EE 703: Digital Message Transmission

Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay
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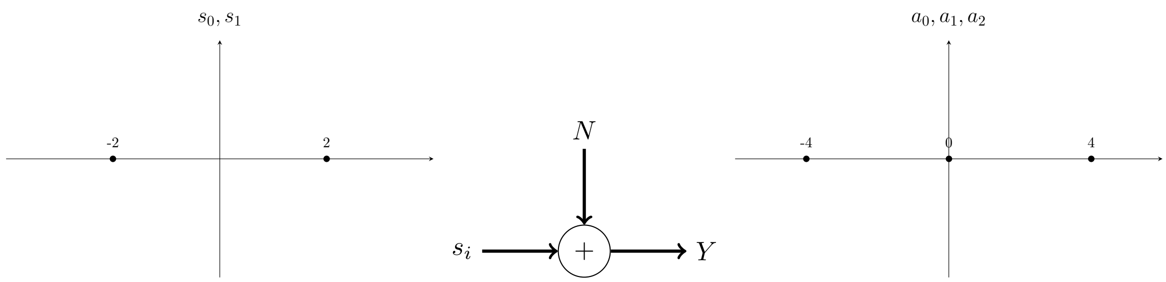
Midsemester Exam : **30 points** (120 min) September 14, 2012

Each question is worth 5 points.

1. The optimal receiver achieves a probability of decision error of $P_e = 0.01$ when the constellation $s_0 = -2, s_1 = 2$ is corrupted by noise N which is a zero mean Gaussian random variable. What is the probability of decision error when the constellation $a_0 = -4, a_1 = 0, a_2 = 4$ is transmitted over the same channel and the corresponding optimal receiver is used? Assume all constellation points are equally likely to be transmitted. Express your final answer in terms of the Q function and its inverse (numerical answer is not necessary).



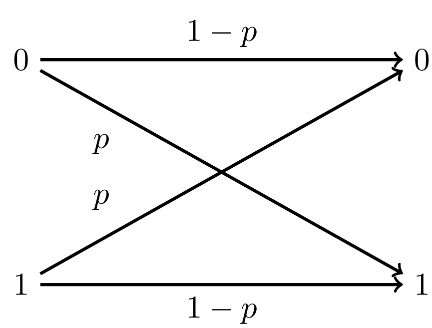
2. For the following hypothesis testing problem, show that Y_2 is an irrelevant statistic when $\rho = 0$ and a relevant statistic when $\rho \neq 0$.

$$\mathbf{Y} = \begin{bmatrix} Y_1 & Y_2 \end{bmatrix}^T$$

$$\begin{aligned} H_1 : &\quad Y_1 = A + N_1, \quad Y_2 = N_2 \\ H_0 : &\quad Y_1 = N_1, \qquad\quad Y_2 = N_2 \end{aligned}$$

where $A > 0$, N_1 and N_2 are jointly Gaussian random variables with zero mean and variance σ^2. The covariance matrix of $\mathbf{Y}$ is given by $\mathbf{C_Y} = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$.

3. Suppose the input to a binary symmetric channel with crossover probability p is equally likely to be 0 or 1. Derive the minimum probability of decision error which can be achieved for this channel as a function of p.



4. M signals $s_1(t), s_2(t), \ldots, s_M(t)$ which are nonzero for $0 \leq t \leq T$ are transmitted over an AWGN channel. Each signal is identical to all the others in the interval $[t_1, t_2]$ where $0 < t_1 < t_2 < T$. Show that the optimal receiver can ignore the signal received in the interval $[t_1, t_2]$ in taking its decision.

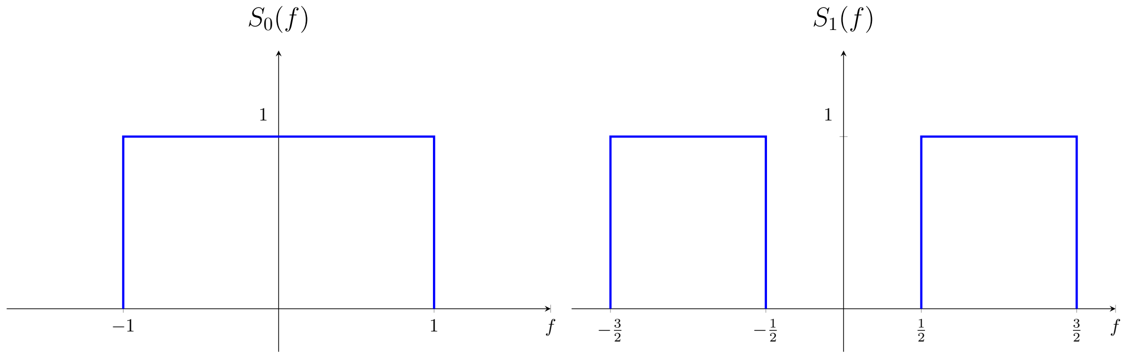


Figure 1: Fourier transforms of signals $s_0(t)$ and $s_1(t)$

5. Calculate the minimum probability of decision error which can be achieved when the equally likely signals shown in Figure 1 are sent over an AWGN channel with PSD $S_n(f) = 0.25$. $S_0(f)$ and $S_1(f)$ are the Fourier transforms of the signals $s_0(t)$ and $s_1(t)$ respectively.

6. A pulse $p(t)$ which is nonzero for $0 \leq t < T$ is transmitted through a channel which adds WGN $n(t)$ having PSD σ^2 and also induces a random delay D as shown in the figure below. If the delay D is equally likely to be 0, T or $2T$, what is the best estimator for the delay?