Indian Institute of Technology Bombay

Department of Electrical Engineering

**Handout 7** EE 706 Communication Networks

Assignment 2 : **30 points** **Due date**: February 8, 2010

**Please write up your solutions in detail. Solutions missing explanations or crucial steps will not get full credit.**

1. Suppose a six-faced die is tossed and every face is equally likely to show up. The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. Let $X$ be the random variable which is equal to the number on the face which is pointing up.

   (a) What is the probability mass function (pmf) of $X$? [1 point]

   (b) What is the expected value $E(X)$ of $X$? [1 point]

   (c) The variance of a random variable $Y$ is defined as $E\left[(Y - E[Y])^2\right]$. What is the variance of $X$? [2 points]

2. A *Bernoulli* random variable $X$ has pmf

$$\begin{aligned} p(0) &= P[X = 0] = 1 - q \\ p(1) &= P[X = 1] = q \end{aligned}$$

   where $0 \leq q \leq 1$. What is the mean (expected value) and variance of $X$?

   [2 points]

3. A *binomial* random variable is defined as the number of successes in $n$ independent experiments, each of which results in a success with probability $q$ and in a failure with probability $1 - q$.

   (a) Argue that the number of sequences of $n$ outcomes with $i$ successes and $n - i$ failures is

$$\frac{n(n-1)\cdots(n-i+1)}{i(i-1)(i-2)\cdots 1}.$$

   [1 point]

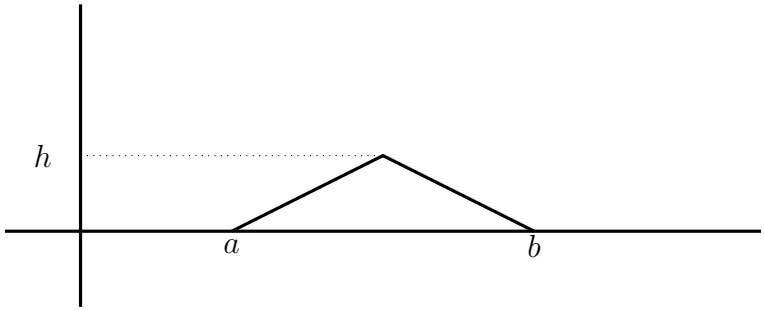
   (b) Using the above argument, derive the pmf of a binomial random variable.

   [1 point]

   (c) Derive the expected value of the binomial random variable in the following two ways.

      i. Directly from the pmf. [2 points]

      ii. Writing the binomial random variable as a sum of *some other* random variables and using the linearity property of expectation, $E[X + Y] = E[X] + E[Y]$ where $X$ and $Y$ are random variables. [2 points]




4. The probability mass function of a geometric random variable having parameter $q$ is given by

$$p(n) = P[X = n] = (1 - q)^{n-1}q, \quad n = 1, 2, \ldots$$

Derive its mean (expected value) and variance. [4 points]

5. A continous random variable has probability density function (pdf) as shown in the above figure. The dotted line is not part of the pdf. It indicates that the height of the triangle is $h$.

   (a) If the triangle is isosceles, what should the value of $h$ be in terms of $a$ and $b$ for the function plotted in the figure to be a valid pdf? [2 points]

   (b) What is the expected value of this random variable if $a = 0$ and $b = 2$? [2 points]

6. Let the generator polynomial $g(X) = X^4 + X^3 + X^2 + 1$ be used to generate CRC check bits. Without doing actual division, argue why each of the following error patterns can be detected by the CRC error detection procedure. In each case, the number of the transmitted bits is equal to the number of bits in the error pattern. [10 points]

   (a) 0100000000

   (b) 0100100

   (c) 1010000000001101

   (d) 00000000111100000000