EE 325: Probability and Random Processes (Spring 2015)
Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay

Assignment 1: 25 points | Due Date: January 28, 2015

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1. [5 points] Consider the set B of all binary sequences given by

$$B = \{ b_1 b_2 b_3 b_4 \cdots \mid b_i \in \{0, 1\} \text{ for } i \in \mathbb{N} \}.$$

   Show that B is uncountable. *Hint: Use Cantor's diagnolization argument.*

2. [5 points] Let $A_1, A_2, A_3, A_4, \ldots$ be a sequence of countable sets. Show that $\bigcup_{i=1}^{\infty} A_i$ is countable.

3. [5 points] Consider the equivalence relation R on the interval $[0, 1]$ given by $x \sim y$ if $x - y$ is rational. This equivalence relation partitions $[0, 1]$ into disjoint equivalence classes. Let $H \subset [0, 1]$ be the set consisting of exactly one element from each of the equivalence classes. Show that H is uncountable. *Hint: Use the result from the previous problem.*

4. [5 points] Suppose $\Omega = [0, 1]$ and $\mathcal{F}$ is the set of subsets A of Ω such that either A or A^c is finite. Let $P : \mathcal{F} \mapsto [0, 1]$ be defined by $P(A) = 0$ if A is finite and $P(A) = 1$ if A^c is finite. Answer the following with **justification**.
   (a) Is $\mathcal{F}$ an algebra?
   (b) Is $\mathcal{F}$ a σ-algebra?
   (c) Is P finitely additive?
   (d) Is P countably additive?

5. [5 points] Consider the Monty Hall problem with four doors. One of the doors has a car behind it and the other three have goats. The car is equally likely to be behind any of the four doors. A contestant picks a door at random. The game show host then reveals one of the other doors which do not have the car. If the contestant always switches from his currently chosen door to one of the two doors which are not open, what is the probability that he wins the car? Assume that both the host and contestant choose randomly when faced with multiple choices for doors.