EE 706: Communication Networks

Instructor: Saravanan Vijayakumaran
Indian Institute of Technology Bombay
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Assignment 1 : **34 points** **Due date**: January 21, 2011

The first 11 questions are worth 2 points each. The remaining 4 questions are worth 3 points each.

1. In an experiment, a coin is tossed until two consecutive heads appear.
   (a) What is the sample space of this experiment?
   (b) If the coin is fair, what is the probability that the experiment will stop after *exactly* five tosses?
   (c) What is the probability that the experiment will stop before the sixth toss?
2. If A, B and C are three events in a sample space S, write the expressions for the following events in terms of A, B, C using the operators $\cup$, $\cap$ and $(\cdot)^c$ (complement).
   (a) All the three events A, B, C occur.
   (b) At least two of the three occur.
   (c) At least one of the three occurs.
   (d) None of the three occur.
   (e) At most one of the three occurs.
   (f) At most two of the three occur.
   (g) Both A and C occur but B does not occur.
3. For two events E and F in a sample space S prove that $P(E \cup F) = P(E) + P(F) - P(E \cap F)$. You should use only the fact that for mutually exclusive events A and B $(A \cap B = \phi)$, $P(A \cup B) = P(A) + P(B)$.
4. Using the result proved in the above question, derive an expression for $P(E \cup F \cup G)$ where E, F, G are events in a sample space.
5. Two fair dice are rolled. What is the probability that at least one of the die values is a five? If the two die values are different, what is the probability that one of the die values is a five?
6. For two events E and F, if the occurrence of F makes E more likely, then does the occurrence of E make F more likely? A more likely event has a higher probability of occurring.
7. For two events E and F, given $P(F) = 0.4$, what can you say about $P(E|F)$ when
   (a) E and F are mutually exclusive.

(b) $F \subset E$.

(c) $E \subset F$.

8. Ram and Shyam go hunting. Both shoot at a target simultaneously. Suppose Ram hits the target with probability 0.7 while Shyam, independently, hits the target with probability 0.4.

   (a) Given that exactly one shot hit the target, what is the probability that it was Shyam's shot?

   (b) Given that the target is hit, what is the probability that Shyam hit it?

9. A course has four BTech boys, six BTech girls, and six MTech boys. How many MTech girls need to be there in the course to make gender and program (BTech or MTech) to be independent when a student is selected at random?

10. There are two boxes. Box 1 contains one gold coin and one silver coin. Box 2 contains two gold coins and one silver coin. A box is selected at random and a coin is selected at random from the box.

    (a) What is the probability that the selected coin is gold?

    (b) Given that the selected coin is silver, what is the probability that the first box was selected?

11. Suppose there are ten coins with the property that the ith coin ($i = 1, 2, \ldots, 10$) will show tails with probability $\frac{i}{10}$ when tossed. One of the coins is chosen at random and tossed resulting in a heads. What is the probability that it is the fourth coin ($i = 4$)?

12. A source node S wants to send 100 bits of information to a destination node D.

    (a) S uses a forward error correction (FEC) scheme which adds 400 bits of redundancy to the information bit string. What is the rate of the FEC scheme?

    (b) If an FEC scheme of rate $\frac{1}{3}$ is used by S, what is the amount of redundancy added, i.e. the number of check bits added?

    (c) If the channel between S and D has data rate equal to 20 bits per second, what is the time duration of transmission in the above two cases?

13. The 3-repetition code maps 0 to 000 and 1 to 111. It can correct one bit error. The 5-repetition code which maps 0 to 00000 and 1 to 11111 can correct 2 bit errors.

    (a) How many bit errors can a 6-repetition code correct? Explain your answer.

    (b) How many bit errors can a n-repetition code correct when n is odd? Explain your answer.

    (c) How many bit errors can a n-repetition code correct when n is even? Explain your answer.

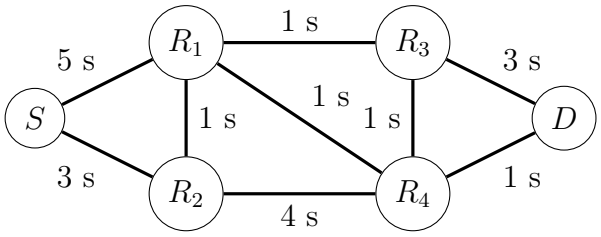
    n is a positive integer in the above two cases.

14. A single parity check is an error detection code which appends a single parity bit to an information bit string. The parity bit is set to 1 if the number of ones in the information bit string is odd and is set to 0 otherwise. Let the information bit string be 10101. If a single parity check bit is added to it and the resulting bit string is sent over a noisy channel, list all possible received bit strings which are declared error free at the destination. Does the set of bit strings which are declared error free at the destination depend on the transmitted bit string?

15. Consider the six-node communication network shown in the below figure.

    (a) List all routes from node S to node D.

    (b) The number alongside a link indicates the packet delay incurred on that link in seconds. Taking the routing cost of a route to be the sum of the delays of the links which constitute the route, write down the minimum-delay routing tables for the nodes S, R_1, R_2 and R_3 in the format shown in the table below.



Routing table for S		
Reachable Node	Next Hop	Routing Cost