to zero in probability? Explain why or why not.

4. Consider the random process $X(t)$ resulting from sinusoid with random phase.

$$X(t) = A\cos\left(2\pi f_c t + \Theta\right)$$

   where A and f_c are constants and Θ is uniformly distributed on $[-\pi, \pi]$.

   (a) Show that $X(t)$ is wide-sense stationary.

   (b) Find the power spectral density of $X(t)$.

5. Consider the random process $X(t)$ resulting from an amplitude modulated pulse train given by

$$X(t) = \sum_{i=-\infty}^{\infty} A_i p(t - iT)$$

   where the A_i's are independent and identically distributed discrete random variables which are equally likely to be ± 1 and $p(t)$ is a unit pulse of duration T

$$p(t) = \begin{cases} 1 & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

   (a) Is the random process $X(t)$ strict-sense stationary? Why or why not?

   (b) Is the random process $X(t)$ wide-sense stationary? Why or why not?