EE 325: Probability and Random Processes (Spring 2014)
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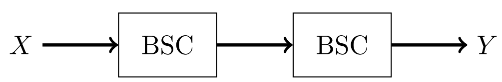
Quiz 1 : 16 points — February 7, 2014

1. (2 points) Let $B_1, B_2, \ldots$ be a decreasing sequence of events, so that $B_1 \supseteq B_2 \supseteq \cdots$. Let B be their limit
$$B = \bigcap_{i=1}^{\infty} B_i = \lim_{i \to \infty} B_i.$$
Prove that $P(B) = \lim_{i \to \infty} P(B_i)$.

2. (2 points) Prove the following:
   (a) If $P(B) = 1$, then $P(A|B) = P(A)$ for any A.
   (b) If $A \subset B$, then $P(B|A) = 1$ and $P(A|B) = \frac{P(A)}{P(B)}$.
   (c) If A and B are mutually exclusive, then $P(A|A \cup B) = \frac{P(A)}{P(A)+P(B)}$.
   (d) $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$.

3. (2 points) Consider a random bit X which is equally likely to be 0 or 1. It is passed through a cascade of two binary symmetric channels having crossover probabilities $\frac{1}{6}$ and $\frac{1}{8}$ respectively. Let the output be Y.
   (a) What is the probability of $X = 1$ given $Y = 0$?
   (b) What is the probability of $X = 0$ given $Y = 0$?



4. (2 points) A shipment of sixty shirts has six defective shirts. The quality control division wants to save time in checking for defective shirts. Instead of checking every shirt in the shipment, they choose ten shirts at random for testing and reject the entire shipment if one or more shirts are found to be defective. What is the probability that this shipment containing six defective shirts passes the inspection?

5. (2 points) We toss n coins, and each one shows heads with probability p, independently of the others. Each coin which shows heads is tossed again. What is the probability mass function of the number of heads resulting from the second round of tosses?

6. (2 points) Consider the following joint probability mass function for a pair of random variables X and Y which take values in the sets $\{x_1, x_2, x_3\}$ and $\{y_1, y_2, y_3\}$ respectively. Prove that X and Y are independent.

Y/X	x_1	x_2	x_3
y_1	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$
y_2	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{8}$
y_3	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{8}$

Table 1: Joint probability mass function $f_{X,Y}(x, y)$

7. (2 points) Let X be a discrete random variable such that $P(X = 1) = \frac{1}{2}$, $P(X = 2) = \frac{1}{4}$ and $P(X = 3) = \frac{1}{4}$. Let Y be a discrete random variable such that $P(Y = 2) = \frac{1}{6}$, $P(Y = 3) = \frac{2}{3}$ and $P(Y = 4) = \frac{1}{6}$. If X and Y are independent, sketch the probability distribution function and probability mass function of $\min(X, Y)$.

8. (2 points) Let X and Y be independent discrete random variables taking values in the positive integers. Both of them have the same probability mass function given by
$$P[X = k] = P[Y = k] = \frac{1}{2^k} \qquad \text{for } k = 1, 2, 3, \ldots$$
Find the following.
   (a) $P[X = Y]$
   (b) $P[Y > X]$