EE 605: Error Correcting Codes

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Assignment 2 : **20 points** **Due date**: August 30, 2011

Each of the following exercises is worth 5 points. Every nontrivial step in a proof should be accompanied by justification.

1. Construct the standard array and syndrome decoding table for the $(7, 4)$ linear block code with generator matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

2. A *burst error* of length l is an error pattern which causes l consecutive locations in the transmitted code to be corrupted. Let H be the parity check matrix of a binary linear block code.

   (a) What is the necessary and sufficient condition on the columns of H so that every burst error of length up to t can be detected?

   (b) What is the necessary and sufficient condition on the columns of H so that every burst error of length up to t can be corrected?

3. Let C be (n, k) linear code. Let T be a set of coordinates of the codewords i.e. $T \subseteq \{1, 2, \ldots, n\}$. Let C^T be the code obtained by puncturing C on the coordinates in T and let C_T be the code obtained by shortening C on the coordinates in T. Prove that

   (a) $(C^\perp)_T = (C^T)^\perp$

   (b) $(C^\perp)^T = (C_T)^\perp$

4. Let C_1 be a (n_1, k_1) binary linear block code with minimum distance d_1 and let C_2 be a (n_2, k_2) binary linear block code with minimum distance d_2. The direct sum of C_1 and C_2 is defined as

$$C_1 \oplus C_2 = \{(\mathbf{c}_1, \mathbf{c}_2) \| \mathbf{c}_1 \in C_1, \mathbf{c}_2 \in C_2\}.$$

   Show that $C_1 \oplus C_2$ is a $(n_1 + n_2, k_1 + k_2)$ linear block code with minimum distance $\min(d_1, d_2)$. Derive the generator matrix of $C_1 \oplus C_2$ in terms of the generator matrices of C_1 and C_2. Derive the parity check matrix of $C_1 \oplus C_2$ in terms of the parity check matrices of C_1 and C_2.