Symbol Detection in CDMA-OFDM Coexistence

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Problem Statement

- One user transmits the QPSK symbol b using CDMA (spreading code). Another user transmits a complex vector $\mathbf{c}$ using OFDM. Both use the same channel (carrier frequency).
- The transmitted OFDM signal

$$c(t) = \sum_{m=0}^{M-1} c_m e^{j2\pi mt/T} \qquad 0 \leq t \leq T \tag{1}$$

  where c_m are the complex data symbols being transmitted.
- The signal transmitted by the CDMA user

$$s(t) = \frac{b}{\sqrt{N}} \sum_{n=0}^{N-1} s_n p(t - nT_c) \qquad 0 \leq t \leq T \tag{2}$$

  where b is the transmitted symbol, s_n the spreading code, T_c the chip duration and $p(t)$ the pulse shaping function.
- OFDM demodulator output, assuming that the noise is AWGN

$$\mathbf{y} = b\mathbf{r} + \mathbf{c} + \sigma\mathbf{n}$$

  where $\mathbf{r}$ is the DFT of s_n and $\mathbf{c} = \{c\}_{m=1}^{M-1}$. The noise $\mathbf{n}$, is complex circularly Gaussian with covariance matrix 2I.
- We need to detect b and $\mathbf{c}$.

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- We would like to exploit the Gaussian nature of the additive noise to do this.

The Adaptive Algorithm: MCPOE

- The mean and variance of the real part of D, conditioned on b are

$$\begin{aligned} \mu_D &= Re\left\{\mathbf{w}^H(b\mathbf{r}+\mathbf{c})\right\} \\ \sigma_D^2 &= \sigma^2\|\mathbf{w}\|^2 \end{aligned} \tag{4}$$

- If b^+ and b^- are obtained from b by making its real part $+1$ and -1 respectively, the conditional probability of error is

$$P_{e|b} = \frac{1}{2}Q\left(\frac{\mu_{D+}}{\sigma_D}\right) + \frac{1}{2}Q\left(\frac{\mu_{D-}}{\sigma_D}\right) \tag{5}$$

where μ_{D+} and μ_{D-} are obtained from μ_D by substituting b by b^+ and b^- respectively.

- Using the gradient descent approach,

$$\mathbf{w}^{(i+1)} = \mathbf{w}^{(i)} - \lambda\nabla P_{e|b}^{(i)}, \tag{6}$$

where

$$\begin{aligned} \nabla P_{e|b} &= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu_{D-}^2}{\sigma_D^2}\right)\frac{\|\mathbf{w}\|^2\left(b^-\mathbf{r}+\mathbf{c}\right) - \mu_{D-}\mathbf{w}}{2\sigma\|\mathbf{w}\|^3} \\ &- \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\mu_{D+}^2}{\sigma_D^2}\right)\frac{\|\mathbf{w}\|^2\left(b^+\mathbf{r}+\mathbf{c}\right) - \mu_{D+}\mathbf{w}}{2\sigma\|\mathbf{w}\|^3} \end{aligned} \tag{7}$$

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