EE 325: Probability and Random Processes (Spring 2015)
Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay

Assignment 3: 25 points — Due Date: March 20, 2015

1. [5 points] Let Y and Z be independent random variables such that Z is equally likely to be 1 or -1 and Y is equally likely to be 1 or 2. Let $X = YZ$. Prove that X and Y are uncorrelated but not independent.

2. [5 points] Prove that if $E(X^2) = 0$ then $P(X = 0) = 1$.

3. [5 points] Prove that $P(X = c) = 1 \iff \text{var}(X) = 0$.

4. [5 points] For random variables X and Y, the Cauchy-Schwarz inequality says that

$$|E(XY)| \leq \sqrt{E(X^2)}\sqrt{E(Y^2)}$$

   Prove that equality holds if and only if $P(X = cY) = 1$ for some constant c. *If the discriminant of a quadratic equation is zero, it has a repeated root. Let c be the root and use question 2 above.*

5. [5 points] If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ are independent Gaussian random variables, show that $X_1 + X_2 \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. *Hints: What is the pdf of the sum of independent random variables? It is enough to show that $X_1 + X_2 - \mu_1 - \mu_2 \sim \mathcal{N}(0, \sigma_1^2 + \sigma_2^2)$.*