

1. [5 points] Let \mathbf{H} be the parity check matrix of a Hamming code of length $n = 2^m - 1$. Consider a matrix \mathbf{H}' obtained by removing all columns of even weight from \mathbf{H} . Let C be the code whose parity check matrix is \mathbf{H}' ?
 - (a) Find the length and dimension of C .
 - (b) Show that C can correct all single bit errors and detect all two-bit errors.
2. [5 points] Find the generator matrices corresponding to the following Reed-Muller codes.
 - (a) RM(1, 3)
 - (b) RM(2, 3)
 - (c) RM(1, 4)
3. [10 points] Suppose a codeword from the RM(2, 4) code is transmitted over a noisy channel and the vector $[1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0]$ is received. Write down the steps of majority-logic decoding and find the 11-bit transmitted message.