Indian Institute of Technology Bombay

Department of Electrical Engineering

Handout 10 EE 706 Communication Networks

Solutions to Quiz 4 February 10, 2010

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1. When are two events A and B in a sample space S said to be mutually exclusive? Give an example. [2 points]

   Ans.

   Two events are A and B in a sample space are said to mutually exclusive if $A \cap B = \phi$ where ϕ denotes the null event. Example: Consider an experiment where a coin is tossed. The sample space is $S = \{H, T\}$. Let $A = \{H\}$ and $B = \{T\}$. The $A \cap B = \phi$ and the events are mutually exclusive.

   $P(A \cap B) = 0$ *does not* imply that the events are mutually exclusive. For example, let X be a uniform random variable taking values in the interval $[0, 1]$. Let $A = \{0 \leq X \leq \frac{1}{2}\}$ and $B = \{X = \frac{1}{2}\}$. In this case $A \cap B = \{X = \frac{1}{2}\} \neq \phi$ but $P(X = \frac{1}{2}) = 0$.

2. When are two events A and B in a sample space S said to be independent? Give an example. [2 points]

   Ans.

   Two events A and B are independent if $P(A \cap B) = P(A)P(B)$. Example: A fair coin is tossed twice. $S = \{HH, HT, TH, TT\}$. Let A be the event that a heads appears in the first toss and B is the event that a heads appears in the second toss. Then $A = \{HH, HT\}$, $B = \{TH, HH\}$ and $A \cap B = \{HH\}$. Now $P(A \cap B) = \frac{1}{4} = \frac{1}{2}\frac{1}{2} = P(A)P(B)$.

3. Give an example of two events A and B in a sample space S which are mutually exclusive and independent at the same time. [2 points]

   Ans.

   If A and B are mutually exclusive, $A \cap B = \phi$ which implies that $P(A \cap B) = 0$. If they are independent, then $P(A \cap B) = P(A)P(B)$. So for both situations to occur simultaneously $P(A)P(B) = 0$. This can happen only if either $P(A)$ or $P(B)$ or both are equal to zero. We know that a null event has probability zero. So choose $A = \phi$ (i.e. an impossible event) and B to be an arbitrary event in any experiment and you will have an example.

   Note that $P(A) = 0$ does not imply that $A = \phi$. Consider a uniform random variable X which takes values in the interval $[0, 1]$. Here $B = \{X = \frac{3}{4}\} \neq \phi$ but $P(B) = P(X = \frac{3}{4}) = 0$. This leads to another example of events which are simultaneously mutually exclusive and independent. Let $A = \{0 \leq X \leq \frac{1}{2}\}$ and $B = \{X = \frac{3}{4}\}$. In this case $A \cap B = \phi$ and hence $P(A \cap B) = 0$. Also $P(A)P(B) = P(X = \frac{3}{4}) = 0$.

4. A course has four BTech boys, six BTech girls, and six MTech boys. How many MTech girls need to be there in the course to make gender and program (BTech or MTech) to be independent when a student is selected at random? [4 points]

Ans.

Let the number of MTech girls be n. For gender and program to be independent when a student is selected at random, we should have

$$\begin{aligned}
P(\text{Student is a girl} \cap \text{Student is in MTech program}) &= P(\text{Student is a girl}) \\
&\quad \times P(\text{Student is in MTech program}) \\
\Longrightarrow \frac{n}{n+16} &= \frac{n+6}{n+16}\frac{n+6}{n+16} \\
\Longrightarrow n &= 9
\end{aligned}$$

You can verify that this value of n is consistent in all other cases of the student's gender and program.