EE 703: Digital Message Transmission (Autumn 2020)
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Assignment 1: 20 points — Date: August 28, 2020

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1. [5 points] Let $\hat{s}_p(t)$ be the Hilbert transform of a passband signal $s_p(t)$. Show that $\langle s_p, \hat{s}_p \rangle = 0$.

2. [5 points] Suppose we define the complex envelope of a passband signal $s_p(t)$ centered at $\pm f_c$ as

$$S(f) = 2S_p(f - f_c)u(-f + f_c)$$

where $S_p(f)$ is the Fourier transform of $s_p(t)$. Derive the following with explanations for each step.

   (a) $s_p(t)$ in terms of $s(t)$

   (b) $s_p(t)$ in terms of $s_c(t)$ and $s_s(t)$ (the in-phase and quadrature components of $s(t)$)

   (c) $s(t)$ in terms of $s_p(t)$

   (d) $S_p(f)$ in terms of $S(f)$

   (e) The relationship between $\|s\|^2$ and $\|s_p\|^2$.

3. [5 points] Consider the passband signals $s_1(t) = \sqrt{2}\cos(2\pi f_1 t)$ and $s_2(t) = \sqrt{2}\cos(2\pi f_2 t)$ where $f_1 \neq f_2$. Calculate the complex baseband representations of these signals for $f_c = f_1$.

4. Let a random process be defined as $X(t) = A\cos(2\pi f_c t) + B\sin(2\pi f_c t)$ where f_c is a constant and A and B are independent **real** random variables with mean zero and variance σ^2. Assume that $E[A^3] \neq 0$ and $E[B^3] \neq 0$.

   (a) [1 point] Find the mean function of $X(t)$.

   (b) [1 point] Find the autocorrelation function of $X(t)$.

   (c) [1½ points] Prove or disprove the wide-sense stationarity of $X(t)$.

   (d) [1½ points] Prove or disprove the strict-sense stationarity of $X(t)$. *Hint: Try calculating* $E\left[X^3(t)\right]$.