EE 605: Error Correcting Codes (Autumn 2014)
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Assignment 1: 20 points — Due Date: July 31, 2014

1. [5 points] Derive the optimal decoder for the n-repetition code used over a binary symmetric channel (BSC) with crossover probability $p < \frac{1}{2}$. Assume that the inputs are equally likely. Calculate the probability of decision error for this decoder. Here n can be any integer greater than 2 (even or odd).

2. [5 points] Write a program to simulate the performance of the optimal decoder for the n-repetition code over a BSC. Assume that the inputs are equally likely. For $n = 5$ and $n = 10$, generate plots containing both the theoretical probability of decision error and simulated probability of decision error. A link will be provided in Moodle for uploading the plots.

3. [5 points] Suppose a binary source generates bits which are equally likely to be 0 or 1. Suppose the source output is encoded by an n-repetition code, before transmission over a time-varying BSC which operates in the following manner. Given that $n = n_1 + n_2$, the time-varying BSC behaves like a regular BSC with crossover probability p_1 for the first n_1 bits which are transmitted through it and it behaves like a regular BSC with crossover probability p_2 for the remaining n_2 bits which are transmitted through it. If $p_1 + p_2 = 1$, what is the optimal decoding rule for this scenario? Can this rule be interpreted in terms of the optimal decoder for a regular BSC?

4. [5 points] Specify a decoder for the 5-repetition code which can *simultaneously* correct one error and detect two or three errors in a codeword. *Hint: Such a decoder is a partition of the set of binary 5-tuples into three parts.*