Mimblewimble

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Mimblewimble

Mimblewimble, which prevents your opponent from accurately casting their next spell.

Gilderoy Lockhart

- A tongue-tying curse from the Harry Potter universe
- A scalable cryptocurrency design with hidden amounts and obscured transaction graph
- Brief history
    - **Aug 2016:** "Tom Elvis Jedusor" posted an onion link to a text file describing Mimblewimble on bitcoin-wizards IRC channel
    - **Oct 2016:** Andrew Poelstra presents formalization of Mimblewimble at Scaling Bitcoin 2016
    - **Oct 2016:** "Ignotus Peverell" announces a project implementing the Mimblewimble protocol called Grin
    - **Jul 2018**: Another Mimblewimble implementation called BEAM announced
    - **Jan 2019:** BEAM launched on Jan 3, 2019 and Grin launched on Jan 15, 2019

Mimblewimble Outputs

- Recall the structure of Monero outputs
  - A public key P acting as destination address
  - A Pedersen commitment C to the amount stored in the output
  - A range proof proving the amount in C is in the right range
- Mimblewimble output structure
  - A Pedersen commitment C where

    $$C = kG + vH$$

    where G and H are generators of an elliptic curve of prime order n and the discrete logarithm of H wrt G is unknown
  - A range proof proving the amount in C is in a range like $\{0, 1, 2, \ldots, 2^{64} - 1\}$
- Features of Mimblewimble output variables
  - The order n is typically a 256-bit prime, i.e. $n \approx 2^{256}$
  - The scalar $v \in \mathbb{F}_n$ is the amount
  - The scalar $k \in \mathbb{F}_n$ is the blinding factor (will play role of **secret key**)

Proving Statements About Commitments

- How to prove that C is a commitment to the zero amount without revealing blinding factor?

  Ans: If $C = C(0, x) = xG$, then give a digital signature verifiable by C as the public key

  If C is a commitment to a non-zero amount a, signature with C as public key will mean discrete log of H is known

  $$C = xG + aH = yG \implies H = a^{-1}(y - x)G$$

- How to prove that C is a commitment to the an amount a without revealing blinding factor?

  Ans: If $C = C(a, x) = xG + aH$, then give a digital signature verifiable by $C - aH$ as the public key

- How to prove that two commitments C_1 and C_2 are commitments to the same amount a without revealing blinding factors?

  Ans:

  $$C_1 = C(a, x_1) = x_1 G + aH$$
  $$C_2 = C(a, x_2) = x_2 G + aH$$

  Give a digital signature verifiable by $C_1 - C_2$ as the public key

Proving the Balance Condition

- Suppose $C_1^{\text{in}}, C_2^{\text{in}}, C_3^{\text{in}}$ are commitments to input amounts a_1, a_2, a_3
- Suppose $C_1^{\text{out}}, C_2^{\text{out}}$ are commitments to output amounts b_1, b_2
- Suppose we want to prove

  $$a_1 + a_2 + a_3 = b_1 + b_2 + f$$

  for some public $f \geq 0$
- A digital signature with

  $$C_1^{\text{in}} + C_2^{\text{in}} + C_3^{\text{in}} - C_1^{\text{out}} - C_2^{\text{out}} - fH$$

  as public key is enough
- **Almost enough!** It only shows that

  $$\begin{aligned} a_1 H + a_2 H + a_3 H &= b_1 H + b_2 H + fH \\ \implies a_1 + a_2 + a_3 &= b_1 + b_2 + f \bmod n, \end{aligned}$$

  since $nH = \mathcal{O}$ (the identity of the elliptic curve group)

Preventing Exploitation of the Modular Balance Condition

$$a_1 + a_2 + a_3 = b_1 + b_2 + f \bmod n$$

- **Example:** $a_1 = 1, a_2 = 1, a_3 = 1$ and $b_1 = n - 4, b_2 = 6, f = 1$
- Typically $n \approx 2^{256}$ and amounts are in a smaller range like $\{0, 1, 2, \ldots, 2^{64} - 1\}$
- Proving that C_1^{out} and C_2^{out} commit to amounts in the range $\{0, 1, 2, \ldots, 2^{64} - 1\}$ solves the problem
- Each output should be accompanied by a range proof

Mimblewimble Transactions

- Each transaction has
  - L input commitments $C_1^{\text{in}}, C_2^{\text{in}}, \ldots, C_L^{\text{in}}$
  - M output commitments $C_1^{\text{out}}, C_2^{\text{out}}, \ldots, C_M^{\text{out}}$ with range proofs
  - N **transaction kernels**
  - A scalar $k_{\text{off}} \in \mathbb{F}_n$ called the **kernel offset**
- Each transaction kernel has the following
  - A scalar $f_i \in \mathbb{F}_n$ representing a fee
  - A curve point $X_i = x_i G$ called the **kernel excess**
  - A Schnorr signature verifiable with X_i as the public key
- For $f = \sum_{i=1}^{N} f_i$, the following equality is checked

$$\sum_{i=1}^{M} C_i^{\text{out}} + fH - \sum_{i=1}^{L} C_i^{\text{in}} = \sum_{i=1}^{N} X_i + k_{\text{off}} G$$

- This ensures

$$\sum_{i=1}^{L} v_i^{\text{in}} = \sum_{i=1}^{M} v_i^{\text{out}} + f \quad \text{and} \quad \sum_{i=1}^{M} k_i^{\text{out}} - \sum_{i=1}^{L} k_i^{\text{in}} = \sum_{i=1}^{N} x_i + k_{\text{off}}$$

- The offset k_{off} is used to hide relationship between specific inputs and outputs of a transaction during block creation

Schnorr Signature Algorithm

- Let $\mathcal{G}$ be a cyclic group of order q with generator G
- Let Hash : $\{0,1\}^* \mapsto \mathbb{Z}_q$ be a cryptographic hash function
- Signer knows $k \in \mathbb{Z}_q$ such that public key $P = kG$
- **Signer:**
  1. On input $m \in \{0,1\}^*$, chooses $r \leftarrow \mathbb{Z}_q$
  2. Computes nonce public key $R = rG$
  3. Computes $e = \text{Hash}(R\|P\|m)$
  4. Computes $s = r + ek \bmod q$
  5. Outputs (s, R) as signature for m
- **Verifier**
  1. On input m and (s, R)
  2. Computes $e = \text{Hash}(R\|P\|m)$
  3. Signature valid if $sG = R + eP$

Schnorr Signature Aggregation

- Suppose Alice and Bob want to create a 2-of-2 multisignature on a message
- Naïve signature aggregation
  - Alice and Bob reveal public keys P_a, P_b and nonce keys R_a, R_b
  - For $e = \text{Hash}(R_a + R_b \| P_a + P_b \| m)$, Alice and Bob respectively compute

$$s_a = r_a + ek_a$$
$$s_b = r_b + ek_b$$

  - Aggregate signature is $(s_a + s_b, R_a + R_b)$ with aggregate public key $P_a + P_b$
  - Signature valid if $(s_a + s_b)\,G = R_a + R_b + e\,(P_a + P_b)$
- Key cancellation attack
  - Bob can choose his public key and nonce key as $P_b' = P_b - P_a$ and $R_b' = R_b - R_a$
  - A valid signature for $P_a + P_b'$ only requires knowing k_b
  - **Solution:** Ask Bob to show signature for public key P_b'

Mimblewimble Transaction Construction

- Unlike other cryptocurrencies, sender and receiver have to **interact** to construct a Mimblewimble transaction
- Interaction can be via email, chat, forum posts
- Suppose Alice owns unspent output $C_{\text{in}} = k_A G + v_A H$
- She wants to send v_B coins to Bob where $v_B < v_A$
- She will be paying transaction fees f
- She wants the remaining $v_A - v_B - f$ coins to be stored in a change output $C_{\text{chg}} = k_C G + (v_A - v_B - f)H$
- Bob wants his new output to have blinding factor k_B, i.e. $C_{\text{out}} = k_B G + v_B H$
- Alice and Bob will exchange a data structure called a **slate**
- **Step 1**
  - Alice adds C_{in}, amount v_B, fees f to the slate
  - She chooses $k_C \xleftarrow{\$} \mathbb{F}_n$, calculates $C_{\text{chg}} = k_C G + (v_A - v_B - f)H$ and a range proof
  - She chooses kernel offset $k_{\text{off}} \xleftarrow{\$} \mathbb{F}_n$ and calculates the **sender kernel excess secret key** as $k'_A = k_C - k_A - k_{\text{off}}$
  - k_{off} and the **sender kernel excess** $X_A = k'_A G$ are added to the slate
  - She chooses nonce $r_A \xleftarrow{\$} \mathbb{F}_n$ and adds the nonce public key $R_A = r_A G$ to the slate.
  - Alice sends slate to Bob

Mimblewimble Transaction Construction

- **Step 2**
  - Bob chooses $k_B \xleftarrow{\$} \mathbb{F}_n$, calculates $C_{\text{out}} = k_B G + v_B H$ and a range proof. He adds C_{out} to the slate.
  - He adds **receiver kernel excess** $X_B = k_B G$ to the slate
  - He chooses nonce $r_B \xleftarrow{\$} \mathbb{F}_n$ and adds the nonce public key $R_B = r_B G$ to the slate.
  - Bob calculates the receiver Schnorr signature on message m as (s_B, R_B) where $s_B = r_B + ek_B$ and

    $$e = \text{Hash}(R_A + R_B \| X_A + X_B \| m).$$

    He adds the signature to the slate. It can be verified using the public key X_B.
  - Bob sends slate to Alice
- **Step 3**
  - Alice verifies Bob's signature (s_B, R_B) by checking the equality

    $$s_B G = R_B + eX_B,$$

  - She calculates the sender Schnorr signature (s_A, R_A) on the same message m as $s_A = r_A + ek'_A$
  - She sets the transaction kernel excess to be equal to $X_A + X_B$.
  - She sets the signature in the transaction kernel to be equal to $(s_A + s_B, R_A + R_B)$.

Mimblewimble Transaction Construction

- Alice broadcasts transaction k_{off}, C_{in}, C_{out}, C_{chg}, and the transaction kernel
- Kernel contains fee f, the kernel excess $X_A + X_B$, and the signature $(s_A + s_B, R_A + R_B)$
- Transaction satisfies

$$
\begin{aligned}
& C_{\text{out}} + C_{\text{chg}} + fH - C_{\text{in}} \\
&= k_B G + v_B H + k_C G + (v_A - v_B - f)H + fH - k_A G - v_A H \\
&= k_B G + (k_C - k_A)G \\
&= k_B G + (k_C - k_A - k_{\text{off}})G + k_{\text{off}} G \\
&= k_B G + k'_A G + k_{\text{off}} G = X_B + X_A + k_{\text{off}} G.
\end{aligned}
$$

- Alice does not learn Bob's blinding factor k_B
- Bob learns neither change amount $v_A - v_B - f$ nor blinding factor k_C

Mimblewimble Scalability

- Cut-through
  - Every Mimblewimble transaction satisfies

$$\sum_{i=1}^{M} C_i^{\text{out}} + fH - \sum_{i=1}^{L} C_i^{\text{in}} = \sum_{i=1}^{N} X_i + k_{\text{off}} G$$

  - Suppose T_1 and T_2 are waiting in the transaction mempool
  - If an output of T_1 is an input of T_2, it can be removed if T_1 and T_2 are included in the same block
- Pruning
  - If an output in a previous block is spent, it can be removed from the block
  - At any point, the following invariant holds

$$\sum_{i \in \text{UTXO}} C_i - (\text{all coins mined})\, H = \sum_{j \in \text{all kernels}} X_j + k_{\text{off}} G$$

  - To verify the above equation, spent outputs are not needed
- Grin team estimate: Assuming 10 million transactions with 100,000 UTXOs
  - 128 GB of Tx data, 1 GB proof data, 250 MB block headers
  - **After cut-through and pruning:** UTXO size 520 MB, 1 GB proof data, 250 MB block headers

References

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