EE 703: Digital Message Transmission (Autumn 2016)
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Assignment 3: 20 points — Date: September 27, 2016

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1. [5 points] Consider the following hypothesis testing problem.

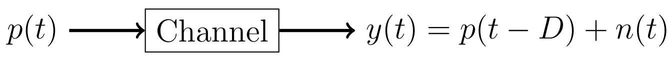
$$\begin{aligned} H_1 &: \quad Y_1 = A + N_1, \quad Y_2 = N_2 \\ H_0 &: \quad Y_1 = N_1, \qquad\quad Y_2 = N_2 \end{aligned}$$

where $A > 0$, $\mathbf{Y} \sim N(\mathbf{m}_i, \mathbf{C})$ under hypothesis H_i, $\mathbf{Y} = \begin{bmatrix} Y_1 & Y_2 \end{bmatrix}^T$, $\mathbf{m}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $\mathbf{m}_1 = \begin{bmatrix} A & 0 \end{bmatrix}^T$, $\mathbf{C} = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$. Show that Y_2 is a relevant statistic by deriving the following conditional densities.

$$\begin{aligned} p(y_2|y_1, H_0) &= \frac{1}{\sqrt{2\pi(1-\rho^2)\sigma^2}} e^{-\frac{(y_2 - \rho y_1)^2}{2(1-\rho^2)\sigma^2}}, \\ p(y_2|y_1, H_1) &= \frac{1}{\sqrt{2\pi(1-\rho^2)\sigma^2}} e^{-\frac{[y_2 - \rho(y_1 - A)]^2}{2(1-\rho^2)\sigma^2}} \end{aligned}$$

2. [5 points] M signals $s_1(t), s_2(t), \ldots, s_M(t)$ which are nonzero for $0 \le t \le T$ are transmitted over an AWGN channel. Each signal is identical to all the others in the interval $[t_1, t_2]$ where $0 < t_1 < t_2 < T$. Show that the optimal receiver can ignore the signal received in the interval $[t_1, t_2]$ in taking its decision.

3. [5 points] A pulse $p(t)$ which is nonzero for $0 \le t < T$ is transmitted through a channel which adds WGN $n(t)$ having PSD σ^2 and also induces a random delay D as shown in the figure below. If the delay D is equally likely to be 0, T or $2T$, what is the best estimator for the delay?



4. [5 points] Consider the M-ary hypothesis testing problem in AWGN where $s_i(t) = A_i p(t)$ for $A_i \in \mathbb{R}$ such that $A_1 < A_2 < \cdots < A_M$ and unit energy pulse $p(t)$ which is nonzero for $0 \le t \le T$.

$$\begin{aligned} H_1 &: \quad y(t) = s_1(t) + n(t) \\ H_2 &: \quad y(t) = s_2(t) + n(t) \\ &\ \ \vdots \qquad\qquad \vdots \\ H_M &: \quad y(t) = s_M(t) + n(t) \end{aligned}$$

If all the hypotheses are equally likely, show that the optimal receiver compares the output of a matched filter to a set of thresholds.