1. In an experiment, a coin is tossed until two consecutive heads appear.

   (a) What is the sample space of this experiment?[1]   [1 point]

   (b) If the coin is fair, what is the probability that the experiment will stop after *exactly* five tosses?   [2 points]

   (c) What is the probability that the experiment will stop before the sixth toss?
   [2 points]

2. If A, B and C are three events in a sample space S, write the expressions for the following events in terms of A, B, C using the operators $\cup$, $\cap$ and $(\cdot)^c$ (complement).

   (a) All the three events A, B, C occur.   [1 point]

   (b) At least two of the three occur.   [1 point]

   (c) At least one of the three occurs.   [1 point]

   (d) None of the three occur.   [1 point]

   (e) At most one of the three occurs.   [1 point]

   (f) At most two of the three occur.   [1 point]

   (g) Both A and C occur but B does not occur.   [1 point]

3. For two events E and F in a sample space S prove that $P(E \cup F) = P(E) + P(F) - P(E \cap F)$. You should use only the fact that for mutually exclusive events A and B ($A \cap B = \phi$), $P(A \cup B) = P(A) + P(B)$.   [2 points]

4. Using the result proved in the above question, derive an expression for $P(E \cup F \cup G)$ where E, F, G are events in a sample space.   [2 points]

5. Two fair dice are rolled. What is the probability that at least one of the die values is a five? If the two die values are different, what is the probability that one of the die values is a five?   [2 points]

6. For two events E and F, if the occurrence of F makes E more likely, then does the occurrence of E make F more likely? A more likely event has a higher probability of occurring.   [2 points]

7. For two events E and F, given $P(F) = 0.4$, what can you say about $P(E|F)$ when

   (a) E and F are mutually exclusive.   [1 point]

   (b) $F \subset E$.   [1 point]

---

[1]Points are not directly proportional to difficulty. More value is given to knowledge of basic concepts.

(c) $E \subset F$. [1 point]

8. Ram and Shyam go hunting. Both shoot at a target simultaneously. Suppose Ram hits the target with probability 0.7 while Shyam, independently, hits the target with probability 0.4.

   (a) Given that exactly one shot hit the target, what is the probability that it was Shyam's shot? [2 points]

   (b) Given that the target is hit, what is the probability that Shyam hit it?

   [2 points]

9. A course has four BTech boys, six BTech girls, and six MTech boys. How many MTech girls need to be there in the course to make gender and program (BTech or MTech) to be independent when a student is selected at random? [3 points]

10. A fair coin is tossed continously.

   (a) What is the probability that the first four tosses result in

      i. H, H, H, H? [1 point]
      ii. T, H, H, H? [1 point]

   (b) What is the probability that the pattern T, H, H, H occurs before the pattern H, H, H, H? [2 points]

11. There are two boxes. Box 1 contains one gold coin and one silver coin. Box 2 contains two gold coins and one silver coin. A box is selected at random and a coin is selected at random from the box.

   (a) What is the probability that the selected coin is gold? [2 points]

   (b) Given that the selected coin is silver, what is the probability that the first box was selected? [2 points]

12. Suppose there are ten coins with the property that the ith coin ($i = 1, 2, \ldots, 10$) will show tails with probability $\frac{i}{10}$ when tossed. One of the coins is chosen at random and tossed resulting in a heads. What is the probability that it is the fourth coin ($i = 4$)?

   [2 points]