

1. [10 points] Derive the ML estimator of the parameter p given M independent observations $Y_i \sim \text{Bernoulli}(p)$ where $i = 1, 2, \dots, M$.
2. [10 points] Suppose X and Y are jointly Gaussian random variables. Let the joint pdf be given by

$$p_{XY}(x, y) = \frac{1}{2\pi|\mathbf{C}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{s} - \boldsymbol{\mu})^T \mathbf{C}^{-1}(\mathbf{s} - \boldsymbol{\mu})\right)$$

where $\mathbf{s} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\boldsymbol{\mu} = \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$.

Suppose Y is observed and we want to estimate X . Derive the MMSE estimator of X .