EE 703: Digital Message Transmission (Autumn 2022)
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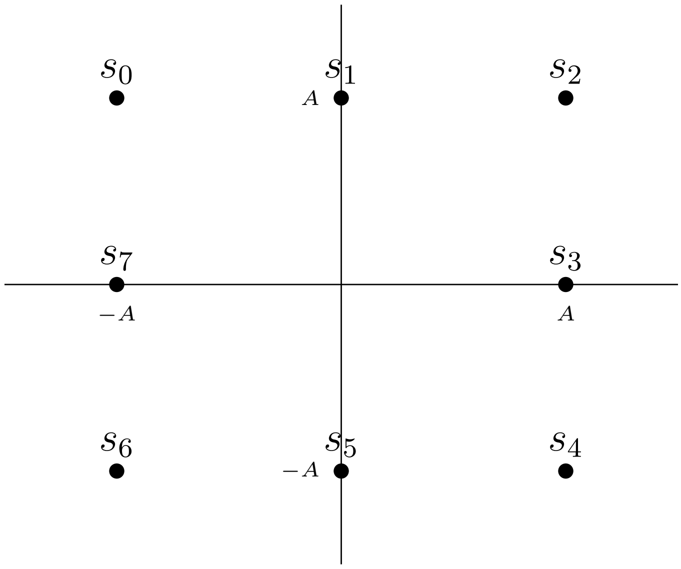
Quiz 2: 10 points — Date: November 1, 2022

1. [6 points] For the below constellation of 8 symbols, assume that the transmitted symbol is corrupted by adding $N = N_c + jN_s$ where N_c and N_s are independent Gaussian random variables with zero mean and variance σ^2. All the constellation points are equally likely to be transmitted. Calculate the following for the optimal decision rule in terms of A and σ.

   (a) The union bound on the exact error probability.

   (b) The intelligent union bound on the exact error probability. *Hint: Draw the decision regions of the optimal decision rule.*

   (c) The nearest neighbor approximation of the exact error probability.



2. [4 points] Suppose we have two biased coins C_1 and C_2. Let the probability that C_2 shows Heads when tossed be two times the probability C_1 shows Heads when it is tossed. Each coin is tossed M times. Let the observations be given by the following, where X_i is the random variable representing the ith toss of C_1 and Y_i is the random variable representing the ith toss of C_2. For both X_i and Y_i, the value 1 corresponds to Heads and the value 0 corresponds to Tails.

$$X_i \sim \text{Bernoulli}(p), \quad i = 1, 2, \dots, M,$$
$$Y_i \sim \text{Bernoulli}(2p), \quad i = 1, 2, \dots, M.$$

   The parameter p is the probability that C_1 shows Heads when tossed. Assume that $0 < p < \frac{1}{2}$. Also assume that the X_i's and Y_i's are pairwise independent, and that the X_i's are independent of the Y_i's.

   Find the ML estimator of the parameter p. Show your steps.