EE 703: Digital Message Transmission (Autumn 2022)
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Assignment 5: 20 points — Date: October 13, 2022

1. [10 points] Let $\psi_1(t), \psi_2(t)$ be a complex orthonormal basis. Let $n(t) = n_c(t) + jn_s(t)$ be complex white Gaussian noise with PSD $2\sigma^2$. Then $n_c(t)$ and $n_s(t)$ are independent real WGN processes with PSD σ^2. Consider the projection of $n(t)$ onto the orthonormal basis given by

$$\mathbf{N} = \begin{bmatrix} \langle n, \psi_1 \rangle \\ \langle n, \psi_2 \rangle \end{bmatrix} = \begin{bmatrix} N_{1,c} + jN_{1,s} \\ N_{2,c} + jN_{2,s} \end{bmatrix}.$$

   (a) Show that $N_{1,c}$ and $N_{2,c}$ are independent random variables.
   (b) Show that $N_{1,c}$ and $N_{2,s}$ are independent random variables.

2. [10 points] For the 16-QAM constellation shown below calculate E_b in terms of A. Assume that the transmitted symbol is corrupted by adding $N = N_c + jN_s$ where N_c and N_s are independent zero-mean Gaussian random variables with variance $\frac{N_0}{2}$. If all the constellation points are equally likely to be transmitted, calculate the following in terms of E_b and N_0.
   (a) The exact error probability of the optimal decision rule.
   (b) The union bound on the exact error probability.
   (c) The intelligent union bound on the exact error probability.
   (d) The nearest neighbor approximation of the exact error probability.