BER Performance of ML Receiver

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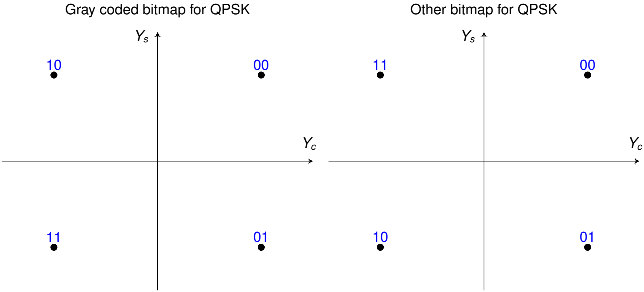
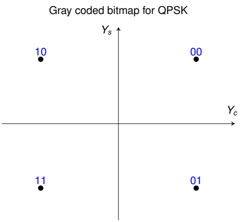
Bit Error Rate of ML Decision Rule

- Average probability of bit error is also called bit error rate (BER)
- For fixed SNR, symbol error probability in AWGN depends only on constellation geometry
- For fixed SNR, BER depends on both constellation geometry and the bits to signal mapping



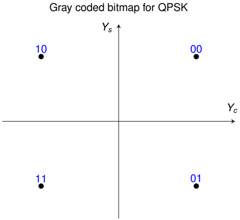
- For an M-ary constellation, number of possible bitmaps is $M! = M(M-1)\cdots 3\cdot 2\cdot 1$

Bit Error Rate for QPSK using Gray Bitmap



- Let $b[1]b[2]$ be the transmitted symbol
- Let $\hat{b}[1]\hat{b}[2]$ be the decoded symbol
- Let $P_1 = \Pr\left(\hat{b}[1] \neq b[1]\right)$ and $P_2 = \Pr\left(\hat{b}[2] \neq b[2]\right)$
- Average probability of bit error is $P_b = \frac{P_1+P_2}{2}$

Bit Error Rate for QPSK using Gray Bitmap

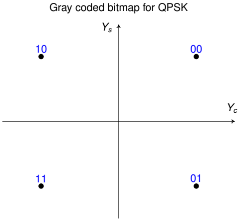


- Probability of making error on $b[1]$ when $b[1]b[2] = 00$ is

$$\begin{aligned} P_{1|00} &= \Pr\left[\hat{b}[1] = 1 \middle| b[1]b[2] = 00\right] \\ &= \Pr\left[\hat{b}[1]\hat{b}[2] = 10 \text{ or } \hat{b}[1]\hat{b}[2] = 11 \middle| b[1]b[2] = 00\right] \\ &= \Pr\left[Y_c < 0 \middle| b[1]b[2] = 00\right] = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \end{aligned}$$

- By symmetry, $P_1 = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

Bit Error Rate for QPSK using Gray Bitmap

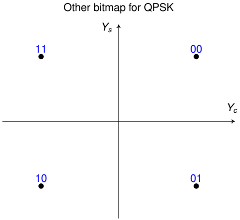


- Probability of making error on $b[2]$ when $b[1]b[2] = 00$ is

$$
\begin{aligned}
P_{2|00} &= \Pr\left[\hat{b}[2] = 1 \middle| b[1]b[2] = 00\right] \\
&= \Pr\left[\hat{b}[1]\hat{b}[2] = 01 \text{ or } \hat{b}[1]\hat{b}[2] = 11 \middle| b[1]b[2] = 00\right] \\
&= \Pr\left[Y_s < 0 \middle| b[1]b[2] = 00\right] = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)
\end{aligned}
$$

- By symmetry, $P_2 = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$. $P_b = (P_1 + P_2)/2 = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

Bit Error Rate for QPSK using Other Bitmap

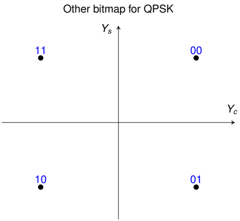


- Probability of making error on $b[1]$ when $b[1]b[2] = 00$ is

$$
\begin{aligned}
P_{1|00} &= \Pr\left[\hat{b}[1] = 1 \middle| b[1]b[2] = 00\right] \\
&= \Pr\left[\hat{b}[1]\hat{b}[2] = 10 \text{ or } \hat{b}[1]\hat{b}[2] = 11 \middle| b[1]b[2] = 00\right] \\
&= \Pr\left[Y_c < 0 \middle| b[1]b[2] = 00\right] = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)
\end{aligned}
$$

- Can we use symmetry? Yes. $P_1 = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

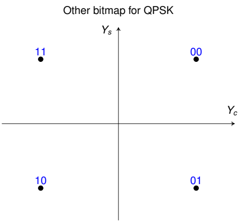
Bit Error Rate for QPSK using Other Bitmap



- Probability of making error on $b[2]$ when $b[1]b[2] = 00$ is

$$
\begin{aligned}
P_{2|00} &= \Pr\left[\hat{b}[2] = 1 \middle| b[1]b[2] = 00\right] \\
&= \Pr\left[\hat{b}[1]\hat{b}[2] = 01 \text{ or } \hat{b}[1]\hat{b}[2] = 11 \middle| b[1]b[2] = 00\right] \\
&= \Pr\left[(Y_c > 0 \cap Y_s < 0) \bigcup (Y_c < 0 \cap Y_s > 0) \middle| b[1]b[2] = 00\right] \\
&= 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]
\end{aligned}
$$

Bit Error Rate for QPSK using Other Bitmap



- Can we use symmetry? Yes. $P_2 = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]$
- Average probability of bit error is

$$P_b = \frac{P_1 + P_2}{2} = \frac{3}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right) - Q^2\left(\sqrt{\frac{2E_b}{N_0}}\right) \approx \frac{3}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- Average bit error probability is increased by about 50%

References

- Section 3.6, *Fundamentals of Digital Communication*, Upamanyu Madhow, 2008