EE 703: Digital Message Transmission

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Quiz 3 : **12 points + 3 bonus points** (90 min) November 9, 2012

The first three questions are worth 4 points each. The last question is a bonus question. You can score full points in this quiz even if you don't attempt the bonus question.

1. (a) Suppose we observe $Y_i,\ i = 1, 2, \dots, M$ such that

$$Y_i \sim \text{Uniform}[-\theta, \theta]$$

where Y_i's are independent and θ is unknown. Assume $\theta \geq 0$. Derive the ML estimator of θ.

   (b) Suppose we observe $Y_i,\ i = 1, 2, \dots, M$ such that

$$Y_i \sim \mathcal{N}(\mu, \sigma^2)$$

where the Y_i's are independent, μ is **known** and σ^2 is **unknown**. Derive the ML estimator of σ^2.

2. The following set of eight signals is used to send three bits over a baseband AWGN channel with PSD $\frac{N_0}{2}$.

$$s_m(t) = A_m p(t), \quad 1 \leq m \leq 8$$

where $p(t) = I_{[0,1]}(t)$ and

$$A_m = (2m - 1 - 8)A, \quad 1 \leq m \leq 8$$

Assume that all the eight signals are equally likely to be transmitted.

   (a) Derive the power efficiency of this modulation scheme.

   (b) Specify a Gray coding bitmap for mapping each symbol to 3 bits. Is it unique? If not, specify one more Gray coding bitmap.

   (c) Calculate the conditional bit error probability of the ML receiver when $3A$ is transmitted as a function of E_b and N_0. Assume the Gray bitmap you specified in part (b) of this question.

3. Suppose we observe a sampled complex baseband signal given by

$$Y_i = s_i e^{j\theta} + N_i, \quad i = 1, \dots, M$$

where s_i is a known sequence, θ is unknown and $\mathbf{N} = \begin{bmatrix} N_1 & \cdots & N_M \end{bmatrix}^T$ is a proper Gaussian random vector with mean $\begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}^T$ and known covariance matrix $2\sigma^2 I$. Derive the ML estimator of θ. Recall that if $\mathbf{Z}$ is proper and Gaussian, its pdf is given by

$$p(\mathbf{z}) = \frac{1}{\pi^n \det(\mathbf{C_Z})} \exp\left(-(\mathbf{z} - \mathbf{m})^H \mathbf{C_Z}^{-1} (\mathbf{z} - \mathbf{m})\right)$$

4. **[Bonus Points Question]** Repeat question 1(a) when $Y_i \sim \text{Uniform}[-\theta, 2\theta]$.