EE 605: Error Correcting Codes

Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay
Autumn 2010

Assignment 5 : **35 points** **Due date**: October 25, 2010

Each of the following exercises is worth 5 points. Every nontrivial step in a proof should be accompanied by justification.

1. Calculate the weight distribution of the $(8, 7)$ single parity check code. Also calculate the probability of undetected error when this code is used on a BSC with crossover probability p.

2. Write down the systematic generator and parity check matrices for the $(15, 11)$ Hamming code. Let the parity check matrix be H. Consider a new parity check matrix H_1 formed by appending a column of zeros to the matrix H and then adding a row of ones. Show that the code with H_1 as the parity check matrix can correct single errors and detect double errors. What is the rate of this new code?

3. Suppose the Reed-Muller code RM(2,4) is used on a noisy channel. Decode the following three received vectors using the Reed decoding algorithm.

   (a) $(1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0)$

   (b) $(1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0)$

   (c) $(0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 1)$

4. Consider a (15,11) cyclic code generated by $g(X) = 1 + X + X^4$.

   (a) Determine the parity polynomial $h(X)$ of this code.

   (b) Determine the generator polynomial of its dual code.

   (c) Find the generator and parity check matrices in systematic form for this code.

5. Let $g(X)$ be the generator polynomial of a binary cyclic code of length n.

   (a) Show that if $g(X)$ has $X + 1$ as a factor, the code contains no codewords of odd weight.

   (b) If n is odd and $X + 1$ is not a factor of $g(X)$, show that the code contains a codeword consisting of all 1's.

   (c) Show that the code has minimum weight of at least 3 if n is the smallest integer such that $g(X)$ divides $X^n + 1$.

6. Show that $g(X) = 1 + X^2 + X^4 + X^6 + X^7 + X^10$ generates a $(21, 11)$ cyclic code. Devise a syndrome computation circuit for this code. Compute the syndrome of $r(X) = 1 + X^5 + X^{17}$.

7. Devise a systematic encoding circuit for the code described in the above question. Compute the codeword corresponding to the input $u(X) = 1 + X + X^3 + X^9$.