EE 703: Digital Message Transmission

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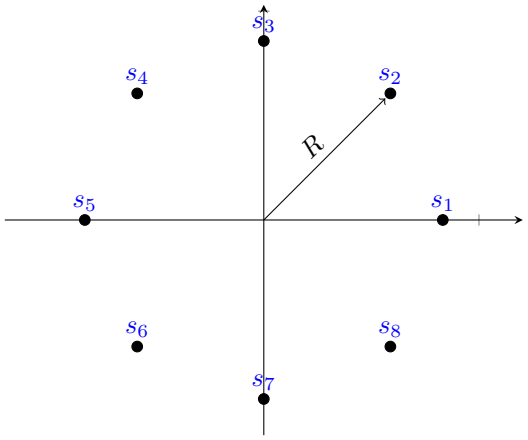
Quiz 3: **12 points** Date: November 11, 2013

1. Suppose that the 8-PSK constellation shown below is used over a complex AWGN channel with PSD N_0. Assume that all the eight signals are equally likely to be transmitted.



   (a) Express E_b as a function of R.
   (b) Derive the power efficiency of this modulation scheme. *Hint:* $\sin \frac{\pi}{8} \approx \frac{3}{8}$.
   (c) Calculate **any two** of the following three quantities as a function of E_b and N_0.
      i. The exact symbol error probability of the ML receiver.
      ii. The intelligent union bound on the symbol error probability of the ML receiver.
      iii. The nearest neighbor approximation to the symbol error probability of the ML receiver.

2. Suppose observations Y_i, $i = 1, 2, \ldots, N$ are Poisson distributed with parameter λ. Assume that the Y_i's are independent.

   (a) Derive the ML estimator for λ.
   (b) Find the mean and variance of the ML estimate.

   Recall that a Poisson distributed random variable with parameter λ has a probability mass function given by

$$\Pr(Y = n) = \frac{e^{-\lambda}\lambda^n}{n!}, n = 0, 1, 2, \ldots$$

   with mean and variance both equal to λ.

3. The following set of four signals is used to send two bits over a baseband AWGN channel with PSD $\frac{N_0}{2}$.

$$s_1(t) = -2Ap(t), s_2(t) = -Ap(t), s_3(t) = Ap(t), s_4(t) = 2Ap(t)$$

   where $p(t) = I_{[0,1]}(t)$ and Assume that all the four signals are equally likely to be transmitted.

   (a) Derive the power efficiency of this modulation scheme.
   (b) Specify a Gray code for mapping each symbol to 2 bits.
   (c) Calculate the bit error probability of the ML receiver in terms of E_b and N_0 assuming the Gray code in part (b).