EE 703: Digital Message Transmission (Autumn 2022)
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Assignment 6: 20 points Date: October 22, 2022

1. (a) [5 points] Let $b \geq 1$ be an integer. For $M = 2^b$, suppose M **orthogonal** real signals $s_i(t),\ i = 1, \ldots, M$ are used for transmitting b bits over a real AWGN channel with PSD $\frac{N_0}{2}$. If all the signals have the same energy E and are equally likely to be transmitted, derive the following as a function of E, N_0, b or M when the optimal receiver is used.
   i. The union bound on the symbol error probability
   ii. The nearest neighbor approximation of the symbol error probability

   (b) [5 points] Suppose we use the M signals in the previous part to form a set of $2M$ real signals

$$\{s_1(t), s_2(t), \ldots, s_M(t), -s_1(t), -s_2(t), \ldots, -s_M(t)\}.$$

   So the set contains M signals and their negative versions. These $2M$ signals are used for transmitting $b+1$ bits over a real AWGN channel with PSD $\frac{N_0}{2}$. If all the $2M$ signals are equally likely to be transmitted, derive the following as a function of E, N_0, b or M when the optimal receiver is used.
   i. The union bound on the symbol error probability
   ii. The nearest neighbor approximation of the symbol error probability

2. [10 points] Suppose N_1, N_2 are independent Gaussian random variables each having mean 0 and variance $\sigma^2 > 0$. The variance σ^2 is assumed to be known. We observe two observations Y_1, Y_2 given by

$$\begin{aligned} Y_1 &= \lambda + N_1 - N_2, \\ Y_2 &= 2\lambda + N_1 + N_2. \end{aligned}$$

   (a) Find the ML estimator of the parameter λ.
   (b) Find the mean and variance of the ML estimator.