EE 605: Error Correcting Codes
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1. [3 points] Let C be a linear block code and $C^\perp$ be its dual code. A code is said to be *self-orthogonal* if $C \subseteq C^\perp$.
   (a) Prove that each codeword in a binary self-orthogonal code C has even weight and $C^\perp$ contains the all-ones codeword $\mathbf{1} = 111\cdots 1$.
   (b) Prove that if every codeword of a binary linear block code C has weight divisible by 4, then C is self-orthogonal.

2. [3 points] Let $\mathbf{H}$ be the parity check matrix of a Hamming code of length $n = 2^m - 1$. Consider a matrix $\mathbf{H}'$ obtained by removing all columns of even weight from $\mathbf{H}$. Let C be the code whose parity check matrix is $\mathbf{H}'$?
   (a) Find the length and dimension of C.
   (b) Show that C can correct all single bit errors and detect all two-bit errors.

3. [3 points] Find the generator matrices corresponding to the following Reed-Muller codes.
   (a) $\text{RM}(1,3)$
   (b) $\text{RM}(2,3)$
   (c) $\text{RM}(1,4)$

4. [3 points] Let C_1 and C_2 be two cyclic codes of same length n with generator polynomials $g_1(X)$ and $g_2(X)$ respectively. Show that $C_1 \cap C_2$ is a cyclic code. What is its generator polynomial?

5. [3 points] For an (n,k) binary cyclic code, show the following.
   (a) The fraction of undetectable bursts of length $n-k+1$ is $2^{-(n-k-1)}$.
   (b) For $m > n-k+1$, the fraction of undetectable bursts of length m is $2^{-(n-k)}$.

6. [5 points] Let C be a binary cyclic code of length n with generator polynomial $g(X)$. Let $r(X)$ be a received polynomial and let $r^{(1)}(X)$ be the polynomial corresponding to a single cyclic shift of $r(X)$. Let $s(X)$ be the syndrome corresponding to $r(X)$. Find the syndrome of $r^{(1)}(X)$ as a function of $s(X)$ and $g(X)$. Explain your answer.

7. [5 points] Show that the binary cyclic code of length 15 having generator polynomial $g(X) = X^4 + X^3 + 1$ is a Hamming code.

8. [5 points] Show that the binary linear block code with the following generator matrix is a cyclic code.

$$\begin{bmatrix}
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}$$