EE 703: Digital Message Transmission

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Quiz 2 : **12 points** (90 min) October 19, 2012

Each question is worth 4 points.

1. The in-phase and quadrature components of a bandpass random process $X_p(t)$ are given by

$$\begin{aligned} X_c(t) &= \frac{1}{\sqrt{2}}\left[X_p(t)\cos 2\pi f_c t + \hat{X}_p(t)\sin 2\pi f_c t\right] \\ X_s(t) &= \frac{1}{\sqrt{2}}\left[\hat{X}_p(t)\cos 2\pi f_c t - X_p(t)\sin 2\pi f_c t\right] \end{aligned}$$

   where $\hat{X}_p(t)$ is the Hilbert transform of $X_p(t)$. Show that the power spectral densities of $X_c(t)$ and $X_s(t)$ satisfy the following equation.

$$S_{X_c}(f) = S_{X_s}(f) = \begin{cases} \frac{1}{2}\left[S_{X_p}(f-f_c) + S_{X_p}(f+f_c)\right] & |f| < f_c \\ 0 & \text{otherwise} \end{cases}$$

2. The following set of eight signals is used to send three bits over a baseband AWGN channel with PSD $\frac{N_0}{2}$.

$$s_m(t) = A_m p(t), \quad 1 \le m \le 8$$

   where $p(t) = I_{[0,1]}(t)$ and

$$A_m = (2m - 1 - 8)A, \quad 1 \le m \le 8$$

   Assume that all the eight signals are equally likely to be transmitted.

   (a) Express E_b as a function of A.
   (b) Calculate the exact symbol error probability of the ML receiver as a function of E_b and N_0.
   (c) Calculate the intelligent union bound on the symbol error probability.
   (d) Calculate the nearest neighbor approximation to the symbol error probability.

3. The following set of eight signals is used to send three bits over a passband AWGN channel with PSD σ^2.

$$s_m(t) = \sqrt{2}A_m p(t)\cos 2\pi f_c t - \sqrt{2}B_m p(t)\sin 2\pi f_c t, \quad 1 \le m \le 8$$

   where $p(t) = I_{[0,1]}(t)$ and the values of A_m and B_m are given the following table.

m	A_m	B_m
1	$-3A$	$3A$
2	$-3A$	A
3	$-3A$	$-A$
4	$-3A$	$-3A$
5	$-A$	$-A$
6	$-A$	$-3A$
7	A	$-3A$
8	$3A$	$-3A$

   Assume that all the eight signals are equally likely to be transmitted.

   (a) Calculate the exact symbol error probability of the ML receiver as a function of A and σ.
   (b) Calculate the intelligent union bound on the symbol error probability.
   (c) Calculate the nearest neighbor approximation to the symbol error probability.